3-D Gains from Trade*

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Abstract

I quantify gains from trade in a multi-country dynamic stochastic environment, including contributions from trade across states of the world and over time, as well as from trade within dates and states (3-D gains). For developing countries, which have volatile productivity, trade across states is an important source of gains. These are particularly large under complete markets, but even with balanced trade, endogenous risk sharing through the terms of trade contributes nontrivially to 3-D gains. Because developed countries' productivity is less volatile, their 3-D gains are only modestly bigger than static gains, even under complete markets.

1 Introduction

The ability to trade internationally allows countries to consume a different bundle of goods and services from the bundle of goods and services they produce. This is true both at a point in time, and across dates and states of the world. Overall gains from trade, i.e. the welfare gains from moving from full autarky to the current level of trade openness, therefore depend not only on the contribution to welfare of trade within dates and states, but also on the contributions to welfare of trade across states of the world and over time. Static quantitative trade models imply that given the current level of trade openness, gains from trade are modest. I quantify the gains from trade taking into account all three dimensions of trade (3-D gains). For developing countries, which face volatile productivity, 3-D gains from trade are at least 2/3 bigger than static gains. Even under balanced trade, endogenous

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risk sharing through the terms of trade is an important source of gains for these countries. Developed country productivity is less volatile and more correlated across countries. Their 3-D gains are only modestly bigger than static gains, even when international asset markets are complete.

I first generalize the formula for welfare gains from trade popularized by Arkolakis, Costinot and Rodriguez-Clare (2012). Under CRRA utility over consumption at different dates and states of the world, 3-D gains from trade can be expressed as a weighted power mean of static gains from trade at all dates and states, where the weights are functions of autarky consumption. Intuitively, agents value static gains from trade more when the marginal utility of autarky consumption is high than when it is low, and 3-D gains take this into account. I use a simple example to illustrate that static gains from trade and therefore the joint distribution of static gains and autarky consumption, depend on international asset markets.

In a dynamic stochastic environment it is not possible to quantify ex-ante gains from trade using summary statistics. Instead it is necessary to calibrate a full model. I therefore lay out a model with multiple countries, inelastic labor supply, Armington specialization, iceberg costs of trade within dates and states of the world, and roundabout production. An exogenous stochastic process for productivity is the source of uncertainty. There are no within-country distortions, so the autarky equilibrium is Pareto efficient.

I then construct a 50-year time-series for productivity for a large number of countries in a model-consistent way. To do so, I "invert" the model to infer cross-sectional relative output prices and trade costs from data on bilateral import shares and expenditure PPPs. This procedure is independent of preferences for consumption across dates and states of the world, as well as the structure of international asset markets. I benchmark time-series variation in output prices using the US output price deflator. The resulting output price and trade cost series allow me to calculate labor productivity consistent with the model assumption of roundabout production.

I use these productivity series to estimate a process for productivity that allows for (a) long-run growth that is the same for all countries, and equal to the US long-run growth rate (b) a rate of convergence to the productivity frontier that differs between developing and developed countries, (c) deviations of growth from trend that may be persistent, correlated across countries, and where the relevant parameters differ between developing and developed countries, and (d) a standard deviation of innovations to deviations of growth from trend that differs between developing and developed countries. The estimates imply convergence that is slower for developing than developed countries. Deviations of growth from trend are less correlated across developing and developed countries than they are within developed countries. Crucially, the standard deviation of innovations to deviations of growth from trend is three times higher for developing than for developed countries.

Using this process for productivity I calculate ex-ante gains from trade from the perspective of 2019 for the 156 countries for which I have the required data, holding trade costs and population fixed at their 2019 levels. I assume an Armington elasticity of 4 and a coefficient of relative risk aversion of 4. I do this under two polar assumptions about international asset markets in the trade equilibrium: (a) balanced trade date-by-date and state-by-state, and (b) complete markets with zero net foreign asset positions in 2019.

For developing countries, the 3-D gains from trade are notably greater than static gains. In 2019, the static gains from trade under balanced trade imply that for the average developing country, being able to trade is equivalent to consumption that is 17% higher than autarky consumption. But the 3-D gains under balanced trade imply that for the average developing country, being able to trade is equivalent to consumption that is permanently 30% higher than autarky consumption. For developed countries, the corresponding numbers are 21% and 22%.

Meanwhile, for developing countries, the 3-D gains from trade under complete markets are dramatically bigger than static gains. For the average developing country, being able to trade under complete markets is equivalent to consumption that is permanently 84% higher than autarky consumption. For the average developed country, the corresponding number is 27% higher than autarky consumption.

Why are 3-D gains bigger than static gains for developing countries, even under balanced trade? The key is that developing countries face a lot of idiosyncratic productivity risk, and agents are risk averse. Because the goods produced by different countries are imperfect substitutes, endogenous movements in the terms of trade provide partial insurance even under balanced trade, as in Cole and Obstfeld (1991). Of course under complete markets, there is optimal insurance (conditional on trade costs) and gains from trade are dramatically bigger. In contrast, developed countries face much less productivity risk, and some of this risk is uninsurable because it is correlated with aggregate risk. As a result, for these countries, 3-D gains from trade are accounted for mainly by static gains.

Related literature

This paper is closely related to Arkolakis et al. (2012), who highlight that in a large class of quantitative multi-country trade models, under balanced trade gains from trade are a parsimonious function of the share of expenditure that is sourced domestically and the trade elasticity. I show that redefining the commodity space to include goods indexed by state and time adds additional potential sources of gains from trade, and I quantify these gains. The role I identify for complementarity in generating gains from trade across states of the world is related to Ossa (2015), who notes the importance of sectors with very low trade elasticities in driving welfare gains in a static environment.

While much of the quantitative literature on the gains from trade focuses on static models, the literature on trade and endogenous growth examines dynamic environments. Sampson (2016), Buera and Oberfield (2020) and Perla, Tonetti and Waugh (2021) find additional sources of gains from trade in second best economies where productivity growth is endogenous to the extent of trade. In contrast to this literature, I assume productivity growth is exogenous to trade, and stochastic, and focus on an environment where the autarky equilibrium is Pareto efficient. Alvarez (2017) and Ravikumar, Santacreu and Sposi (2019) quantify the gains from reducing trade costs in deterministic neoclassical environments with trade costs and capital accumulation. They focus on the difference between steady state-to-steady state comparisons, and comparisons which take into account transition dynamics. Alessandria and Choi (2014), Alessandria, Choi and Ruhl (2021) and Boehm, Levchenko, Pandalai-Nayar and Toma (2025) examine how differences between short- and long-run trade elasticities affect gains from trade. But they do not address the possibility of gains from trade across time between countries with different long-run growth trajectories, nor do they incorporate gains from trade across states of the world into their frameworks.

An older theoretical literature e.g. Helpman and Razin (1978) and Newbery and Stiglitz (1984), addresses potential gains from trade in environments with uncertainty, and in the case of Newbery and Stiglitz, departures from Pareto optimality within countries. More recently, Caselli, Koren, Lisicky and Tenreyro (2019) and Allen and Atkin (2022) quantify the relationship between trade and output volatility in environments with endogenous specialization.[1] Fan and Luo (2025) and Kleinman, Liu and Redding (2025) examine how uncertainty affects the pattern of trade when linkages must be decided before uncertainty is realized. However none of these authors quantify the contribution to gains from trade of trade across states of the world.

This paper is closely related to a literature in international macro that calculates the wel-

[1] I assume that the volatility of productivity and the pattern of specialization are exogenous to trade openness in order to focus on gains from trade between countries with comparative advantage in different states of the world and at different time horizons.

fare gains from moving from frictional asset markets to optimal international consumption risk sharing in environments with uncertainty, i.e. an *asset market* counterfactual instead of the *trade cost* counterfactual I perform here. See Van Wincoop (1999) and Heathcote and Perri (2014) for summaries, and Coeurdacier, Rey and Winant (2020) for a recent example. This literature typically assumes costless trade. An exception is Fitzgerald (2012), where I calculate ex-post gains from moving from historical asset markets to optimal international consumption risk sharing in an environment with international specialization and trade costs. The results I present here on ex-ante gains from trade are consistent with my previous finding that ex-post gains from moving from historical asset markets to optimal risk sharing are bigger for developing countries, which face a lot of idiosyncratic risk, than for developed countries, which face mainly aggregate risk. Also related is Gourinchas and Jeanne (2006), who focus on the welfare gains from moving from incomplete asset markets to optimal intertemporal trade in the absence of idiosyncratic risk. They find small gains from optimal intertemporal trade between countries at different distances from steady state, consistent with the results presented here.

Outline

Section 2 develops results on the relationship between 3-D gains and static gains from trade. Section 3 lays out the full quantitative model. Section 4 describes the data. Section 5 explains how the model is inverted to recover realized productivity and trade costs from the data. Section 6 describes the estimation of the productivity process. Section 7 quantifies 3-D gains from trade under balanced trade and complete markets. The final section concludes.

2 Gains from trade in three dimensions

Uncertainty

In each period t, the world economy experiences one event, $s_t \in S$. Denote by s^t the history of events from date 0 to date t, $\{s_0, \ldots, s_t\}$. The probability at date 0 that history s^t will be realized at date t is given by $\pi_{0,t}(s^t)$, with $\sum_{s^t} \pi_{0,t}(s^t) = 1 \; \forall t$.

Preferences

Assume that the L^i agents in country i have CRRA expected utility over consumption at all future dates and states. It is convenient to write the problem as if country i had a single agent with date-0 (ex-ante) utility given by:[2]

$$U_0^i = L^i \left(\sum_{t=0}^{\infty} \sum_{s^t} (1-\beta)\, \beta^t \pi_{0,t}\left(s^t\right) \left(\frac{C_t^i\left(s^t\right)}{L^i} \right)^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}} \tag{1}$$

where $C_t^i\left(s^t\right)$ is aggregate consumption in country i at date t given that history s^t is realized, L^i is population (and labor supply), assumed fixed,[3] so $C_t^i\left(s^t\right)/L^i$ is per capita consumption. The weights $(1-\beta)\,\beta^t\pi_{0,t}\left(s^t\right)$ sum to 1. Note that ρ is the elasticity of substitution between consumption at different dates and states, while the coefficient of relative risk aversion is given by $1/\rho$. When $\rho < 1$, consumption at different dates and states are complements. Log utility is the case where $\rho = 1$. When $\rho > 1$, consumption at different dates and states are substitutes. As $\rho \to \infty$, utility becomes linear in consumption at different dates and states.

3-D gains from trade

Define the 3-D gains from trade as the amount by which autarky consumption must be increased *at all dates and states* in order for agents to be indifferent between being in the trade equilibrium (the details of which are yet to be specified) and autarky. Given the CRRA form for the utility function, 3-D gains from trade are given by:

$$\kappa_3^i = \frac{U_0^{i,trade}}{U_0^{i,aut}} = \left(\frac{\sum_{t=0}^{\infty} \sum_{s^t} (1-\beta)\, \beta^t \pi_{0,t}\left(s^t\right) \left(C_t^i\left(s^t\right)^{trade} \right)^{\frac{\rho-1}{\rho}}}{\sum_{t=0}^{\infty} \sum_{s^t} (1-\beta)\, \beta^t \pi_{0,t}\left(s^t\right) \left(C_t^i\left(s^t\right)^{aut} \right)^{\frac{\rho-1}{\rho}}} \right)^{\frac{\rho}{\rho-1}} \tag{2}$$

where $C_t^i\left(s^t\right)^{trade}$ is consumption in country i at t, s^t in the trade equilibrium, and $C_t^i\left(s^t\right)^{aut}$ is consumption in country i at t, s^t in autarky.

Denote the static gain from trade at t, s^t (i.e. the amount by which consumption must be increased at t, s^t in order to make agents in country i indifferent between autarky at t, s^t and the trade equilibrium at t, s^t) by $\lambda_t^i\left(s^t\right)$:

$$\lambda_t^i\left(s^t\right) = \frac{C_t^i\left(s^t\right)^{trade}}{C_t^i\left(s^t\right)^{aut}}$$

3-D gains from trade can then be expressed as a weighted power mean of date-by-date and

[2] I abstract from within-country heterogeneity.

[3] This avoids valuing the utility of future agents who do not currently exist.

state-by-state static gains from trade:

$$\kappa_3^i = \left(\sum_{t=0}^{\infty} \sum_{s^t} w_{0,t}^i \left(s^t\right) \lambda_t^i \left(s^t\right)^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}} \tag{3}$$

where the weights are functions of autarky consumption at all dates and states:

$$\begin{aligned} w_{0,t'}^i \left(s^{t'}\right) &= \frac{(1-\beta)\, \beta^{t'} \pi_{0,t'} \left(s^{t'}\right) \left(C_{t'}^i \left(s^{t'}\right)^{aut}\right)^{\frac{\rho-1}{\rho}}}{\sum_{t=0}^{\infty} \sum_{s^t} (1-\beta)\, \beta^t \pi_{0,t} \left(s^t\right) \left(C_t^i \left(s^t\right)^{aut}\right)^{\frac{\rho-1}{\rho}}} \\ &= (1-\beta)\, \beta^{t'} \pi_{0,t'} \left(s^{t'}\right) \left(\frac{C_{t'}^i \left(s^{t'}\right)^{aut}}{U_0^{i,aut}} \right)^{\frac{\rho-1}{\rho}} \end{aligned}$$

Notice that for a given value of ρ, $w_{0,t}^i \left(s^t\right) \in [0,1]$ and

$$\sum_{t=0}^{\infty} \sum_{s^t} w_{0,t}^i \left(s^t\right) = 1$$

This implies that 3-D gains are bounded by the minimum and maximum static gains across all dates and states, i.e.

$$\min_{t,s^t} \left\{ \lambda_t^i \left(s^t\right) \right\} \leq \kappa_3^i \leq \max_{t,s^t} \left\{ \lambda_t^i \left(s^t\right) \right\}$$

It also implies that if country i weakly gains from trade at all dates and states, i.e. $\lambda_t^i \left(s^t\right) \geq 1$ $\forall t, s^t$, then 3-D gains are also weakly positive, i.e. $\kappa_3^i \geq 1$.

To build some intuition, rewrite further:

$$\kappa_3^i = \left(E\left(\lambda_t^i \left(s^t\right)^{\frac{\rho-1}{\rho}} \right) + COV\left(\left(\frac{C_t^i \left(s^t\right)^{aut}}{U^{i,aut}} \right)^{\frac{\rho-1}{\rho}}, \lambda_t^i \left(s^t\right)^{\frac{\rho-1}{\rho}} \right) \right)^{\frac{\rho}{\rho-1}} \tag{4}$$

where E denotes the expectation given weights $(1-\beta)\, \beta^t \pi_{0,t} \left(s^t\right)$, and COV denotes the covariance under the same weights. In general, when agents are risk-averse, 3-D gains from trade depend not just on the average level of static gains, but on whether static gains are high in dates and states where autarky consumption is high (relative to ex-ante welfare in autarky), or in dates and states where autarky consumption is low.[4] Only in the special case

[4]This is true under any homothetic utility over consumption at all dates and states, including Epstein-Zin nonexpected utility.

where $\rho \to 1$ (i.e. log utility) are 3-D gains independent of the joint distribution of static gains and autarky consumption. 3-D gains in this case are a geometric weighted average of static gains, with weights given by $(1-\beta)\beta^t \pi_{0,t}(s^t)$:

$$\kappa_3^i|_{\rho\to 1} = \prod_{t=0}^{\infty} \prod_{s^t} \lambda_t^i \left(s^t\right)^{(1-\beta)\beta^t \pi_{0,t}\left(s^t\right)}$$

An example illustrating the role of asset markets

Asset markets play an important role in determining date-by-date and state-by-state static gains from trade. To see this, consider an endowment model with Armington specialization, and zero resource costs of within-date-and-state trade. Suppose there are N equally sized countries in the world, indexed $i = 1, \dots, N$. Assume country i is endowed with $Y_t^i(s^t)$ units of good i at t, s^t. Agents in all countries have identical CES preferences over consumption of the N different goods, so utility in country i at t, s^t is given by:

$$C_t^i\left(s^t\right) = \left(\sum_{k=1}^{N} C_t^{ki}\left(s^t\right)^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}}$$

where $C_t^{ik}(s^t)$ is consumption of good k by country i, and $\eta \in (1,\infty)$, i.e. goods are substitutes, but not perfectly substitutable. In the autarky equilibrium, $C_t^i(s^t)^{aut} = Y_t^i(s^t)$. Because there are no trade costs, in the trade equilibrium independent of the asset market, the resource constraint for good i is given by:

$$Y_t^i\left(s^t\right) = \sum_{k=1}^{N} C_t^{ik}\left(s^t\right)$$

Consider two polar cases for asset trade: (i) balanced trade, i.e. trade *within* dates and state but no asset trade, sometimes referred to as "financial autarky", and (ii) complete markets, where countries can trade ex-ante claims to each of the N goods at all t, s^t, and all countries face the same vector of relative prices for these claims. In Appendix A, I show that under balanced trade, static gains from trade at t, s^t are given by:

$$\lambda_t^i\left(s^t\right)^{bal} = \left(\frac{Y_t^i\left(s^t\right)^{\frac{\eta-1}{\eta}}}{\sum_{k=1}^{N} Y_t^k\left(s^t\right)^{\frac{\eta-1}{\eta}}}\right)^{\frac{1}{1-\eta}}$$

Notice that since $\eta \in (1,\infty)$ this implies $\lambda_t^i(s^t) > 1\ \forall t, s^t$. As a result, there are ex-ante gains from trade, i.e. $\kappa_3^i > 1$, despite the fact that markets are incomplete. In Appendix A, I also show that under complete markets, static gains from trade at t, s^t are given by:

$$\lambda_t^i\left(s^t\right)^{cm} = \gamma^i\left(\frac{Y_t^i\left(s^t\right)^{\frac{\eta-1}{\eta}}}{\sum_{k=1}^N Y_t^k\left(s^t\right)^{\frac{\eta-1}{\eta}}}\right)^{\frac{\eta}{1-\eta}}$$

where $\gamma^i \in (0,1)$, $\sum_i \gamma^i = 1$, and γ^i is a function of endowments of all countries at all dats and states (see Appendix A). Since $\gamma^i \in (0,1)$, it is possible to have $\lambda_t^i\left(s^t\right) < 1$. However the first welfare theorem guarantees that there are 3-D gains from trade for each country, i.e. $\kappa_3^i \geq 1$. But since $\lambda_t^i\left(s^t\right)^{cm} = \gamma^i\left(\lambda_t^i\left(s^t\right)^{bal}\right)^\eta$, static gains from trade (and therefore also 3-D gains) differ under the two different configurations of international asset markets.

Summary

In a dynamic stochastic environment, there is no straightforward summary statistics approach to calculating the 3-D gains from trade. Firstly, static gains are an exact function of an easily measured expenditure share and the trade elasticity only under balanced trade. Otherwise, it is necessary to calculate counterfactual autarky consumption to obtain exact static gains. Secondly, as equation (4) shows, even if trade is balanced at each date and state, outside of the log utility case, 3-D gains depend on the joint distribution of static gains and normalized autarky consumption. Even in the log utility case, static gains are not observed for all future dates and states. Calculating these objects requires making assumptions about the details of the underlying environment.

3 Quantitative model

Summary

There are N countries in the world, indexed $i = 1, \ldots, N$. Labor in each country is inelastically supplied. Each country produces a distinct tradeable intermediate good, also indexed i, using labor and materials. Productivity is stochastic. There are trade costs for intermediates which take the iceberg form. Intermediates are combined using a CES aggregator, the same in all countries, to produce a non-tradeable final good which is used for consumption, and as the material input into the production of the intermediate. Agents in all countries have identical CRRA expected utility preferences over consumption of the final good. I consider the case of complete and frictionless international asset markets and then the case of balanced trade date-by-date and state-by-state.

Uncertainty

The notation for uncertainty is as in the previous section.

Preferences

Agents have identical CRRA expected utility as described in as described in equation (1).

Technology and trade costs

Country i's output of intermediate good i at date t after history s^t is $Y_t^i(s^t)$, with:

$$Y_t^i\left(s^t\right) = A_t^i\left(s^t\right)\left(L^i\right)^{1-\mu} M_t^i\left(s^t\right)^{\mu} \tag{5}$$

where $A_t^i(s^t)$ is productivity, and $M_t^i(s^t)$ is material input. The production function for the final good, X is:

$$X_t^i\left(s^t\right) = \left(\sum_{k=1}^{N} Z_t^{ki}\left(s^t\right)^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}} \tag{6}$$

where $Z_t^{ki}(s^t)$ is absorption of the k-country intermediate in country i at s^t. The Armington elasticity of substitution between different intermediates ($\eta \neq 1$) determines the within-date-and-state trade elasticity. The resource constraint for the non-traded final good in country i is:

$$X_t^i\left(s^t\right) = C_t^i\left(s^t\right) + M_t^i\left(s^t\right) \tag{7}$$

There are (deterministic) resource costs of trading intermediate goods. In order for one unit of k's intermediate good to arrive in i at t, $\tau_t^{ki} \geq 1$ units must be shipped, with $\tau_t^{kj}\tau_t^{ji} \geq \tau_t^{ki}$. Trade costs need not be symmetric, i.e. it may be the case that $\tau_t^{ik} \neq \tau_t^{ki}$. Trade costs are normalized to zero within a country, i.e. $\tau_t^{ii} = 1\ \forall i$. The resource constraint for each intermediate good must take account of the resource costs of trade:

$$Y_t^i\left(s^t\right) = \sum_{k=1}^{N} \tau_t^{ik} Z_t^{ik}\left(s^t\right) \tag{8}$$

Markets for labor, intermediate and final goods

The market for intermediate goods is perfectly competitive. Let $Q_{t,t}^k(s^t)$ be the price of 1 unit of the country-k intermediate good in country k at date t after history s^t in terms of date-t "dollars" (freely traded). The price of the country-k intermediate in country k is equal to the marginal cost of production:

$$Q^k_{t,t}\left(s^t\right) = \frac{1}{A^k_t\left(s^t\right)}\left(\frac{W^k_{t,t}\left(s^t\right)}{1-\mu}\right)^{1-\mu}\left(\frac{P^k_{t,t}\left(s^t\right)}{\mu}\right)^{\mu}$$

where $W^k_{t,t}\left(s^t\right)$ is the wage in country k at t after s^t expressed in date-t dollars, and $P^k_{t,t}\left(s^t\right)$ is the price of the final good in country k at t after s^t expressed in date-t dollars. Since τ^{ki}_t units of intermediate k must be purchased in country k in order for 1 unit to be delivered in country i, the price of 1 unit of the country-k intermediate in country i at date t after history s^t is $\tau^{ki}_t Q^k_{t,t}\left(s^t\right)$. Within date t and state s^t, all agents face the same relative prices for trading claims to intermediate goods appropriately indexed by country of production and country of delivery.

Markets for final goods are also perfectly competitive, so price is equal to marginal cost. The price of 1 unit of the final good in country i after history s^t expressed in date-t dollars is therefore:

$$P^i_{t,t}\left(s^t\right) = \left(\sum_{k=1}^{N}\left(\tau^{ki}_t Q^k_{t,t}\left(s^t\right)\right)^{1-\eta}\right)^{\frac{1}{1-\eta}} \tag{9}$$

Since the final good is non-traded, there is no additional condition relating final goods prices across countries.

The labor market is perfectly competitive and labor is paid its marginal product:

$$W^k_{t,t}\left(s^t\right) = (1-\mu)\,\mu^{\frac{\mu}{1-\mu}} A^k_t\left(s^t\right)^{\frac{1}{1-\mu}} Q^k_{t,t}\left(s^t\right)^{\frac{1}{1-\mu}} P^i_{t,t}\left(s^t\right)^{\frac{-\mu}{1-\mu}}$$

Asset markets: Complete markets

At date 0 after observing s_0, agents trade the full menu of Arrow-Debreu securities, which are indexed by country of production and country of delivery. All agents face the same prices for these securities. Let $Q^k_{0,t}\left(s^t\right)$ be the price of 1 unit of intermediate good k in country k at date t at s^t in terms of date-0 dollars (date-0 price). Since τ^{ki}_t units must be purchased in country k in order for 1 unit to be delivered in country i, the date-0 price of 1 unit of good k in country i at date t at s^t is $\tau^{ki}_t Q^k_{0,t}\left(s^t\right)$.

Meanwhile, B^i_0 is net wealth of country i entering period 0 expressed in date-0 dollars. Date-0 net wealth of the world as a whole is equal to zero:

$$\sum_{k=1}^{N} B^k_0 = 0$$

This implies that the date-0 budget constraint for country i can be written:

$$\sum_{t=0}^{\infty}\sum_{s^t}\sum_{k=1}^{N}\tau_t^{ki}Q_{0,t}^{k}\left(s^t\right)Z_t^{ki}\left(s^t\right)\leq B_0^i+\sum_{t=0}^{\infty}\sum_{s^t}Q_{0,t}^{i}\left(s^t\right)Y_t^i\left(s^t\right) \tag{10}$$

Competitive equilibrium

A competitive equilibrium is given by sequences of ex-ante prices $\left\{\left\{\left\{Q_{0,t}^{i}\left(s^t\right)\right\}_{i=1}^{N}\right\}_{s^t}\right\}_{t=0}^{\infty}$, intermediate goods absorption $\left\{\left\{\left\{\left\{Z_t^{ki}\left(s^t\right)\right\}_{k=1}^{N}\right\}_{i=1}^{N}\right\}_{s^t}\right\}_{t=1}^{\infty}$, materials $\left\{\left\{\left\{M_t^{i}\left(s^t\right)\right\}_{i=1}^{N}\right\}_{s^t}\right\}_{t=0}^{\infty}$ and consumption $\left\{\left\{\left\{C_t^{i}\left(s^t\right)\right\}_{i=1}^{N}\right\}_{s^t}\right\}_{t=0}^{\infty}$ such that:

1. Given ex-ante prices, country i chooses absorption of each of the N intermediate goods, materials, and consumption, for each date and state, to maximize welfare, subject to the country-i final good resource constraints, and the country-i ex-ante budget constraint.

2. All intermediate and final goods resource constraints are satisfied.

Alvarez and Lucas (2007) and Allen, Arkolakis and Takahashi (2020) provide conditions under which equilibrium exists and is unique in static one-sector gravity models with balanced trade. I conjecture that given the nested CES structure of the environment here, extensions of these results to multi-sector models with a continuum of sectors may be used to prove existence and uniqueness in this case, with each realization $\{t, s^t\}$ interpreted as corresponding to a "sector."

Special case of asset market frictions: Balanced trade

Balanced trade date-by-date and state-by-state is a special case of asset market frictions where it is straightforward to construct the equilibrium consumption allocation. As long as goods are freely traded within dates and states subject to iceberg trade costs, then within dates and states all countries face the same relative prices for intermediate goods indexed appropriately by country of production and country of delivery. Date-by-date and state-by-state the value of output must equal the value of expenditure at these common prices:

$$Q_{t,t}^{i}\left(s^t\right)Y_t^i\left(s^t\right)=\sum_{k=1}^{N}Q_{t,t}^{ki}\left(s^t\right)Z_t^{ki}\left(s^t\right)$$

Shadow date-0 prices for delivery at $t > 0$ may differ across countries. These shadow prices may be obtained by evaluating marginal utilities at the equilibrium consumption allocation.

Some useful expenditure shares

The following expressions for expenditure shares are useful in taking the model to the data, and are independent of the asset market. The value of country i's imports from country k at t, s^t as a share of i's total t, s^t expenditure, all expressed in terms of date-t dollars is:

$$\omega_{t,t}^{ki}\left(s^t\right) = \frac{\tau_t^{ki} Q_{t,t}^k\left(s^t\right) Z_t^{ki}\left(s^t\right)}{P_{t,t}^i\left(s^t\right) X_t^i\left(s^t\right)} = \left(\frac{\tau_t^{ki} Q_{t,t}^k\left(s^t\right)}{P_{t,t}^i\left(s^t\right)}\right)^{1-\eta} \tag{11}$$

Expenditure on materials in i at t, s^t is a constant share of the value of i's t, s^t output:

$$\frac{P_{t,t}^i\left(s^t\right) M_t^i\left(s^t\right)}{Q_{t,t}^i\left(s^t\right) Y_t^i\left(s^t\right)} = \mu \tag{12}$$

Approximate change in welfare

In Appendix B, I derive a second order approximation to the change in welfare induced by changes in trade costs or foreign productivity, expressed as a function of expenditure shares. Let Π_0^i be the date-0 price of utility in country i in terms of date-0 dollars:

$$\Pi_0^i = \left[\sum_{t=0}^{\infty}\sum_{s^t}\left(\left((1-\beta)\beta^t\right)\pi_{0,t}\left(s^t\right)\right)^{\rho}\left(P_{0,t}^i\left(s^t\right)\right)^{1-\rho}\right]^{\frac{1}{1-\rho}}$$

Define:

$$\phi_{0,t}^i\left(s^t\right) = \frac{P_{0,t}^i\left(s^t\right) C_{0,t}^i\left(s^t\right)}{\Pi_0^i U_0^i} = \left(\left((1-\beta)\beta^t\right)\pi_{0,t}\left(s^t\right)\right)^{\rho}\left(\frac{P_{0,t}^i\left(s^t\right)}{\Pi_0^i}\right)^{1-\rho} \tag{13}$$

and

$$\theta_{0,t}^i\left(s^t\right) = \frac{(1-\mu) Q_{0,t}^i\left(s^t\right) Y_t^i\left(s^t\right)}{\Pi_0^i U_0^i} \tag{14}$$

That is, $\phi_{0,t}^i\left(s^t\right)$ is expenditure on date-t history-s^t consumption as a share of date-0 expenditure on consumption in all dates and states, expressed in terms of date-0 prices. Meanwhile, $\theta_{0,t}^i\left(s^t\right)$ is the ratio of date-t history-s^t value added to date-0 expenditure on consumption at all dates and states, expressed in terms of date-0 prices. Assume $B_0^i = 0$ (i.e. country i

has zero net wealth at date 0). Then for any asset market:

$$d \ln U_0^i = \sum_{t=0}^{\infty} \sum_{s^t} \theta_{0,t}^i \left(s^t\right) \left[\frac{d \ln \omega_{t,t}^{ii} \left(s^t\right)}{(1-\mu)(1-\eta)} + \frac{d \ln \phi_{0,t}^i \left(s^t\right)}{(1-\rho)} + \right.$$
$$\left. \frac{1}{2} \left(\frac{d \ln \omega_{t,t}^{ii} \left(s^t\right)}{(1-\mu)(1-\eta)} \right)^2 + \frac{d \ln \omega_{t,t}^{ii} \left(s^t\right) d \ln \phi_{0,t}^i \left(s^t\right)}{(1-\mu)(1-\eta)(1-\rho)} + \frac{1}{2} \left(\frac{d \ln \phi_{0,t}^i \left(s^t\right)}{(1-\rho)} \right)^2 \right] \tag{15}$$

Equation (15) generalizes the formula for welfare gains from trade in a static environment popularized by Arkolakis et al (2012). However as noted above, there is no sufficient statistics approach to evaluating (15). The domestic share in expenditure $\omega_{t,t}^{ii}\left(s^t\right)$ is observed only for realized histories. Even for realized histories, the expenditure shares $\phi_{0,t}^i\left(s^t\right)$ and $\theta_{0,t}^i\left(s^t\right)$ are unobserved, because total ex-ante expenditure, $\Pi_0^i U_0^i$ is unobserved. This is also true of the corresponding shares $\phi_{0,t}^i\left(s^t\right)$ under the counterfactual of infinite trade costs.

As a result, the model must be calibrated and solved for both baseline and counterfactual in order to calculate the gains from trade. Of course, once the full baseline and counterfactual model have been solved, it is straightforward to calculate exact changes in welfare using equation (2), and approximations are not necessary. Moreover, in the quantitative analysis, I find that for large changes (such as the changes in trade costs required to bring about autarky) the second-order approximation is in addition quite inaccurate.

4 Data

I use the following data to recover time series of productivity and trade costs for a large panel of countries: (a) national income accounts expressed in current US$ at market exchange rates (b) the matrix of bilateral trade flows expressed in current US$ at market exchange rates, (c) labor supply, proxied by population, (d) relative expenditure prices across countries, and (e) time series of real gross output for the U.S. I also make use of data on the ratio of gross output to value added. I construct an unbalanced panel dataset with variables (a)-(d) covering the period 1970-2019. Subject to some restrictions, I include countries for which there are at least 10 consecutive years of complete data on the required variables. The total number of countries covered is 162. The number of countries present in each cross-section ranges from 85 in 1970 to 160 in the 2010s, with 156 countries present in 2019. I also construct a Rest of the World (ROW) aggregate. Full details of data sources and coverage are reported in Appendix C.

4.1 Data sources

4.1.1 National income accounts

I obtain national income accounts data (GDP, exports, and imports measured in current US$ at market exchange rates, along with the merchandise share in total balance of payments imports) for most countries in the sample from the World Development Indicators (WDI). I supplement this with data from the Organization for Economic Cooperation and Development (OECD) for a number of countries for which OECD time-series coverage is better. I use data from the OECD on national income accounts for the former West Germany, 1970-1990. I obtain data for Taiwan directly from the Taiwanese statistical agency. For a small number of developing countries, WDI reports total imports and exports based on the balance of payments, but not national income accounts. In these cases, I use imports and exports from the balance of payments. For details, see Appendix C.

4.1.2 Bilateral trade

I obtain bilateral merchandise trade data for 1970-1994 from Feenstra and Lipsey's NBER-United Nations Trade Data. I obtain bilateral merchandise trade data for 1995-2019 from the CEPII's Baci dataset. I obtain bilateral services trade data for 1995-2004 from WTO-OECD BaTiS, version BPM5. I obtain bilateral services trade data for 2005-2019 from WTO-OECD BaTiS, version BPM6. All of these data sets measure bilateral import flows in current US$ at market exchange rates.

4.1.3 Labor supply

I use population as a proxy for labor supply. I obtain data on population for most countries in the sample from the WDI. I supplement this with OECD data for a number of countries for which coverage is better. I use the Penn World Tables 5.6 to obtain population for the former West Germany, 1970-1990. I obtain population data for Taiwan directly from the Taiwanese statistical agency.

4.1.4 Expenditure prices

I obtain expenditure PPPs for all countries from the Penn World Tables 10.01. The variable I use is "pl_da."

4.1.5 US real GDP

I obtain time series data on US constant price real gross output, used to benchmark output growth over time, from the Bureau of Economic Analysis (BEA).

4.1.6 Gross output to value added ratio

I use the OECD's national accounts data to obtain the ratio of nominal gross output to nominal value added for a limited subsample of the countries and years. This guides my choice for μ, the materials share in gross output.

4.1.7 Gravity variables

To validate the calibration, I make use of use a standard set of gravity variables provided by CEPII.

4.2 Expenditure shares and expenditure prices

Recovering productivity and trade costs consistent with the model requires bilateral (gross) expenditure shares, including the share of a country's gross expenditure devoted to its own output. Information on gross output is available for only a limited set of countries and years. The average ratio of gross output to value added in the OECD national accounts data is approximately 2. Time variation in this ratio is modest for most covered countries (see Appendix C). Guided by this fact and by equation (12), I use the value $\mu = 0.5$ for the materials share. I then use data on GDP to impute gross output of country i at date t using:

$$OUT_t^i = \frac{GDP_t^i}{1-\mu}$$

For Singapore, Hong Kong, and Luxembourg this implies that exports exceed gross output in at least some years. I drop these countries from the sample. I measure a country's imports from itself as gross output less total exports from national income accounts:

$$IM_t^{ii} = OUT_t^i - EXna_t^i$$

I measure a country's total absorption as gross output less exports from national income accounts plus imports from national income accounts:

$$ABS_t^i = OUT_t^i - EXna_t^i - IMna_t^i$$

For the period 1970-1994 I use bilateral merchandise trade data to calculate bilateral import shares in total imports:

$$\frac{IM_t^{ki}}{\sum_{j\neq i} IM_t^{ji}} = \frac{Merchim_t^{ki}}{\sum_{j\neq i} Merchim_t^{ji}}$$

For the period 1995-2019, for countries for which I observe the share of merchandise in BOP imports, I construct bilateral import shares in total imports as follows:

$$\frac{IM_t^{ki}}{\sum_{j\neq i} IM_t^{ji}} = \frac{Merchim_t^{ki}}{\sum_{j\neq i} Merchim_t^{ji}} \frac{BOPmerchim_t^i}{BOPim_t^i} + \frac{Servim_t^{ki}}{\sum_{j\neq i} Servim_t^{ji}} \left(1 - \frac{BOPmerchim_t^i}{BOPim_t^i}\right)$$

If the share of merchandise in BOP imports is not available, I continue to use bilateral merchandise import shares to measure bilateral import shares. Shares in absorption are then calculated by multiplying the bilateral import shares in total imports by the ratio of imports from national income accounts to absorption:

$$\frac{IM_t^{ki}}{ABS_t^i} = \frac{IMna_t^i}{ABS_t^i} \frac{IM_t^{ki}}{\sum_{j\neq i} IM_t^{ji}}$$

I construct a rest of the world (ROW) aggregate composed of all countries not included in the baseline sample using WDI data on world GDP and bilateral trade data on world imports and exports. The ROW aggregate accounts for close to 12% of world GDP in the 1970s, falling to just over 2% from 2000. It accounts for around 12% of imports of in-sample countries in the 1970s and early 1980s, falling to between 4 and 5% of imports of in-sample countries for the remainder of the sample period (see Appendix C).

Recovering productivity and trade costs requires a measure of relative expenditure prices. Expenditure PPPs from the PWT are used to measure the price of the expenditure basket in each country at date t relative to that in the US:

$$\frac{\hat{P}_t^i}{\hat{P}_t^{US}} = \frac{PPP_t^i}{PPP_t^{US}}$$

5 Recovering productivity and trade costs

With expenditure shares and relative expenditure prices in hand, I "invert" the quantitative model described in Section 3 to recover productivity and trade costs. This requires values for the parameters μ and η. As explained in the previous section, guided by data on the ratio of gross output to value added, I set the materials share $\mu = 0.5$. Following the trade literature, I choose a baseline value of $\eta = 4$, implying a trade elasticity of 3.

Recovering output prices and trade costs

Dropping state-contingent notation and using prices expressed in date-t dollars, the model tells us (see equation (11)) that irrespective of intertemporal preferences and the nature of the asset market:

$$\frac{IM_t^{ki}}{ABS_t^i} = \frac{\tau^{ki} Q_t^k Z_t^{ki}}{P_t^i X_t^i} = \frac{\left(\tau_t^{ki} Q_t^k\right)^{1-\eta}}{\sum_{j=1}^N \left(\tau_t^{ji} Q_t^j\right)^{1-\eta}}$$

with

$$\left(P_t^i\right)^{1-\eta} = \sum_{j=1}^N \left(\tau_t^{ji} Q_t^j\right)^{1-\eta}$$

These expressions imply that given information on own import shares and expenditure prices, the vector of relative output prices is given by:

$$\left(\hat{Q}_t^i\right)^{1-\eta} = \frac{IM_t^{ii}}{ABS_t^i} \left(\hat{P}_t^i\right)^{1-\eta}$$

Then bilateral trade costs can be recovered from bilateral import shares using:

$$\left(\hat{\tau}_t^{ki}\right)^{1-\eta} = \frac{IM_t^{ki}}{ABS_t^i} \left(\frac{\hat{P}_t^i}{\hat{Q}_t^k}\right)^{1-\eta} = \frac{IM_t^{ki}}{ABS_t^i} \frac{ABS_t^k}{IM_t^{kk}} \left(\frac{\hat{P}_t^i}{\hat{P}_t^k}\right)^{1-\eta}$$

The bilateral trade costs recovered using this approach do not always satisfy the restriction $\hat{\tau}_t^{ki} \geq 1$. Negative implied trade costs are clustered in developing countries importing from developed countries. They are relatively infrequent at the beginning and end of the sample. They are most frequent in the 1990s (associated with importing countries newly formed after the break-up of the Soviet Union), when they account for between 0.4% and 1% of bilateral pairs (see Appendix D). Negative trade costs are a sign of either model misspecification, or data mismeasurement, as they imply that output can be created costlessly by shipping intermediate k from k to i. I take the stand that these negative implied trade costs are the result of systematic measurement error.

As a baseline, I assume expenditure prices are mismeasured. Consistent with this assumption, I apply an adjustment to measured expenditure PPPs for problematic importer-time pairs. In particular, I impose $\tilde{\tau}_t^{ki} \geq 1$ for all partners $k \neq i$, with equality for the pair for which $\hat{\tau}_t^{ki}$ is smallest (i.e. most negative trade costs):

$$\left(\tilde{P}_t^i\right)^{1-\eta} = \left(\hat{P}_t^i\right)^{1-\eta} \max_k \left\{ \frac{ABS_t^i}{IM_t^{ki}} \left(\frac{\hat{P}_t^i}{\hat{Q}_t^k}\right)^{\eta-1} \right\}$$

I then construct output prices and iceberg trade costs using the adjusted expenditure prices:

$$\left(\tilde{Q}_t^i\right)^{1-\eta} = \frac{IM_t^{ii}}{ABS_t^i} \left(\tilde{P}_t^i\right)^{1-\eta}$$

$$\left(\tilde{\tau}_t^{ki}\right)^{1-\eta} = \frac{IM_t^{ki}}{ABS_t^i} \left(\frac{\tilde{P}_t^i}{\tilde{Q}_t^k}\right)^{1-\eta}$$

I iterate until $\tilde{\tau}_t^{ki} \geq 1 \ \forall i, k$. This procedure recovers trade costs and cross-sectional relative output prices which exactly fit import shares, but the implied expenditure prices do not exactly match measured PPPs. In Appendix D, I report more details on this adjustment, as well as on an alternative which assumes instead that bilateral imports are mismeasured.

The model inversion does not pin down how output prices (and therefore real quantities) evolve over time. To do so, I use time series data on US real output from the BEA. I recover time-series variation in the US output price using:

$$\frac{Q_t^{US}}{Q_{1970}^{US}} = \frac{OUT_t^{US}/Realout_t^{US}}{OUT_{1970}^{US}/Realout_{1970}^{US}}$$

I normalize $Q_{1970}^{US} = 1$. The output prices used to calculate productivity are constructed using:

$$\left(\hat{\tilde{Q}}_t^i\right)^{1-\eta} = \left(\frac{\tilde{Q}_t^i}{\tilde{Q}_t^{US}} \frac{Q_t^{US}}{Q_{1970}^{US}}\right)^{1-\eta}$$

Corresponding expenditure prices are constructed using

$$\left(\hat{\tilde{P}}_t^i\right)^{1-\eta} = \frac{ABS_t^i}{IM_t^{ii}} \left(\hat{\tilde{Q}}_t^i\right)^{1-\eta}$$

Productivity

Real output is obtained by dividing the current dollar value of output by the appropriate output price:

$$\hat{\tilde{Y}}_t^i = \frac{OUT_t^i}{\hat{\tilde{Q}}_t^i}$$

Given the value of μ, materials inputs to production are calculated using:

$$\hat{\tilde{M}}_t^i = \mu \frac{\hat{\tilde{Q}}_t^i \hat{\tilde{Y}}_t^i}{\hat{\tilde{P}}_t^i}$$

Using these implied materials inputs to production, and data on labor input (i.e. population), productivity is given by:

$$\hat{\tilde{A}}_t^i = \frac{\hat{\tilde{Y}}_t^i}{(L_t^i)^{1-\mu} \left(\hat{\tilde{M}}_t^i\right)^{\mu}}$$

Results

Figure 1 plots the resulting time-series of productivity for all countries in the sample, with US productivity normalized to 1 in 1970. As the figure shows, the productivity series can be quite volatile. Some of the largest productivity swings, and most of the cases where productivity is above that of the US, are associated with oil-producing countries. Figure 2 plots the distribution of the standard deviation of the log change in productivity at the country level, for OECD and non-OECD countries (Appendix C lists which countries are OECD members). Figure 3 plots the distribution of the correlation of the log change in productivity at the country level with the log change in US productivity. These figures illustrate a key feature of the productivity series: productivity behaves differently for OECD and non-OECD countries. Productivity growth is more volatile in non-OECD countries than in OECD countries, and productivity growth is less correlated with US productivity growth in non-OECD countries than OECD countries.

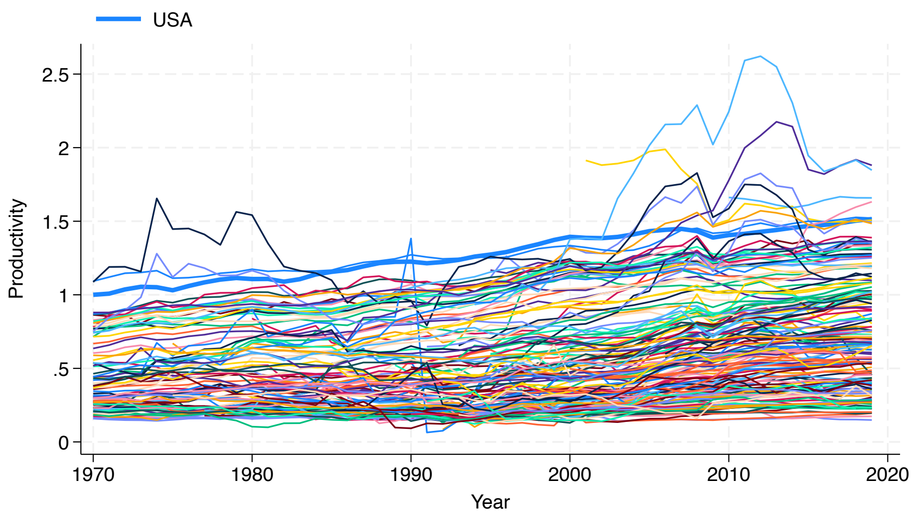

Figure 1: Model-consistent productivity

Note: Figure plots model-consistent productivity, constructed as described in the text, for all observations in the unbalanced panel. US productivity is normalized to 1 in 1970. Countries and period covered for each country are listed in Appendix C.

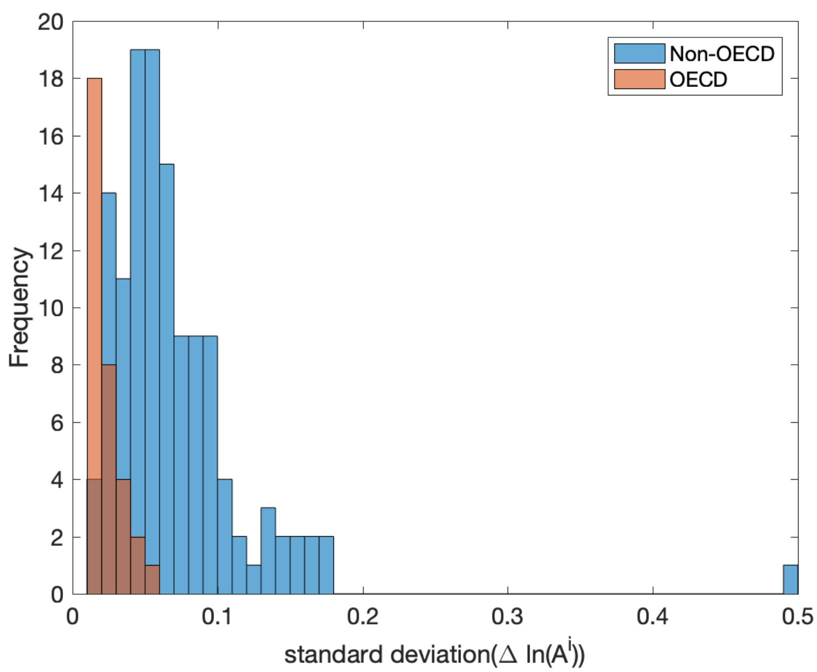

Figure 2: Distribution of standard deviation of $\Delta \ln A_t^i$

Note: Figure plots the distribution across all countries in the unbalanced panel of the standard deviation of $\Delta \ln A_t^i$, i.e. the log change in model-based productivity, calculated at the country level. Countries, period covered for each country, and OECD membership, are listed in Appendix C.

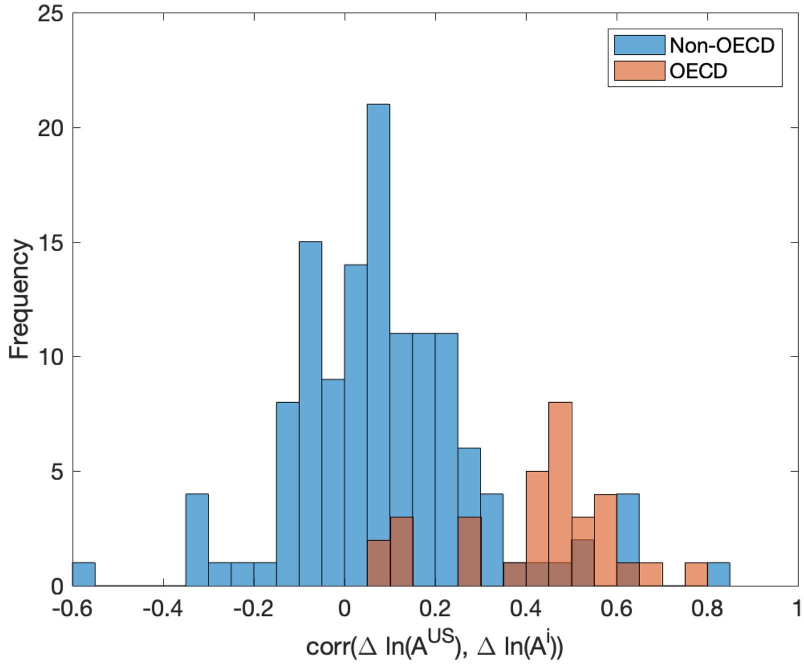

Figure 3: Distribution of $corr\left(\Delta \ln A_t^{US}, \Delta \ln A_t^i\right)$

Note: Figure plots the distribution across all countries in the unbalanced panel of $corr\left(\Delta \ln A_t^{US}, \Delta \ln A_t^i\right)$, i.e. the correlation of the log change in model-based productivity with the log change in model-based productivity for the US. Countries, period covered for each country, and OECD membership, are listed in Appendix C.

As is well-known in the literature, for standard values of η, the wedges $\left(\tilde{\tau}_t^{ki}\right)^{1-\eta}$ necessary to rationalize observed patterns of trade imply high average levels of trade costs. However the distribution of these wedges is correlated with bilateral distance, contiguity, common language, etc., in the expected way. In Appendix D I report results from regressing $\left(\tilde{\tau}_t^{ki}\right)^{1-\eta}$ on a standard set of gravity variables from CEPII. The coefficients have the expected sign, and are all strongly statistically significant.

6 Productivity process

The next step is to estimate a stochastic process for productivity. The ideal process should be flexible enough to account for (a) long-run growth, (b) conditional convergence at rates that may differ across countries, (c) persistent deviations of growth from trend, (d) correlations in deviations of growth from trend across countries, and (e) heterogeneous volatility of deviations of growth from trend across countries.

I implement this by assuming that the US is the world technology leader. Let $a_t^i = \ln A_t^i$.

I assume that a_t^{US} grows at constant rate g^{US}. Actual US productivity growth equals trend growth g^{US}, plus deviations from trend ε_t^{US}, which may be persistent, i.e.

$$\Delta a_t^{US} = g^{US} + \varepsilon_t^{US}$$

where

$$\varepsilon_t^{US} = \psi^{US} \varepsilon_{t-1}^{US} + \sigma^{US} \nu_t^{US}$$

with $\nu_t^{US} \sim N(0,1)$.

Meanwhile, trend productivity in $i \neq US$ grows at the same average rate as the US (i.e. the rate of growth of the technology frontier) plus a term reflecting convergence to the frontier. Actual productivity growth again equals trend growth, plus deviations from trend.

$$\Delta a_t^i = g^{US} + \gamma^i \left(a_{t-1}^{US} - a_{t-1}^i\right) + \varepsilon_t^i$$

Here, γ^i governs the speed of convergence to the frontier. In the case of $i \neq US$, deviations from trend may depend on lagged deviations of US productivity growth from trend, as well as innovations to US deviations of growth from trend, in addition to dependence on own lagged deviations and innovations:

$$\varepsilon_t^i = \psi^i \varepsilon_{t-1}^i + \psi^{i,US} \varepsilon_{t-1}^{US} + \sigma^i \nu_t^i + \sigma^{i,US} \nu_t^{US}$$

with $\nu_t^i \sim N(0,1)$. The dependence of deviations from trend on US deviations induces correlation in deviations of productivity growth from trend across non-US countries.

In the interest of parsimony, I divide countries (excluding the US) into two groups: OECD and non-OECD countries, and impose the same parameters within-group, but allow for different parameters for each group. This division is motivated by the evidence in Figures 2 and 3 that the process for productivity growth differs between OECD and non-OECD countries. I restrict the estimation sample to 80 countries (plus the US) for which I have a balanced panel of data.[5] Appendix C lists the countries included as well as OECD membership. I first estimate the parameters of the US productivity process and then estimate the process for the remaining countries by maximum likelihood, conditioning on $\left\{g^{US}, \psi^{US}, \sigma^{US}\right\}$. Details are reported in Appendix E.

The parameter estimates are reported in Table 1 along with bootstrapped 95% confidence

[5] Iraq is an outlier in the standard deviation of productivity due to the behavior of GDP around the invasion of Kuwait: see Figure 2. Although data is available for Iraq for the full sample period, I drop it from the estimation sample.

intervals. Unsurprisingly, given Figures 2 and 3, the estimates differ between developed and developing countries. Most notably, the standard deviation of innovations to deviations from trend is three times higher in developing than developed countries. In addition, comovement with US deviations of growth from trend is lower in developing than developed countries. Meanwhile convergence is weak for both groups, but more so for developing than developed countries.

Table 1: Parameters of estimated productivity process

Parameter		Value	95% CI
US			
US LR growth rate	g^{US}	0.0085	[0.0031, 0.0138]
Autocorrelation of deviations from trend, US	ψ^{US}	0.31	[0.02, 0.52]
SD of innovations to deviations from trend, US	σ^{US}	0.012	[0.007, 0.017]
OECD countries			
Rate of convergence to US	γ^{O}	0.0103	[0.0053, 0.0114]
Autocorrelation of deviations from trend	ψ^{O}	0.11	[0.04, 0.15]
Correlation with lagged US deviations from trend	$\psi^{O,US}$	0.38	[0.17, 0.55]
SD of innovations to deviations from trend	σ^{O}	0.022	[0.020, 0.025]
Correlation with US innovations to deviations from trend	$\sigma^{O,US}$	0.008	[0.004, 0.012]
Non-OECD countries			
Rate of convergence to US	γ^{N}	0.0016	[-0.0016, 0.0047]
Autocorrelation of deviations from trend	ψ^{N}	0.00	[-0.03, 0.04]
Correlation with lagged US deviations from trend	$\psi^{N,US}$	0.15	[-0.24, 0.35]
SD of innovations to deviations from trend	σ^{N}	0.066	[0.063, 0.072]
Correlation with US innovations to deviations from trend	$\sigma^{N,US}$	0.003	[-0.000, 0.006]

Notes: Estimation restricted to balanced panel 1970-2019, excluding Iraq. Appendix C lists included countries along with OECD membership. OECD and non-OECD coefficients estimated by maximum likelihood conditional on coefficients for US. Bootstrapped 95% confidence intervals based on 100 draws with replacement.

7 Gains from trade

Calculating the 3-D gains from trade requires taking a stand on trade costs and international asset markets in the trade equilibrium. I fix trade costs in the trade equilibrium at their calibrated levels in 2019, the last year in my sample, and the year for which I calculate ex-ante gains from trade. The literature on international macroeconomics has not converged on a way of modeling frictions in asset markets that performs well in matching international comovements at a business cycle frequency, or observed portfolios. Clearly from the fact that net exports deviate from zero, trade across dates and states is possible. Yet Heathcote and Perri (2002) conclude that balanced trade (what they call “financial autarky”) does a better job of matching features of international business cycles in developed countries than either complete markets, or trade in a single non-contingent nominal bond, another popular benchmark. With this in mind, I focus on two polar cases, balanced trade, and complete markets, which have the advantage of being straightforward to compute. I leave analysis of gains from trade under alternative asset market structures to future research.

I therefore solve for equilibrium consumption allocations and prices under three different scenarios:

1. Infinite trade costs (full autarky) at all dates and states, including 2019. Because trade costs are infinite, the structure of international asset markets is irrelevant for allocations.

2. Trade costs held fixed at their 2019 levels at all dates and states; balanced trade in all dates and states, including 2019.

3. Trade costs held fixed at their 2019 levels at all dates and states; complete and frictionless asset markets with zero initial net wealth (i.e. $B_0^i = 0$) for all countries in 2019.

In all three scenarios, productivity in 2018 and 2019 is held fixed at the levels calculated in Section 5.[6] From 2020 forward, productivity is assumed to follow the process estimated in Section 6. In addition, labor supply is held fixed at 2019 levels.

Under scenarios (1) and (2), there are no dynamic decisions. Under scenario (3), re-opening markets at any date or state does not lead countries to change their asset positions vis-a-vis the choices made at date 0. So in all three cases, the problem can be treated as if all choices are made at date 0. Given this, I can approximate the solution to the date-0 problem under each scenario by simulating S paths of length T from the process for $\mathbf{a}_t = \ln \mathbf{A}_t$, starting at $t = 2019$, and conditioning on $\{\mathbf{a}_{2019}, \Delta\mathbf{a}_{2019}\}$. I then solve for the equilibrium from a date-0 perspective under the assumption that each of these S paths has equal probability. I now describe how equilibrium is calculated in each of the three scenarios.

1. Autarky equilibrium

In autarky, the sum of consumption and materials equals output in each country at each date and state. Given the optimal choice of materials, the consumption allocation is given

[6] Because productivity growth is persistent, productivity growth post-2019 depends on productivity growth between 2018 and 2019.

by:

$$C_t^i\left(s^t\right)^{aut} = (1-\mu)\,\mu^{\frac{\mu}{1-\mu}} A_t^i\left(s^t\right)^{\frac{1}{1-\mu}} L_{2019}^i$$

and this allows $U_0^{i,aut}$ to be calculated. In autarky, the country-i output price is equal to the country-i expenditure price at each date and state. Given the consumption allocation, the prices of output and expenditure at date t after history s^t relative to the price of ex-ante utility in i, $\Pi_0^{i,aut}$, can be recovered from the first order condition for consumption:

$$\frac{Q_{0,t}^i\left(s^t\right)^{aut}}{\Pi_0^{i,aut}} = \frac{P_{0,t}^i\left(s^t\right)^{aut}}{\Pi_0^{i,aut}} = \left(\frac{1-\beta}{1-\beta^{T+1}}\right)\beta^t \pi_{0,t}\left(s^t\right)\left(\frac{C_t^i\left(s^t\right)^{aut}}{U_0^{i,aut}}\right)^{-\frac{1}{\rho}}$$

2. Trade equilibrium with balanced trade

I use a tatonnement algorithm to solve for the competitive equilibrium quantities and prices. At each date and state, I normalize one output price to 1, guess a vector $\left\{Q_{t,t}^k\left(s^t\right)^{bal}\right\}_{k=1}^N$ of relative output prices, and use excess demands to update the output price vector, increasing prices when excess demand is positive and reducing them when excess demand is negative, and iterating until excess demands converge to zero. Excess demand for good k at t, s^t is given by:

$$ED_t^k\left(s^t\right)^{bal} = \sum_{i=1}^N \tau_{2019}^{ki} Z_t^{ki}\left(s^t\right)^{bal} - Y_t^k\left(s^t\right)^{bal}$$

where

$$Y_t^k\left(s^t\right)^{bal} = L_{2019}^k\left(A_t^k\left(s^t\right)\right)^{\frac{1}{1-\mu}}\left(\frac{\mu Q_{t,t}^k\left(s^t\right)^{bal}}{P_{t,t}^k\left(s^t\right)^{bal}}\right)^{\frac{\mu}{1-\mu}}$$

and

$$Z_t^{ki}\left(s^t\right)^{bal} = \left(\frac{P_{t,t}^i\left(s^t\right)^{bal}}{\tau_{2019}^{ki} Q_{t,t}^k\left(s^t\right)^{bal}}\right)^{\eta}\left(C_t^i\left(s^t\right)^{bal} + \mu^{\frac{1}{1-\mu}}\left(\frac{Q_{t,t}^i\left(s^t\right)^{bal} A_t^i\left(s^t\right)}{P_{t,t}^i\left(s^t\right)^{bal}}\right)^{\frac{1}{1-\mu}} L_{2019}^i\right)$$

and, imposing balanced trade:

$$C_t^i\left(s^t\right)^{bal} = (1-\mu)\frac{Q_{t,t}^i\left(s^t\right)^{bal} Y_t^i\left(s^t\right)^{bal}}{P_{t,t}^i\left(s^t\right)^{bal}}$$

Given the resulting consumption allocation, date-0 expenditure prices for country i relative to $\Pi_0^{i,bal}$ can be recovered from the the first order condition for consumption:

$$\frac{P_{0,t}^i\left(s^t\right)^{bal}}{\Pi_0^{i,bal}} = \left(\frac{1-\beta}{1-\beta^{T+1}}\right)\beta^t \pi_{0,t}\left(s^t\right)\left(\frac{C_t^i\left(s^t\right)^{bal}}{U_0^{i,bal}}\right)^{-\frac{1}{\rho}}$$

and date-0 output prices for country i can be recovered using:

$$\frac{Q_{0,t}^i\left(s^t\right)^{bal}}{\Pi_0^{i,bal}} = (1-\mu)\frac{P_{0,t}^i\left(s^t\right)^{bal}}{\Pi_0^{i,bal}}\frac{C_t^i\left(s^t\right)^{bal}}{Y_t^i\left(s^t\right)^{bal}}$$

3. Trade equilibrium with complete and frictionless asset markets

I use a tatonnement algorithm to solve for the competitive equilibrium quantities and prices under complete markets. I normalize one price to 1, guess a vector $\left\{\left\{\left\{Q_{0,t}^k\left(s^t\right)^{cm}\right\}_{k=1}^N\right\}_{s^t\in S}\right\}_{t=1}^T$ of date-0 relative output prices. I use this vector to calculate expenditure prices in each country at each date and state, and therefore $\Pi_0^{i,cm}$ as outlined in Section 3. I then use excess demands for each intermediate at each date and state to update the output price vector, increasing prices when excess demand is positive and reducing them when excess demand is negative, and iterating until excess demands converge to zero. Excess demand for good k at t, s^t is given by:

$$ED_t^{k,cm}\left(s^t\right) = \sum_{i=1}^N \tau_{2019}^{ki} Z_t^{ki}\left(s^t\right)^{cm} - Y_t^k\left(s^t\right)^{cm}$$

where

$$Y_t^k\left(s^t\right)^{cm} = L_{2019}^k\left(A_t^k\left(s^t\right)\right)^{\frac{1}{1-\mu}}\left(\frac{\mu Q_{0,t}^k\left(s^t\right)^{cm}}{P_{0,t}^k\left(s^t\right)^{cm}}\right)^{\frac{\mu}{1-\mu}}$$

and

$$Z_t^{ki}\left(s^t\right)^{cm} = \left(\frac{P_{0,t}^i\left(s^t\right)^{cm}}{\tau_{2019}^{ki} Q_{0,t}^k\left(s^t\right)^{cm}}\right)^{\eta}\left(C_t^i\left(s^t\right)^{cm} + \mu^{\frac{1}{1-\mu}}\left(\frac{Q_{0,t}^i\left(s^t\right)^{cm} A_t^i\left(s^t\right)}{P_{0,t}^i\left(s^t\right)^{cm}}\right)^{\frac{1}{1-\mu}} L_{2019}^i\right)$$

and making use of the CES structure of preferences, complete markets, and $B_0^i = 0$:

$$C_t^i\left(s^t\right)^{cm} = \left(\frac{\Pi_0^{i,cm}\left(\frac{1-\beta}{1-\beta^{T+1}}\right)\beta^t\pi_{0,t}\left(s^t\right)}{P_{0,t}^i\left(s^t\right)^{cm}}\right)^{\rho}\left(\frac{(1-\mu)\sum_{t=1}^{\infty}\sum_{s^t} Q_{0,t}^i\left(s^t\right)^{cm} Y_t^i\left(s^t\right)^{cm}}{\Pi_0^{i,cm}}\right)$$

The solution to this problem delivers both equilibrium allocations and date-0 prices.

Welfare gains from trade

I use the equilibrium consumption allocations under autarky and trade (with the two different configurations of asset markets) to calculate the 3-D gains from trade defined in equation (2). This notion of welfare corresponds to the measure of welfare costs of business cycles in Lucas (1987).

Parameters

As already noted, it is necessary to assume values for η and μ in order to calculate productivity and trade costs, and the baseline values I choose are $\eta = 4$ and $\mu = 0.5$. For the counterfactuals, values for $\{\beta, \rho\}$ are also required. I set $\beta = 0.96$, and $\rho = 1/4$. I set $T = 100$ and $S = 200$.

Baseline results

Table 2 reports summary statistics on the distribution of 3-D gains from trade, for the case of balanced trade and for the case of complete markets. Statistics are reported for the full sample, and for the OECD and non-OECD subsamples. The table also reports summary statistics on the static gains from trade for 2019 under balanced trade.[7] The results are also illustrated in Figure 4. The left panel plots 3-D gains under balanced trade against static gains from trade evaluated in 2019. The right panel plots 3-D gains under complete markets against static gains in 2019.

Table 2: Distribution of gains from trade

	Static gains			3-D gains					
	2019			Balanced trade			Complete markets		
	All	Rich	Poor	All	Rich	Poor	All	Rich	Poor
Mean	1.18	1.21	1.17	1.28	1.22	1.30	1.72	1.27	1.84
Med	1.15	1.15	1.14	1.22	1.16	1.24	1.69	1.21	1.77
Min	1.04	1.05	1.04	1.05	1.05	1.06	1.06	1.06	1.46
Max	2.22	1.65	2.22	2.90	1.69	2.90	3.56	1.77	3.56
Pop-wgt mean	1.09	1.12	1.08	1.14	1.12	1.15	1.55	1.16	1.63

Notes: "Rich" refers to OECD countries plus the US. "Poor" refers to non-OECD countries. Sample includes the 156 countries (33 OECD, 123 non-OECD) for which data is available to construct productivity and trade costs in 2019. Appendix C lists countries included and OECD membership status.

[7] In Appendix F, I report static gains from trade for 2019 under observed trade balances. For countries which run trade deficits in 2019, gains from trade under observed trade balances are naturally larger than under balanced trade, while for countries which run surpluses in 2019, gains from trade are naturally smaller than under balanced trade.

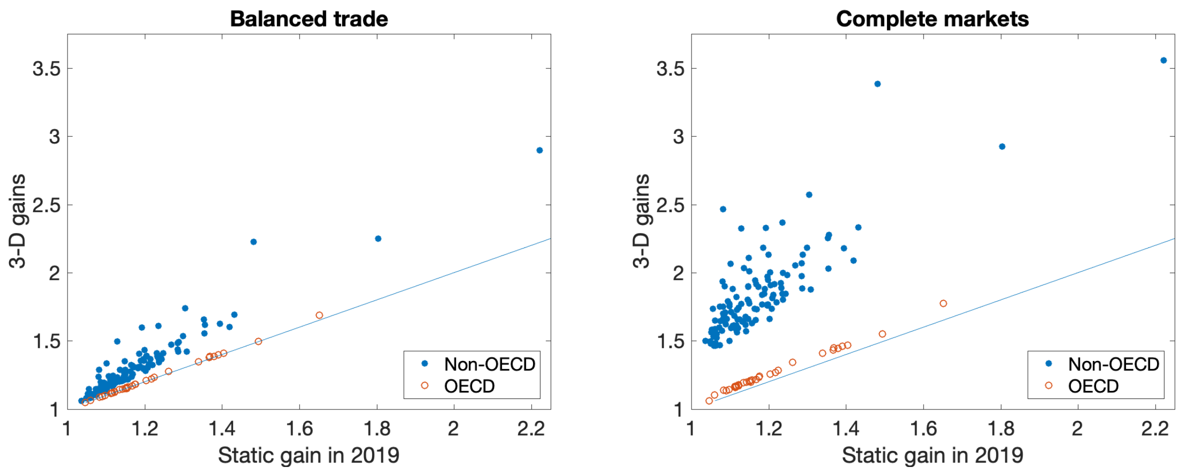

Figure 4: 3-D gains from trade

Notes: Left panel plots 3-D gains under balanced trade (y-axis) against static gains in 2019 (x-axis). Right panel plots 3-D gains under complete markets (y-axis) against static gains in 2019 (x-axis). Sample includes the 156 countries (33 OECD, 123 non-OECD) for which data is available to construct productivity and trade costs in 2019. Appendix C lists countries included and OECD membership status.

Table 2 and Figure 4 show that 3-D gains from trade are bigger than static gains, especially for developing countries, and especially under complete markets. For OECD countries, 3-D gains under balanced trade are 1.04 times static gains on average ($1.22 = 1.21^{1.04}$), while 3-D gains under complete markets are 1.25 times static gains ($1.27 = 1.21^{1.25}$). Meanwhile, for non-OECD countries, 3-D gains under balanced trade are 1.67 times static gains ($1.30 = 1.17^{1.67}$). Under complete markets, 3-D gains for non-OECD countries are on average 3.88 times static gains ($1.84 = 1.17^{3.88}$).

Why are 3-D gains bigger than static gains?

Equation (4), reproduced below, provides some intuition for why 3-D gains are bigger than static gains, especially for developing countries, and especially under complete markets:

$$\kappa_3^i = \left(\underbrace{E\left(\lambda_t^i\left(s^t\right)^{\frac{\rho-1}{\rho}}\right)}_{\kappa_1^i} + \underbrace{COV\left(\left(\frac{C_t^i\left(s^t\right)^{aut}}{U^{i,aut}}\right)^{\frac{\rho-1}{\rho}}, \lambda_t^i\left(s^t\right)^{\frac{\rho-1}{\rho}}\right)}_{\kappa_2^i} \right)^{\frac{\rho}{\rho-1}}$$

Since $\rho = 0.25 \in [0,1]$, 3-D gains are decreasing in the sum $\kappa_1^i + \kappa_2^i$. From equation (3) we know that $\kappa_1^i + \kappa_2^i > 0$. This implies that conditional on κ_1^i, the more negative is κ_2^i, the bigger are 3-D gains from trade. Figure 5 plots $\kappa_1^i + \kappa_2^i$ (on the y-axis) against κ_1^i (on the x-axis). The left panel is for the case of balanced trade, while the right panel is for the case of complete markets. It turns out that κ_1^i differs across countries, but is very similar under balanced trade and complete markets for a given country. Figure 5 therefore shows that 3-D gains are bigger for developing than developed countries, and under complete markets than balanced trade, because in these cases, the covariance term is more negative. That is, static gains from trade are high precisely when autarky consumption is low.

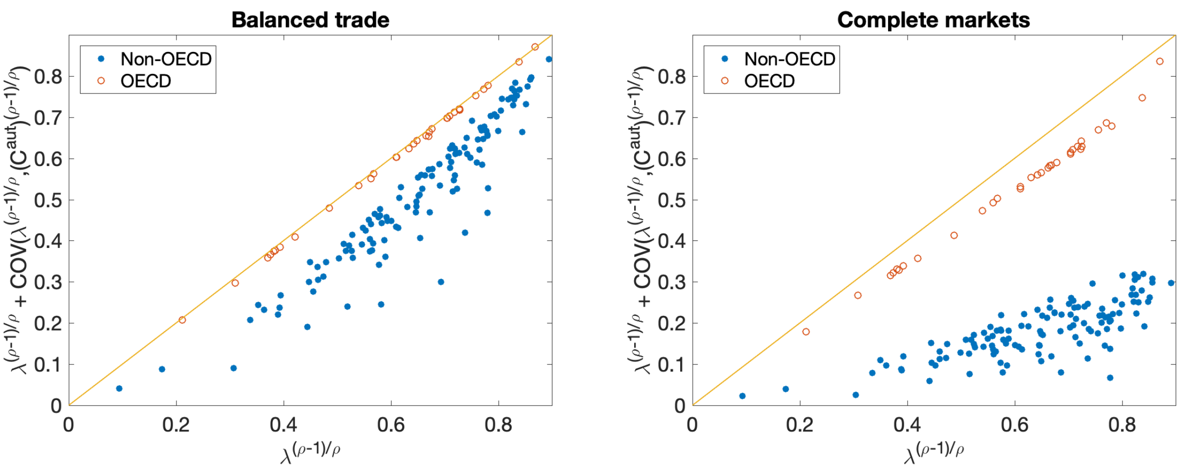

Figure 5: Components of gains from trade

Notes: Left panel plots $COV\left(\left(C_t^i\left(s^t\right)^{aut}/U^{i,aut}\right)^{\frac{\rho-1}{\rho}}, \lambda_t^i\left(s^t\right)^{\frac{\rho-1}{\rho}}\right)$ (y-axis) against $E\left(\lambda_t^i\left(s^t\right)^{\frac{\rho-1}{\rho}}\right)$ (x-axis) under balanced trade. Right panel plots $COV\left(\left(C_t^i\left(s^t\right)^{aut}/U^{i,aut}\right)^{\frac{\rho-1}{\rho}}, \lambda_t^i\left(s^t\right)^{\frac{\rho-1}{\rho}}\right)$ (y-axis) against $E\left(\lambda_t^i\left(s^t\right)^{\frac{\rho-1}{\rho}}\right)$ (x-axis) under under complete markets. Sample includes the 156 countries (33 OECD, 123 non-OECD) for which data is available to construct productivity and trade costs in 2019. Appendix C lists countries included and OECD membership status.

Why are 3-D gains bigger than static gains even under balanced trade?

Under balanced trade, countries cannot run trade deficits or surpluses to smooth consumption across states of the world and over time. So why are 3-D gains for developing countries bigger than static gains even under balanced trade? The reason is that although explicit insurance and intertemporal smoothing trades are not possible, there is still partial insurance through movements in the terms of trade, as in Cole and Obstfeld (1991). At dates and states when a country's productivity is relatively low, because its good is imperfectly substitutable with the goods of other countries, its output price will be high relative to the price of its consumption basket. This mitigates the loss of purchasing power due to lower output, and contributes to the negative covariance of static gains and autarky consumption.

To illustrate this mechanism, I calculate the correlation between log changes in the terms of trade ($\Delta \ln \left(Q_{t,t}^{i}\left(s^{t}\right)/P_{t,t}^{i}\left(s^{t}\right)\right)$), and log changes in productivity ($\Delta \ln A_{t}^{i}\left(s^{t}\right)$), country-by-country. The distribution of these correlations in the model under balanced trade is plotted in the left panel of Figure 6. The correlation is very negative, especially for developing countries. Quantitatively, these correlations are much more negative than those in the historical data (plotted in the right panel of Figure 6). But the fact that they are more negative for developing than developed countries is consistent with the data.[8]

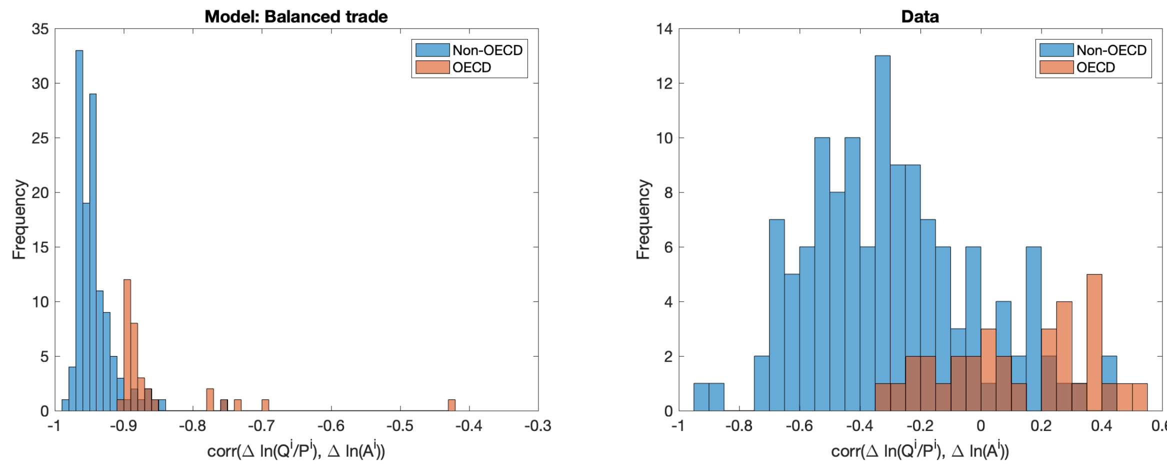

Figure 6: Correlation of terms of trade and productivity growth

Notes: Left panel plots the distribution of the correlation between the log change in terms of trade ($\Delta \ln \left(Q_{t}^{i}/P_{t}^{i}\right)$) and log change in productivity ($\Delta \ln A_{t}^{i}$) in the model under balanced trade. Right panel plots the distribution of the correlation between log change in terms of trade ($\Delta \ln \left(Q_{t}^{i}/P_{t}^{i}\right)$) and log change in productivity ($\Delta \ln A_{t}^{i}$) in the historical data. Sample includes the 156 countries (33 OECD, 123 non-OECD) for which data is available to construct productivity and trade costs in 2019. Appendix C lists countries included and OECD membership status.

Do 3-D gains come from trade across states, or from trade over time?

The only source of fully predictable gains from trade over time in my counterfactuals is differences in long-run growth rates across countries due to different speeds of convergence to the frontier. I isolate this source of gains by shutting down idiosyncratic shocks to productivity. Table 3 reports the 3-D gains from trade under this exercise in the same format as Table 2. The reported gains are almost identical to the static gains from trade. This is not surprising, since estimated rates of convergence are very slow, so there is no strong comparative advantage across countries in different periods.

[8] Under complete markets, the negative correlation is still present, though weaker than under balanced trade: See Appendix F.

Table 3: 3-D gains from trade in the absence of idiosyncratic shocks

	Balanced trade			Complete markets		
	All	Rich	Poor	All	Rich	Poor
Mean	1.18	1.21	1.17	1.18	1.21	1.17
Median	1.15	1.15	1.14	1.15	1.16	1.14
Min	1.04	1.05	1.04	1.04	1.05	1.04
Max	2.23	1.66	2.23	2.23	1.67	2.23

Notes: Table reports summary statistics on 3-D gains from trade when idiosyncratic shocks to productivity are shut down. “Rich” refers to OECD countries plus the US. “Poor” refers to non-OECD countries. Sample includes the 156 countries (33 OECD, 123 non-OECD) for which data is available to construct productivity and trade costs in 2019. Appendix C lists countries included and OECD membership status.

Why are 3-D gains so different for OECD and non-OECD countries?

To examine why 3-D gains are so much bigger for developing than developed countries, I redo the welfare calculations under the assumption that developing countries face the innovation standard deviation of developed countries, i.e. $\sigma^N = \sigma^O = 0.022$. The implied distribution of gains is reported in Table 4. With this modification, the 3-D gains from trade for both developed and developing developing countries under balanced trade are almost exactly equal to the static gains from trade. 3-D gains from trade under complete markets for both developed and developing countries are marginally higher than static gains from trade. For developed countries, these gains are close to those under the estimated productivity process, but for developing countries they are dramatically lower. This indicates that it is the fact that developing countries face more idiosyncratic risk, and therefore gain more from insurance through trade, that explains why they have higher 3-D gains than developed countries in the baseline calibration.

Table 4: 3-D gains when non-OECD countries face OECD innovation volatility

	Balanced trade			Complete markets		
	All	Rich	Poor	All	Rich	Poor
Mean	1.18	1.21	1.18	1.22	1.25	1.22
Med	1.15	1.16	1.14	1.19	1.20	1.18
Min	1.04	1.05	1.04	1.05	1.05	1.07
Max	2.25	1.69	2.25	2.32	1.75	2.32

Notes: Table reports summary statistics on 3-D gains from trade when the productivity process for non-OECD countries has the standard deviation of innovation estimated for OECD countries, i.e. $\sigma^N = \sigma^O = 0.022$. “Rich” refers to OECD countries plus the US. “Poor” refers to non-OECD countries. Sample includes the 156 countries (33 OECD, 123 non-OECD) for which data is available to construct productivity and trade costs in 2019. Appendix C lists countries included and OECD membership status.

How do 3-D gains from trade relate to the welfare cost of business cycles?
Lucas (1987) uses preferences as in (1) and an assumption about the stochastic process for consumption to conclude that the welfare cost of business cycles in the US is very small. Subsequent research points to the importance of the preference specification and the assumed process for consumption in reaching this conclusion. The exercise I perform here is clearly related. However the stochastic process for consumption induced by the productivity shocks in my model differs from that in the Lucas calculation: deviations of productivity growth from trend are persistent, which magnifies the cost of fluctuations. Moreover, even under the standard Lucas calculation, when the stochastic process for consumption is estimated to match developing rather than developed countries, the implied welfare cost of business cycles can be large (see e.g. Pallage and Robe (2003)). In Appendix F I report summary statistics on the Lucas cost of business cycles induced by the baseline process for productivity, under the assumption that countries are in autarky, and there is roundabout production. The US would be willing to pay 1.64% of consumption permanently in order to zero out deviations of productivity growth from trend. The average for OECD countries, is 4%, while the average for non-OECD countries is close to 50%.

How good is the second order approximation?
In addition to calculating the exact 3-D gains from trade using (2), I also calculate the second order approximation to the 3-D gains from trade using expression (15). In Appendix F I plot the first and second order approximations against exact 3-D gains. For small gains from trade, the approximations perform well. But when gains are large, even the second order approximation performs quite poorly.

Robustness to alternative measures of productivity
I check the robustness of the baseline results to using two alternative measures of productivity to estimate the productivity process. The first alternative is constructed similarly to the baseline measure, but assumes that negative trade costs are a result of mismeasurement of bilateral import shares rather than mismeasurement of expenditure prices. The procedure for recovering this productivity measure is described in detail in Appendix D. The second alternative measure is real GDP per capita from the Penn World Tables.

Using these two measures, I estimate the same productivity process using the same panel of countries as in the baseline case. Parameter estimates are reported in Appendix F. Crucially, in the case of the first alternative, the estimate of σ^N is lower than the baseline estimate, while in the case of the second alternative, the estimate of σ^N is higher than the baseline. I then perform the same counterfactuals as in the baseline case, but with the alternative estimates for the productivity process, and in the case of the first alternative, also alternative measures of initial productivity and trade costs. Results are reported in Appendix F in the same format as Table 2. The 3-D gains I calculate for developing countries using the first alternative productivity estimates are smaller than the baseline (closer to static gains), while the 3-D gains I calculate for developing countries using the second alternative are even bigger than in the baseline case. This reflects the different estimates of σ^N.

Robustness to alternative parameter values

The two key parameters in the model are the Armington elasticity η, and the CRRA parameter ρ. I examine the sensitivity of 3-D gains to alternative values for each. For η I consider values in the range $[1.5, 6]$.[9] For ρ I consider values in the range $[0.1, 1.5]$. This corresponds to values of the CRRA parameter in the range $[0.75, 10]$, i.e. from less risk averse than log utility to risk aversion of 10.

The upper panel of Table 5 reports the mean welfare gains for different values of η, holding ρ fixed at its baseline value, while the lower panel reports the mean welfare gains holding η fixed and varying ρ. Note that trade costs and the productivity process depend on η, so when I change η, I recompute trade costs and productivity and re-estimate the productivity process before redoing the counterfactuals. In Appendix F, I report the estimated parameters of the productivity process corresponding to each value of η.

As is well known, holding fixed trade openness, static gains from trade are decreasing in η. This is reflected in the top left panel of Table 5. In addition to this effect, in the 3-D gains calculation there are some additional, sometimes offsetting effects. First, the estimated standard deviation of the innovation to productivity in non-OECD countries is U-shaped in η: within the set of values considered, it is at its lowest for $\eta = 3$. Second, under balanced trade, endogenous risk sharing through the terms of trade increases as $\eta \to 1$.[10] Third, higher η implies lower trade costs are necessary to rationalize observed trade in 2019, and as a result, trade costs are lower throughout the counterfactual. This increases the ability of trade to provide optimal smoothing of consumption across dates and states of the world under complete markets. The net effect of these forces is that 3-D gains from trade under balanced trade are always greater than static gains, but the contribution of trade across dates and states of the world to welfare gains does not vary much with η, and 3-D gains

[9] International macroeconomics often uses values for η on the interval $[0, 1]$, but values less than or equal to 1 do not make sense in the context of a model with iceberg trade costs, as they imply that import shares in expenditure are insensitive to, or increasing in iceberg trade costs.

[10] Cole and Obstfeld (1991) consider the case where $\eta = 1$, where movements in the terms of trade perfectly offset shocks to productivity. In an environment with iceberg trade costs, values of $\eta \leq 1$ do not make sense.

overall are decreasing in η. In the case of complete markets, 3-D gains are always greater than static gains, and for developing countries there is a U-shaped relationship between 3-D gains and η.

Turning to ρ, which governs risk aversion and the elasticity of intertemporal substitution, higher risk aversion (lower ρ) implies higher gains from trade, both under balanced trade and complete markets. The increase in gains from trade with higher risk aversion is most dramatic for developing countries, which have more volatile productivity. A commonly used value of the intertemporal elasticity of substitution corresponds to $\rho = 0.5$. For this value of ρ, 3-D gains under balanced trade are only modestly higher than static gains, even for developing countries. Risk aversion of 8, corresponding to a value of $\rho = 0.125$ is not out of line in the finance literature. For this value of ρ, 3-D gains for developing countries are dramatically higher than static gains, even under balanced trade. For intuition, note that risk aversion implies that consumption is *complementary* across dates and states. This relates to the work of Ossa (2015) which shows that sectors with low elasticities of substitution can play an important role in magnifiying static gains from trade.

Table 5: Average gains from trade under alternative parameter values

		Static gains			3-D gains					
		2019			Balanced trade			Complete markets		
η	ρ	All	Rich	Poor	All	Rich	Poor	All	Rich	Poor
1.5	0.25	3.21	3.59	3.11	3.89	3.64	3.96	4.50	3.77	4.70
3	0.25	1.28	1.33	1.27	1.37	1.34	1.38	1.68	1.39	1.75
4	0.25	1.18	1.21	1.17	1.28	1.22	1.30	1.72	1.27	1.84
5	0.25	1.13	1.15	1.13	1.25	1.16	1.28	1.87	1.23	2.04
6	0.25	1.10	1.12	1.10	1.25	1.13	1.28	2.07	1.21	2.30
4	1.5	1.18	1.21	1.17	1.18	1.21	1.17	1.13	1.22	1.11
4	0.5	1.18	1.21	1.17	1.21	1.21	1.20	1.34	1.24	1.37
4	0.25	1.18	1.21	1.17	1.28	1.22	1.30	1.72	1.27	1.84
4	0.125	1.18	1.21	1.17	1.54	1.22	1.63	3.10	1.27	3.59
4	0.1	1.18	1.21	1.17	1.59	1.22	1.69	3.78	1.28	4.45

Notes: For values of η which differ from the baseline, productivity is re-calculated and the productivity process is re-estimated. "Rich" refers to OECD countries plus the US. "Poor" refers to non-OECD countries. Sample includes the 156 countries (33 OECD, 123 non-OECD) for which data is available to construct productivity and trade costs in 2019. Appendix C lists countries included and OECD membership status.

8 Conclusion

Static multi-country models of international trade imply there are relatively modest gains associated with moving from autarky to the current level of trade openness. But static models ignore a potentially important role for international trade in allowing countries to smooth consumption across states and over time. In addition to the determinants of static gains from trade, the full extent of welfare gains from trade across states and over time therefore depends on the volatility and correlation of productivity growth across countries, and on whether their long-run growth trajectories differ. The size of gains also depends on whether there is endogenous risk sharing through movements in the terms of trade, as well as access to and functioning of international asset markets. Finally it depends on preferences over risk and intertemporal substitution.

I quantify the gains from trade taking into account gains from trade across states and over time as well as gains from trade within dates and states (3-D gains), under two special cases for international asset markets: balanced trade, and complete markets. To do so, I estimate trade costs and the process for productivity to match historical data on trade patterns and GDP.

For developing countries, which have productivity growth that is volatile and not very correlated with that of developed countries, I find substantial gains from trade across states. This is true even under balanced trade. In my calibration the goods produced by different countries are imperfect substitutes, so endogenous movements in the terms of trade provide some insurance even under balanced trade. Under complete markets, the 3-D gains from trade for developing countries are dramatically bigger than static gains. For developed countries, which have less volatile productivity and are more exposed to aggregate risk, gains from trade under balanced trade are very close to static gains. Even under complete markets, 3-D gains for these countries are only modestly bigger than static gains. I estimate growth trajectories to be similar across countries on average, so there is little scope for gains from trade over time.

My quantitative analysis abstracts from several potentially important considerations: features of the economy such as endogenous specialization, elastic labor supply, and capital accumulation which allow for risk mitigation and consumption smoothing in autarky; frictional international trade in assets; nonexpected utility preferences and disaster risk; departures from Pareto efficiency within countries. All of these considerations likely affect the exact size of 3-D gains from trade. Yet it remains true that in ignoring trade across states and over time, potential sources of gains from international trade are ruled out. My contribution is to highlight that these sources of gains may be quantitatively large.

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A Example: An endowment model with zero trade costs

A.1 Balanced trade

Let $Q_{t,t}^{k}(s^{t})$ be the price of 1 unit of the country-k intermediate good (in all countries) at date t after history s^t in terms of date-t "dollars" (freely traded). Then the price of 1 unit of the consumption good (in all countries) is given by:

$$P_{t,t}\left(s^{t}\right)=\left(\sum_{k=1}^{N}Q_{t,t}^{k}\left(s^{t}\right)^{1-\eta}\right)^{\frac{1}{1-\eta}}$$

If trade is balanced date-by-date and state-by-state, then $\forall t, s^t$:

$$P_{t,t}\left(s^{t}\right)C_{t}^{i}\left(s^{t}\right)=Q_{t,t}^{i}\left(s^{t}\right)Y_{t}^{i}\left(s^{t}\right)$$

Since consumption in autarky is given by the endowment, this implies the static gain from trade is given by:

$$\lambda_{t}^{i}\left(s^{t}\right)=\frac{C_{t}^{i}\left(s^{t}\right)}{Y_{t}^{i}\left(s^{t}\right)}=\frac{Q_{t,t}^{i}\left(s^{t}\right)}{P_{t,t}\left(s^{t}\right)}$$

Meanwhile demand of country k for i's good is:

$$C_{t}^{ik}\left(s^{t}\right)=\left(\frac{Q_{t,t}^{i}\left(s^{t}\right)}{P_{t,t}\left(s^{t}\right)}\right)^{-\eta}C_{t}^{k}\left(s^{t}\right)$$

and market clearing for good i implies:

$$Y_{t}^{i}\left(s^{t}\right)=\sum_{k=1}^{N}C_{t}^{ik}\left(s^{t}\right)=\left(\frac{Q_{t,t}^{i}\left(s^{t}\right)}{P_{t,t}\left(s^{t}\right)}\right)^{-\eta}\sum_{k=1}^{N}C_{t}^{k}\left(s^{t}\right)$$

so substituting in and rearranging,

$$\left(\frac{Q_{t,t}^{i}\left(s^{t}\right)}{P_{t,t}\left(s^{t}\right)}\right)^{-\eta}=\left(\frac{Y_{t}^{i}\left(s^{t}\right)^{\frac{\eta-1}{\eta}}}{\sum_{k=1}^{N}Y_{t}^{k}\left(s^{t}\right)^{\frac{\eta-1}{\eta}}}\right)^{\frac{\eta}{\eta-1}}$$

and therefore

$$\lambda_{t}^{i}\left(s^{t}\right)=\frac{Q_{t,t}^{i}\left(s^{t}\right)}{P_{t,t}\left(s^{t}\right)}=\left(\frac{Y_{t}^{i}\left(s^{t}\right)^{\frac{\eta-1}{\eta}}}{\sum_{k=1}^{N}Y_{t}^{k}\left(s^{t}\right)^{\frac{\eta-1}{\eta}}}\right)^{\frac{1}{1-\eta}}$$

A.2 Complete markets

The complete markets allocation coincides with the solution to the planner's problem, i.e.

$$\max_{\left\{C_t^i(s^t)\right\}} \sum_{i=1}^{N} \gamma^i \left(\sum_{t=0}^{\infty} \sum_{s^t} (1-\beta)\,\beta^t \pi_{0,t}\left(s^t\right) C_t^i\left(s^t\right)^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}}$$

subject to

$$C_t^i\left(s^t\right) = \left(\sum_{k=1}^{N} C_t^{ki}\left(s^t\right)^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}$$

and

$$\sum_{k=1}^{N} C_t^{ik}\left(s^t\right) \leq Y_t^i\left(s^t\right)$$

This is satisfied by

$$C_t^i\left(s^t\right) = \gamma^i \left(\sum_{k=1}^{N} Y_t^k\left(s^t\right)^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} = \gamma^i C_t^{world}\left(s^t\right)$$

The values of γ^i consistent with the competitive equilibrium can be obtained by substituting the consumption allocation into the ex-ante budget constraint for country i:

$$\gamma^i = \frac{\sum_{t=0}^{\infty}\sum_{s^t} Q_{0,t}^i\left(s^t\right) Y_t^i\left(s^t\right)}{\sum_{t=0}^{\infty}\sum_{s^t} P_{0,t}\left(s^t\right) C_t^{world}\left(s^t\right)} = \frac{\sum_{t=0}^{\infty}\sum_{s^t} Q_{0,t}^i\left(s^t\right) Y_t^i\left(s^t\right)}{\sum_{t=0}^{\infty}\sum_{s^t}\sum_{k=1}^{N} Q_{0,t}^k\left(s^t\right) Y_t^k\left(s^t\right)}$$

where $Q_{0,t}^i\left(s^t\right)$ and $P_{0,t}\left(s^t\right)$ are prices from a date-0 perspective i.e. in terms of date-0 dollars. Notice that this implies $\gamma^i \in (0,1)$ and $\sum_{k=1}^{N}\gamma^k = 1$. Given homotheticity of preferences, prices are given by the marginal utility of an agent consuming world output:

$$Q_{0,t}^k\left(s^t\right) = (1-\beta)\,\beta^t \pi_{0,t}\left(s^t\right) \left(\frac{Y_t^k\left(s^t\right)}{C_t^{world}\left(s^t\right)} \right)^{-\frac{1}{\eta}} \left(\frac{C_t^{world}\left(s^t\right)}{U_0^{world}} \right)^{-\frac{1}{\rho}}$$

with

$$P_{0,t}\left(s^t\right) = \left(\sum_{k=1}^{N} Q_{0,t}^k\left(s^t\right)^{1-\eta} \right)^{\frac{1}{1-\eta}}$$

so

$$P_{0,t}\left(s^t\right) = (1-\beta)\,\beta^t \pi_{0,t}\left(s^t\right) \left(\frac{C_t^{world}\left(s^t\right)}{U_0^{world}} \right)^{-\frac{1}{\rho}}$$

Substituting in these prices implies:

$$\gamma^i = \frac{\sum_{t=0}^{\infty}\sum_{s^t}(1-\beta)\beta^t\pi_{0,t}(s^t)\left(\sum_{k=1}^{N}Y_t^k(s^t)^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}\left(\frac{1}{\eta}-\frac{1}{\rho}\right)}Y_t^i(s^t)^{\frac{\eta-1}{\eta}}}{\sum_{t=0}^{\infty}\sum_{s^t}(1-\beta)\beta^t\pi_{0,t}(s^t)\left(\sum_{k=1}^{N}Y_t^k(s^t)^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}\frac{\rho-1}{\rho}}}$$

B Derivation of second order welfare formula

WLOG the budget constraint for country i can be written:

$$\Pi_0^i U_0^i = B_0^i + \sum_{t=0}^{\infty}\sum_{s^t}(1-\mu)\mu^{\frac{\mu}{1-\mu}}A_t^i(s^t)^{\frac{1}{1-\mu}}L^i Q_{0,t}^i(s^t)^{\frac{1}{1-\mu}}P_{0,t}^i(s^t)^{-\frac{\mu}{1-\mu}}$$

where $P_{0,t}^i(s^t)$ and $Q_{0,t}^i(s^t)$ are (shadow) prices from the perspective of country i in terms of date-0 dollars of $C_t^i(s^t)$ and $Y_t^i(s^t)$ respectively. Under complete markets, all countries face the same relative price vector. Under arbitrary frictions in asset markets, the budget constraint can be written in this way using the appropriate marginal utilities in the equilibrium allocation as the shadow prices. Π_0^i is the country-i price of U_0^i in terms of date-0 dollars. Assume B_0^i. Normalize $\Pi_0^i = 1$ (this pins down what a dollar is). Taking a second order approximation of the budget constraint yields:

$$\begin{aligned} U_0^i d\ln U_0^i \;=\; & \sum_{t=0}^{\infty}\sum_{s^t}(1-\mu)Q_{0,t}^i(s^t)Y_t^i(s^t)\left[\frac{d\ln Q_{0,t}^i(s^t)}{1-\mu}-\frac{\mu d\ln P_{0,t}^i(s^t)}{1-\mu}+\right.\\ & \left.\frac{1}{2}\left(\frac{d\ln Q_{0,t}^i(s^t)^2}{(1-\mu)^2}-\frac{2\mu d\ln Q_{0,t}^i(s^t)\,d\ln P_{0,t}^i(s^t)}{(1-\mu)^2}+\frac{\mu^2 d\ln P_{0,t}^i(s^t)^2}{(1-\mu)^2}\right)\right]\end{aligned}$$

Define

$$\omega_{0,t}^{ki}(s^t) = \frac{\tau_t^{ki}Q_{0,t}^k(s^t)Z_t^{ki}(s^t)}{P_{0,t}^i(s^t)X_t^i(s^t)} = \left(\frac{\tau_t^{ki}Q_{0,t}^k(s^t)}{P_{0,t}^i(s^t)}\right)^{1-\eta}$$

so

$$d\ln\omega_{0,t}^{ii}(s^t) = (1-\eta)\left(d\ln Q_{0,t}^i(s^t) - d\ln P_{0,t}^i(s^t)\right)$$

Rearrange:

$$d\ln Q_{0,t}^i(s^t) = \frac{d\ln\omega_{0,t}^{ii}(s^t)}{1-\eta} + d\ln P_{0,t}^i(s^t)$$

Define

$$\phi_{0,t}^i(s^t) = \frac{P_{0,t}^i(s^t)C_{0,t}^i(s^t)}{\Pi_0^i U_0^i} = \left(\left((1-\beta)\beta^t\right)\pi_{0,t}(s^t)\right)^{\rho}\left(\frac{P_{0,t}^i(s^t)}{\Pi_0^i}\right)^{1-\rho}$$

so since $\Pi_0^i = 1$, we get

$$d \ln P_{0,t}^i \left(s^t\right) = \frac{d \ln \phi_{0,t}^i \left(s^t\right)}{(1-\rho)}$$

and therefore also

$$d \ln Q_{0,t}^i \left(s^t\right) = \frac{d \ln \omega_{0,t}^{ii} \left(s^t\right)}{(1-\eta)} + \frac{d \ln \phi_{0,t}^i \left(s^t\right)}{(1-\rho)}$$

Define

$$\theta_{0,t}^i \left(s^t\right) = \frac{(1-\mu) \, Q_{0,t}^i \left(s^t\right) Y_t^i \left(s^t\right)}{\Pi_0^i U_0^i}$$

Then

$$\begin{aligned} d \ln U_0^i \;&=\; \sum_{t=0}^{\infty} \sum_{s^t} \theta_{0,t}^i \left(s^t\right) \left[\frac{d \ln \omega_{0,t}^{ii} \left(s^t\right)}{(1-\mu)(1-\eta)} + \frac{d \ln \phi_{0,t}^i \left(s^t\right)}{(1-\rho)} + \right. \\ & \left. \frac{1}{2} \left(\left(\frac{d \ln \omega_{0,t}^{ii} \left(s^t\right)}{(1-\mu)(1-\eta)} \right)^2 + 2 \frac{d \ln \omega_{0,t}^{ii} \left(s^t\right)}{(1-\mu)(1-\eta)} \frac{d \ln \phi_{0,t}^i \left(s^t\right)}{(1-\rho)} + \left(\frac{d \ln \phi_{0,t}^i \left(s^t\right)}{(1-\rho)} \right)^2 \right) \right] \end{aligned}$$

Higher order approximations can be derived analogously.

C Data sources and coverage

Tables 6, 7, 8, 9 and 10 list the countries in the sample used to recover productivity, along with the dates for which data is available. They also note whether the country is present in the balanced panel used to estimate the productivity process, along with OECD membership. If data is available for 2019, the country is part of the sample for which welfare gains are calculated. The final two columns note whether, and for which period, balance of payments (rather than national accounts) data is used to measure total imports and exports, and whether OECD is used instead of WDI as the source for national income accounts.

Table 6: Data coverage I

Country	2019	Balanced	OECD	Years	BOP	OECD
Albania	Yes	No	No	1984-2019		
Algeria	Yes	Yes	No	1970-2019		
Angola	Yes	No	No	1993-2019	1993-2019	
Argentina	Yes	No	No	1983-2019		
Armenia	Yes	No	No	1992-2019		
Aruba	Yes	No	No	1995-2019		
Australia	Yes	Yes	Yes	1970-2019		
Austria	Yes	Yes	Yes	1970-2019		
Azerbaijan	Yes	No	No	1992-2019		
Bahamas	Yes	No	No	1977-2019	1988	
Bahrain	Yes	No	No	1980-2019		
Bangladesh	Yes	No	No	1972-2019		
Barbados	Yes	No	No	1975-2019		
Belarus	Yes	No	No	1992-2019		
Belgium	Yes	Yes	Yes	1970-2019		
Belize	Yes	No	No	1980-2019		
Benin	Yes	Yes	No	1970-2019		
Bermuda	Yes	No	No	2010-2019		
Bhutan	Yes	No	No	1995-2019		
Bolivia	Yes	Yes	No	1970-2019		
Bosnia & Herzegovina	Yes	No	No	1994-2019		
Botswana	Yes	No	No	2000-2019		
Brazil	Yes	No	No	1989-2019		
Brunei	Yes	No	No	1995-2019		
Bulgaria	Yes	No	No	1980-2019		
Burkina Faso	Yes	Yes	No	1970-2019		
Burundi	Yes	Yes	No	1970-2019		
Cabo Verde	Yes	No	No	1995-2019		
Cambodia	Yes	No	No	1993-2019		
Cameroon	Yes	Yes	No	1970-2019		
Canada	Yes	Yes	Yes	1970-2019		
Central African Republic	Yes	Yes	No	1970-2019		
Chad	Yes	Yes	No	1970-2019		

Table 7: Data coverage II

Country	2019	Balanced	OECD	Years	BOP	OECD
Chile	Yes	Yes	Yes	1970-2019		
China	Yes	Yes	No	1970-2019		
Colombia	Yes	Yes	No	1970-2019		
Comoros	Yes	No	No	1995-2019		
Congo, Dem Rep	Yes	No	No	1994-2019		
Congo	Yes	Yes	No	1970-2019		
Costa Rica	Yes	Yes	No	1970-2019		
Cote d'Ivoire	Yes	Yes	No	1970-2019		
Croatia	Yes	No	No	1995-2019		
Cyprus	Yes	No	No	1975-2019		
Czech Republic	Yes	No	Yes	1993-2019		
Denmark	Yes	Yes	Yes	1970-2019		
Dominica	No	No	No	2006-2018		
Dominican Republic	Yes	Yes	No	1970-2019		
Ecuador	Yes	Yes	No	1970-2019		
Egypt	Yes	Yes	No	1970-2019		
El Salvador	Yes	Yes	No	1970-2019		
Equatorial Guinea	Yes	No	No	2005-2019		
Estonia	Yes	No	Yes	1993-2019		yes
Eswatini	Yes	No	No	2000-2019		
Ethiopia	Yes	No	No	1977-2019	1970-2019	
Fiji	Yes	No	No	1980-2019		
Finland	Yes	Yes	Yes	1970-2019		
France	Yes	Yes	Yes	1970-2019		
Gabon	Yes	Yes	No	1970-2019		
Gambia	Yes	Yes	No	1970-2019		
Georgia	Yes	No	No	1992-2019		
Germany[11]	Yes	Yes	Yes	1970-2019		yes[12]
Ghana	Yes	Yes	No	1970-2019		
Greece	Yes	Yes	Yes	1970-2019		
Guatemala	Yes	Yes	No	1970-2019		
Guinea	Yes	No	No	1986-2019		
Guinea-Bissau	Yes	Yes	No	1970-2019		

[11] West Germany 1970-1990.
[12] Population for West Germany 1970-1990 is from PWT 5.6.

Table 8: Data coverage III

Country	2019	Balanced	OECD	Years	BOP	OECD
Guyana	No	No	No	1970-2005		
Haiti	Yes	No	No	1988-2019		
Honduras	Yes	Yes	No	1970-2019		
Hungary	Yes	No	Yes	1991-2019		
Iceland	Yes	Yes	Yes	1970-2019		
India	Yes	Yes	No	1970-2019		
Indonesia	Yes	Yes	No	1970-2019		
Iran	Yes	No	No	1970-2019[13]		
Iraq	Yes	No[14]	No	1970-2019		
Ireland	Yes	Yes	Yes	1970-2019		
Israel	Yes	Yes	Yes	1970-2019		yes
Italy	Yes	Yes	Yes	1970-2019		
Jamaica	Yes	Yes	No	1970-2019		
Japan	Yes	Yes	No	1970-2019		
Jordan	Yes	No	No	1976-2019		
Kazakhstan	Yes	No	No	1992-2019		
Kenya	Yes	Yes	No	1970-2019		
Korea	Yes	Yes	Yes	1970-2019		
Kuwait	Yes	Yes	No	1970-2019		
Kyrgyz Republic	Yes	No	No	1992-2019		
Laos	No	No	No	1984-2016		
Latvia	Yes	No	No	1995-2019		
Lebanon	Yes	No	No	1989-2019		
Lesotho	Yes	No	No	2007-2019		
Lithuania	Yes	No	No	1995-2019		
Macao	Yes	No	No	1982-2019		
Madagascar	Yes	Yes	No	1970-2019		
Malawi	Yes	No	No	1980-2019	1980-2019	
Malaysia	Yes	Yes	No	1970-2019		
Mali	Yes	Yes	No	1970-2019		
Malta	Yes	Yes	No	1970-2019		
Mauritania	Yes	Yes	No	1970-2019		
Mauritius	Yes	No	No	1976-2019		

[13] Data is missing for 1991 and 1992.
[14] Iraq is not included in the estimation sample, though 50 years of data are available.

Table 9: Data coverage IV

Country	2019	Balanced	OECD	Years	BOP	OECD
Mexico	Yes	Yes	Yes	1970-2019		
Moldova	Yes	No	No	1995-2019		
Mongolia	Yes	No	No	1981-2019		
Montenegro	Yes	No	No	2006-2019		
Morocco	Yes	Yes	No	1970-2019		
Mozambique	Yes	No	No	1991-2019	1991-2019	
Namibia	Yes	No	No	2000-2019		
Nepal	Yes	Yes	No	1970-2019		
Netherlands	Yes	Yes	Yes	1970-2019		
New Zealand	Yes	Yes	Yes	1970-2019		yes
Nicaragua	Yes	No	No	1988-2019		
Niger	Yes	Yes	No	1970-2019		
Nigeria	Yes	No	No	1977-2019	1977-2019	
North Macedonia	Yes	No	No	1993-2019		
Norway	Yes	Yes	Yes	1970-2019		
Oman	Yes	Yes	No	1970-2019	1989	
Pakistan	Yes	Yes	No	1970-2019		
Panama	Yes	Yes	No	1970-2019	1970-2019	
Paraguay	Yes	Yes	No	1970-2019		
Peru	Yes	No	No	1982-2019		
Philippines	Yes	No	No	1981-2019		
Poland	Yes	No	Yes	1995-2019		
Portugal	Yes	Yes	Yes	1970-2019		
Qatar	Yes	No	No	1994-2019		
Romania	Yes	No	No	1990-2019		
Russia	Yes	No	No	1995-2019		
Rwanda	Yes	Yes	No	1970-2019		
Saudi Arabia	Yes	Yes	No	1970-2019		
Senegal	Yes	Yes	No	1970-2019		
Serbia	Yes	No	No	2006-2019		
Serbia & Montenegro	No	No	No	1995-2005		
Seychelles	Yes	No	No	1976-2019		
Sierra Leone	Yes	Yes	No	1970-2019		

Table 10: Data coverage V

Country	2019	Balanced	OECD	Years	BOP	OECD
Slovak Republic	Yes	No	Yes	1993-2019		
Slovenia	Yes	No	Yes	1992-2019		yes
South Africa	Yes	Yes	No	1970-2019		
Spain	Yes	Yes	Yes	1970-2019		
Sri Lanka	Yes	Yes	No	1970-2019	yes	
Sudan	Yes	Yes	No	1970-2019		
Sweden	Yes	Yes	Yes	1970-2019		
Switzerland	Yes	Yes	Yes	1970-2019		yes
Syria	Yes	Yes	No	1970-2019		
Taiwan[15]	Yes	Yes	No	1970-2019		
Tajikistan	Yes	No	No	1993-2019		
Tanzania	Yes	No	No	1990-2019		
Thailand	Yes	Yes	No	1970-2019		
Togo	Yes	Yes	No	1970-2019		
Tunisia	Yes	Yes	No	1970-2019		
Turkey	Yes	Yes	Yes	1970-2019		
Turkmenistan	Yes	No	No	1992-2019		
Uganda	Yes	Yes	No	1970-2019		
Ukraine	Yes	No	No	1992-2019		
UAE	Yes	No	No	2001-2019		
UK	Yes	Yes	Yes	1970-2019		
USA	Yes	Yes	Yes	1970-2019		
Uruguay	Yes	Yes	No	1970-2019		
Uzbekistan	Yes	No	No	1997-2019		
Venezuela	No	No	No	1970-2014		
Vietnam	Yes	No	No	1986-2019		
West Bank & Gaza	Yes	No	No	2000-2019		
Yemen	No	No	No	1991-2018		
Zambia	Yes	No	No	1994-2019		
Zimbabwe	Yes	No	No	1975-2019		

[15] National income accounts data taken from the Taiwanese statistical agency.

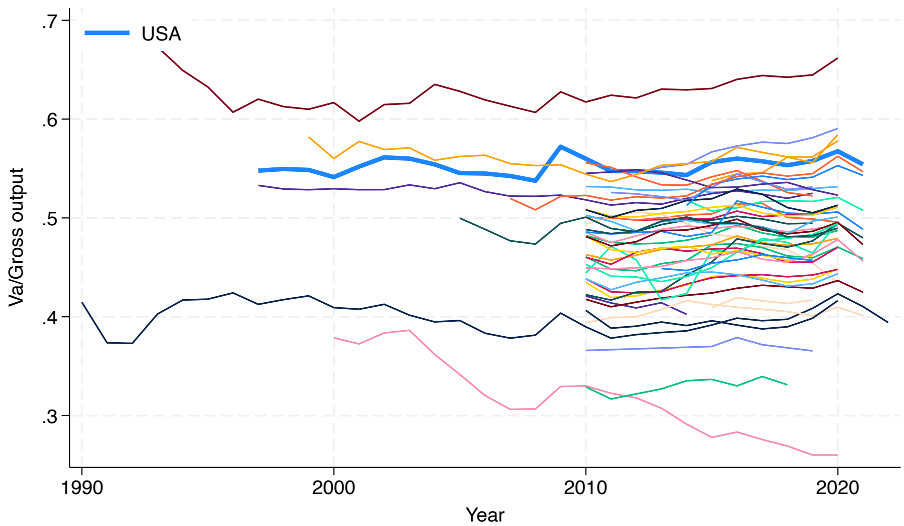

Figure 7: Ratio of GDP to Gross Output

Source: Figure plots ratio of GDP to Gross Output for countries and years for which at least 5 years of Gross Output data is available in OECD National Accounts. Countries included: Australia, Austria, Belgium, Bulgaria, Brazil, Canada, Cameroon, Colombia, Cape Verde, Costa Rica, Cyprus, Czechia, Germany, Denmark, Spain, Estonia, Finland, France, UK, Greece, Croatia, Hungary, Ireland, Italy, Japan, Korea, Morocco, North Macedonia, Malta, Netherlands, Norway, Peru, Poland, Portugal, Romania, Senegal, Singapore, Serbia, Slovak Republic, Slovenia, Sweden, South Africa, USA.

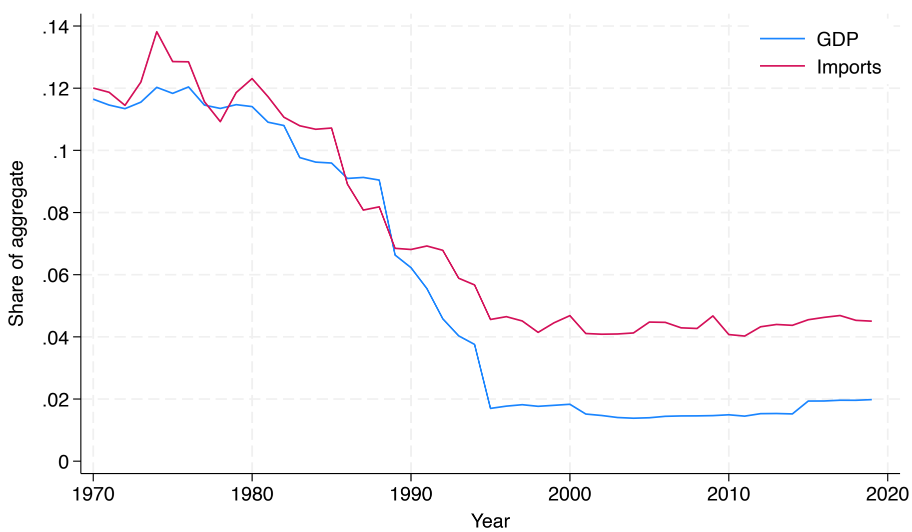

Figure 8: ROW share in World GDP and in-sample imports

Notes: Figure plots share of out-of-sample countries in world GDP, and share of imports from out-of-sample countries in total in-sample country imports.

D Measurement of productivity and trade costs

D.1 Baseline measure of productivity

As noted in the paper, the trade cost wedges, $\hat{\tau}^{ik}$, that exactly match bilateral expenditure shares and expenditure PPPs are not always consistent with positive trade costs. Figure 9 plots a time series of the share of bilateral pairs $i \neq k$ with negative trade costs ($\hat{\tau}^{ik} < 1$) under the baseline parameters. The baseline measure of productivity is constructed under the assumption that any negative trade costs are due to mismeasurement of expenditure PPPs. Expenditure prices are adjusted such that $\tilde{\tau}^{ik} \geq 1 \; \forall i, k$. Figure 10 plots a time series of the share of expenditure PPPs which are adjusted. Figure 11 shows a scatter plot of the resulting adjusted expenditure prices (y-axis) against measured expenditure PPPs (x-axis).

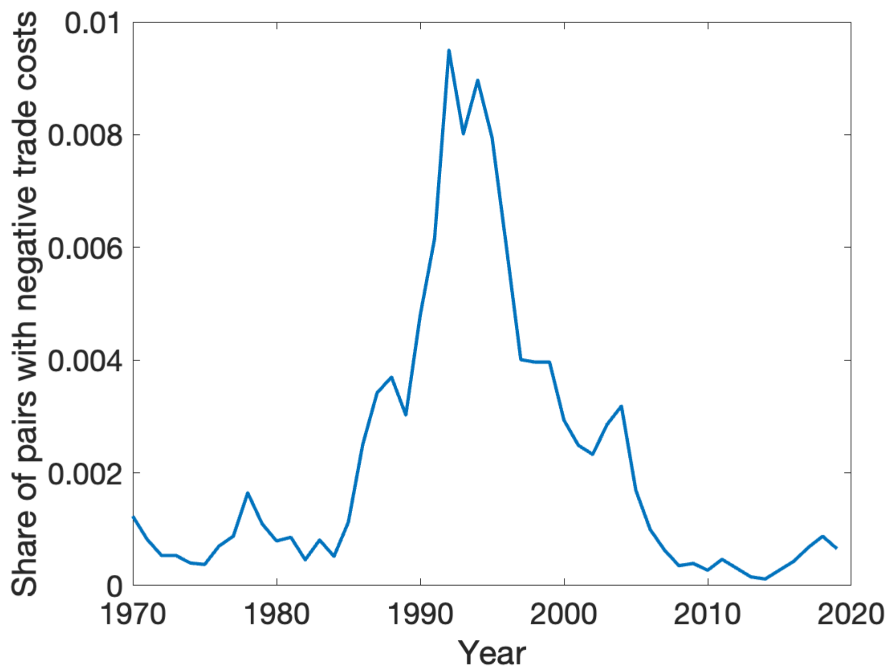

Figure 9: Share of bilateral pairs with negative trade costs under baseline μ, η
Notes: Figure plots share of pairs i, k with $i \neq k$ for which $\hat{\tau}_t^{ik} < 1$ under baseline parameterization ($\mu = 0.5$, $\eta = 4$).

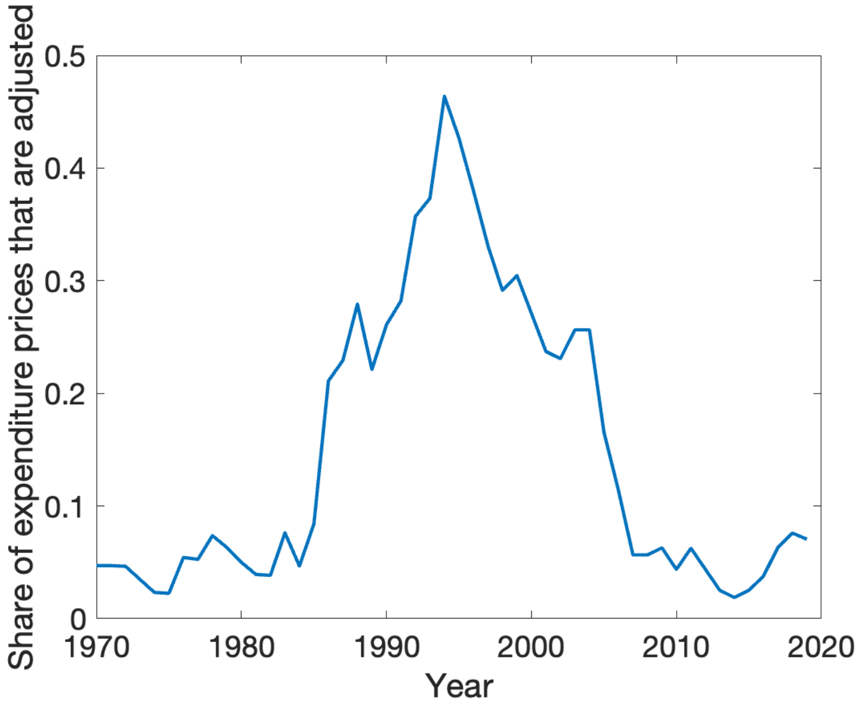

Figure 10: Share of expenditure prices adjusted under baseline μ, η

Notes: Figure plots share of expenditure prices $\hat{P}_t^i$ which must be adjusted to guarantee $\tilde{\tau}_t^{ki} \geq 1 \; \forall k, i$ using baseline parameterization ($\mu = 0.5$, $\eta = 4$) and baseline approach to guaranteeing positive trade costs.

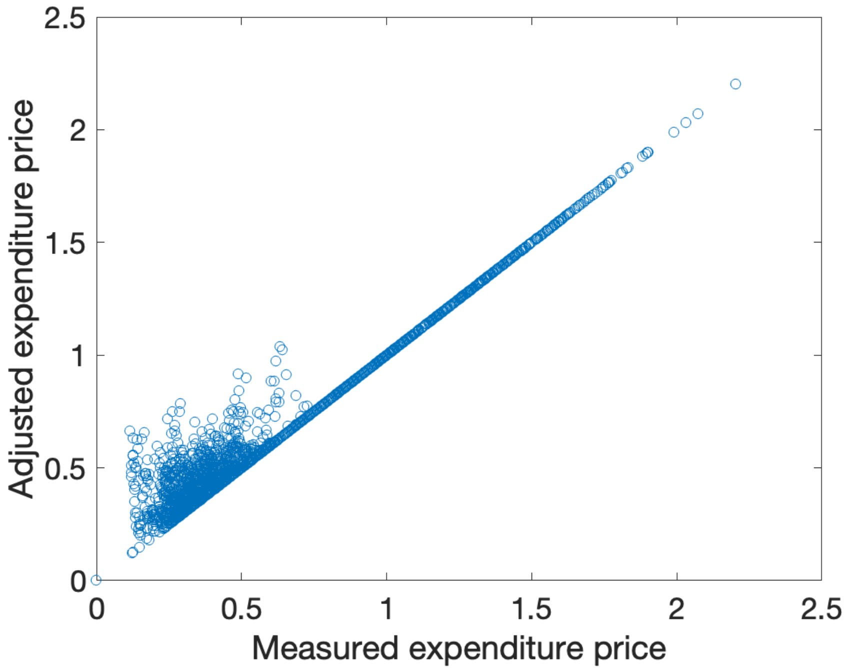

Figure 11: Measured and adjusted expenditure prices under baseline μ, η

Notes: Figure plots adjusted expenditure prices $\tilde{P}_t^i$ (adjusted to guarantee positive trade costs) against measured expenditure prices $\hat{P}_t^i$ for the baseline parameterization ($\mu = 0.5$, $\eta = 4$).

Table 11: Relationship between $\left(\tilde{\tau}_t^{ik}\right)^{1-\eta}$ and gravity variables

	(1)		(2)	
	coeff	s.e.	coeff	s.e.
log distance	-1.15	(0.01)**	-1.00	(0.01)**
contiguous			0.32	(0.03)**
common language			0.42	(0.02)**
colony			0.74	(0.02)**
common colonizer			0.20	(0.02)**
N	880,432		880,292	
importer & exporter f.e.	yes		yes	

Notes: Regression of baseline values of $\left(\tilde{\tau}_t^{ik}\right)^{1-\eta}$ against gravity variables. Estimated by Poisson Pseudo-Maximum Likelihood. Does not include observations where $i = k$.

D.2 Alternative measure of productivity I

I construct an alternative measure of productivity and trade costs under the assumption that any negative trade costs ($\hat{\tau}^{ik} < 1$) implied by

$$\left(\hat{\tau}_t^{ki}\right)^{1-\eta} = \frac{IM_t^{ki}}{ABS_t^i} \frac{ABS_t^k}{IM_t^{kk}} \left(\frac{\hat{P}_t^i}{\hat{P}_t^k}\right)^{1-\eta}$$

are due to mismeasurement of bilateral imports. I impose a floor of 1 on $\tilde{\tau}^{ki}$, but do not adjust expenditure prices. Output prices are then recovered as the solution to the system of equations given by ($\forall i = 1, \ldots, N$):

$$\left(\hat{P}_t^i\right)^{1-\eta} = \sum_{k=1}^{N} \left(\tilde{\tau}_t^{ki}\right)^{1-\eta} \left(\hat{Q}_t^k\right)^{1-\eta}$$

Recovering $\hat{Q}_t^k$ in this way is inconsistent with:

$$\left(Q_t^k\right)^{1-\eta} = \frac{IM_t^{kk}}{ABS_t^k} \left(P_t^k\right)^{1-\eta}$$

As a result, the expenditure shares implied by $\tilde{\tau}_t^{ki}$ and $\hat{Q}_t^k$ do not exactly match measured expenditure shares. This is true not just for the ki-pairs for which I set $\tilde{\tau}_t^{ki} = 1$. Because the vector of output prices $\hat{Q}_t^k$ depends on all $\tilde{\tau}_t^{ki}$, all pairs (except pairs where $IM^{ki} = 0$) are affected to some extent:

$$\left(\frac{\tilde{\tau}_t^{ki} \hat{Q}_t^k}{\sum_{j=1}^{N} \left(\tilde{\tau}_t^{ji}\right)^{1-\eta} \left(\hat{Q}_t^j\right)^{1-\eta}}\right)^{1-\eta} \neq \frac{IM_t^{ki}}{ABS_t^i}$$

Figure 12 shows a scatter plot of the implied expenditure shares (y-axis) against measured expenditure shares (x-axis).

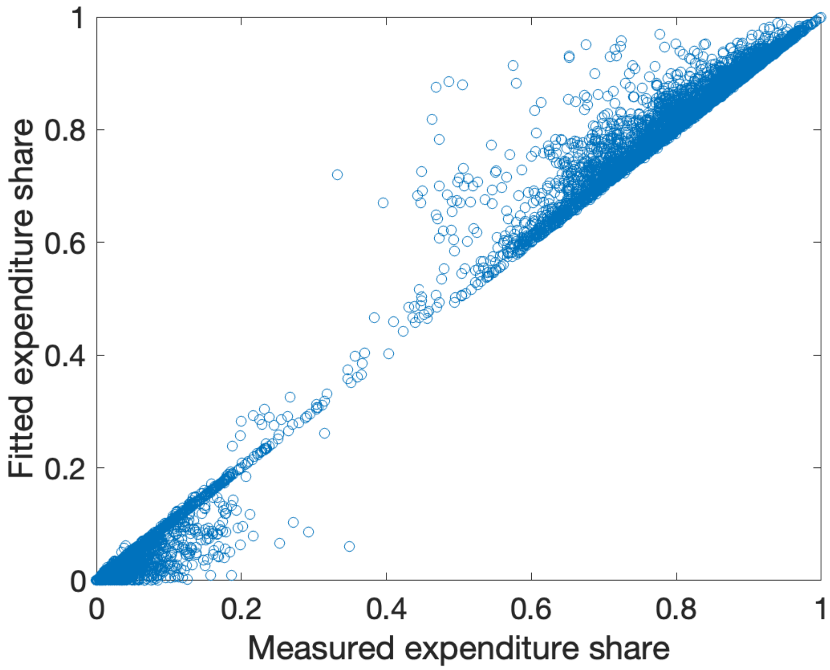

Figure 12: Measured and adjusted expenditure shares under baseline μ, η

Notes: Figure plots predicted expenditure share (y-axis) against measured expenditure share (x-axis) when bilateral trade costs are set to $\left(\tilde{\tau}_t^{ki}\right)^{1-\eta} = \min\left\{\left(\hat{\tau}_t^{ki}\right)^{1-\eta}, 1\right\}$.

D.3 Alternative measure of productivity II

The second alternative measure of productivity is real GDP (output price, "rgdpo") per capita from the Penn World Tables 10.01.

E Estimation of the productivity process

Let $\mathbf{a}_t = \left[\ln A_t^{US}, A_t^{O_1}, \ldots, A_t^{O_{N_O}}, A_t^{N_1}, \ldots, A_t^{N_{N_N}}\right]'$ be the vector of log productivity for all countries in the estimation sample at date t. The first country is the US, then N_O OECD countries, followed by N_N non-OECD countries. The productivity process to be estimated can be written:

$$(\mathbf{a}_t - \mathbf{a}_{t-1}) = \mathbf{g} + \Gamma \mathbf{a}_{t-1} + \boldsymbol{\varepsilon}_t$$

where

$$\boldsymbol{\varepsilon}_t = \Psi \boldsymbol{\varepsilon}_{t-1} + \Omega \boldsymbol{\eta}_t$$

and

$$\mathbf{g} = \begin{bmatrix} g^{US} \\ \begin{bmatrix} g^{US} \\ \vdots \\ g^{US} \end{bmatrix} \\ \begin{bmatrix} g^{US} \\ \vdots \\ g^{US} \end{bmatrix} \end{bmatrix}$$

$$\Gamma = \begin{bmatrix} [0] & \begin{bmatrix} 0 & \dots & 0 \end{bmatrix} & \begin{bmatrix} 0 & \dots & 0 \end{bmatrix} \\ \begin{bmatrix} \gamma^O \\ \vdots \\ \gamma^O \end{bmatrix} & \begin{bmatrix} -\gamma^O & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & -\gamma^O \end{bmatrix} & \begin{bmatrix} 0 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & 0 \end{bmatrix} \\ \begin{bmatrix} \gamma^N \\ \vdots \\ \gamma^N \end{bmatrix} & \begin{bmatrix} 0 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & 0 \end{bmatrix} & \begin{bmatrix} -\gamma^N & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & -\gamma^N \end{bmatrix} \end{bmatrix}$$

$$\Psi = \begin{bmatrix} [\psi^{US}] & \begin{bmatrix} 0 & \dots & 0 \end{bmatrix} & \begin{bmatrix} 0 & \dots & 0 \end{bmatrix} \\ \begin{bmatrix} \psi_O^{US} \\ \vdots \\ \psi_O^{US} \end{bmatrix} & \begin{bmatrix} \psi_O^O & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \psi_O^O \end{bmatrix} & \begin{bmatrix} 0 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & 0 \end{bmatrix} \\ \begin{bmatrix} \psi_N^{US} \\ \vdots \\ \psi_N^{US} \end{bmatrix} & \begin{bmatrix} 0 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & 0 \end{bmatrix} & \begin{bmatrix} \psi_N^N & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \psi_N^N \end{bmatrix} \end{bmatrix}$$

$$\Omega = \begin{bmatrix} [\sigma^{US}] & \begin{bmatrix} 0 & \dots & 0 \end{bmatrix} & \begin{bmatrix} 0 & \dots & 0 \end{bmatrix} \\ \begin{bmatrix} \sigma_O^{US} \\ \vdots \\ \sigma_O^{US} \end{bmatrix} & \begin{bmatrix} \sigma_O^O & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \sigma_O^O \end{bmatrix} & \begin{bmatrix} 0 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & 0 \end{bmatrix} \\ \begin{bmatrix} \sigma_N^{US} \\ \vdots \\ \sigma_N^{US} \end{bmatrix} & \begin{bmatrix} 0 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & 0 \end{bmatrix} & \begin{bmatrix} \sigma_N^N & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \sigma_N^N \end{bmatrix} \end{bmatrix}$$

Condition on $\mathbf{a}_1$. Then at $t = 2$:

$$(\mathbf{a}_2 - \mathbf{a}_1) = \mathbf{g} + \Gamma \mathbf{a}_1 + \boldsymbol{\varepsilon}_2$$

and

$$\boldsymbol{\varepsilon}_2 = \Psi\boldsymbol{\varepsilon}_1 + \Omega\boldsymbol{\eta}_2$$

so

$$(\mathbf{a}_2 - \mathbf{a}_1) = \mathbf{g} + \Gamma\mathbf{a}_1 + \Psi\boldsymbol{\varepsilon}_1 + \Omega\boldsymbol{\eta}_2$$

We don't observe $\boldsymbol{\varepsilon}_1$, but we do know its unconditional distribution:

$$\boldsymbol{\varepsilon}_1 \sim N\left(0, (I-\Psi)^{-1}\Omega\Omega'\left((I-\Psi)^{-1}\right)'\right).$$

Meanwhile, $\boldsymbol{\varepsilon}_1$ and $\boldsymbol{\eta}_2$ are independent. So

$$(\mathbf{a}_2 - \mathbf{a}_1)|\mathbf{a}_1 \sim N\left(\mathbf{g} + \Gamma\mathbf{a}_1, \Omega\Omega' + \Psi(I-\Psi)^{-1}\Omega\Omega'\left((I-\Psi)^{-1}\right)'\Psi'\right).$$

In general, for $t > 2$

$$(\mathbf{a}_t - \mathbf{a}_{t-1}) = \mathbf{g} + \Gamma\mathbf{a}_{t-1} + \boldsymbol{\varepsilon}_t$$

while

$$\boldsymbol{\varepsilon}_t = \Psi\boldsymbol{\varepsilon}_{t-1} + \Omega\boldsymbol{\eta}_t$$

and

$$\boldsymbol{\varepsilon}_{t-1} = (\mathbf{a}_{t-1} - \mathbf{a}_{t-2}) - \mathbf{g} - \Gamma\mathbf{a}_{t-2}$$

so

$$(\mathbf{a}_t - \mathbf{a}_{t-1}) = (I-\Psi)\mathbf{g} + (\Gamma+\Psi)\mathbf{a}_{t-1} - \Psi(I+\Gamma)\mathbf{a}_{t-2} + \Omega\boldsymbol{\eta}_t$$

and

$$(\mathbf{a}_t - \mathbf{a}_{t-1})|\mathbf{a}_1, \ldots, \mathbf{a}_{t-1} \sim N\left((I-\Psi)\mathbf{g} + (\Gamma+\Psi)\mathbf{a}_{t-1} - \Psi(I+\Gamma)\mathbf{a}_{t-2}, \Omega\Omega'\right)$$

Let $\boldsymbol{\theta} = \{\mathbf{g}, \Gamma, \Psi, \Omega\}$.The log likelihood conditional on $\mathbf{a}_1$ is given by:

$$L(\Delta\mathbf{a}_T, \ldots, \Delta\mathbf{a}_2|\mathbf{a}_1; \boldsymbol{\theta}) = -\sum_{t=2}^{T}\frac{1}{2}\left[\begin{array}{c}(\Delta\mathbf{a}_t - \boldsymbol{\mu}_t(\mathbf{a}_1, \ldots, \mathbf{a}_{t-1}; \boldsymbol{\theta}))'\Sigma_t(\boldsymbol{\theta})^{-1}(\Delta\mathbf{a}_t - \boldsymbol{\mu}_t(\mathbf{a}_1, \ldots, \mathbf{a}_{t-1}; \boldsymbol{\theta})) \\ -\ln\left(\left|\Sigma_t(\boldsymbol{\theta})^{-1}\right|\right) + N_t\ln(2\pi)\end{array}\right]$$

with

$$\boldsymbol{\mu}_t(\mathbf{a}_1, \ldots, \mathbf{a}_{t-1}; \boldsymbol{\theta}) = \begin{cases}\mathbf{g} + \Gamma\mathbf{a}_1 & t = 2 \\ (I-\Psi)\mathbf{g} + (\Gamma+\Psi)\mathbf{a}_{t-1} - \Psi(I+\Gamma)\mathbf{a}_{t-2} & t > 2\end{cases}$$

and

$$\Sigma_t(\boldsymbol{\theta}) = \begin{cases} \Omega\Omega' + \Psi (I-\Psi)^{-1} \Omega\Omega' \left((I-\Psi)^{-1}\right)' \Psi' & t=2 \\ \Omega\Omega' & t>2 \end{cases}$$

The idea is to choose $\boldsymbol{\theta} = \{\mathbf{g}, \Gamma, \Psi, \Omega\}$ to maximize the log likelihood $L(\Delta\mathbf{a}_T, \ldots, \Delta\mathbf{a}_2|\mathbf{a}_1; \boldsymbol{\theta})$. This is done in two steps. First, the parameters $\boldsymbol{\theta}^{US} = \left\{g^{US}, \psi^{US}, \sigma^{US}\right\}$ are chosen to maximize $L\left(\Delta a_T^{US}, \ldots, \Delta a_2|a_1; \boldsymbol{\theta}^{US}\right)$. Then the remaining parameters $\left\{\boldsymbol{\theta}^O, \boldsymbol{\theta}^N\right\}$ are chosen to maximize the log likelihood conditional on the estimates of $\boldsymbol{\theta}^{US}$. That is, what is maximized is $L\left(\Delta\mathbf{a}_T^{nonUS}, \ldots, \Delta\mathbf{a}_2^{nonUS}|\mathbf{a}_1^{nonUS}, \boldsymbol{\theta}^{US}; \boldsymbol{\theta}^O, \boldsymbol{\theta}^N\right)$.

E.1 Bootstrapped standard errors

I bootstrap standard errors for $\left\{\boldsymbol{\theta}^{US}, \boldsymbol{\theta}^O, \boldsymbol{\theta}^N\right\}$ as follows. Given the fitted values $\left\{\hat{\boldsymbol{\theta}}^{US}, \hat{\boldsymbol{\theta}}^O, \hat{\boldsymbol{\theta}}^N\right\}$ and $\{\mathbf{a}_1, \mathbf{a}_2\}$, I calculate

$$\hat{\boldsymbol{\varepsilon}}_t = (\mathbf{a}_t - \mathbf{a}_{t-1}) - \hat{\mathbf{g}} - \hat{\Gamma}\mathbf{a}_{t-1}$$

for $t \geq 2$, and

$$\hat{\boldsymbol{\eta}}_t = \hat{\Omega}^{-1}\left(\hat{\boldsymbol{\varepsilon}}_t - \hat{\Psi}\hat{\boldsymbol{\varepsilon}}_{t-1}\right)$$

for $t \geq 3$. I then randomly draw from $\{\hat{\boldsymbol{\eta}}_3, \ldots, \hat{\boldsymbol{\eta}}_T\}$ R times with replacement. For each $r = 1, \ldots, R$, I use $\{\hat{\boldsymbol{\eta}}_3^r, \ldots, \hat{\boldsymbol{\eta}}_T^r\}$, $\left\{\hat{\boldsymbol{\theta}}^{US}, \hat{\boldsymbol{\theta}}^O, \hat{\boldsymbol{\theta}}^N\right\}$ and $\{\mathbf{a}_1, \mathbf{a}_2\}$ to construct $\hat{\mathbf{a}}^r = \{\hat{\mathbf{a}}_3^r, \ldots, \hat{\mathbf{a}}_2^r\}$. I use each simulated sample to estimate $\left\{\hat{\boldsymbol{\theta}}^{US}, \hat{\boldsymbol{\theta}}^O, \hat{\boldsymbol{\theta}}^N\right\}^r$, and use the resulting distribution to construct 95% confidence intervals.

F Additional counterfactual results

Table 12: Distribution of static gains from trade under observed trade balances (2019)

	All	Rich	Poor
Mean	1.21	1.18	1.21
Med	1.17	1.14	1.19
Min	0.62	1.05	0.62
Max	2.11	1.74	2.11

Notes: Table reports summary statistics static gains from trade in 2019 under trade balances observed in 2019. "Rich" refers to OECD countries plus the US. "Poor" refers to non-OECD countries. Sample includes the 156 countries (33 OECD, 123 non-OECD) for which data is available to construct productivity and trade costs in 2019. Appendix C lists countries included and OECD membership status.

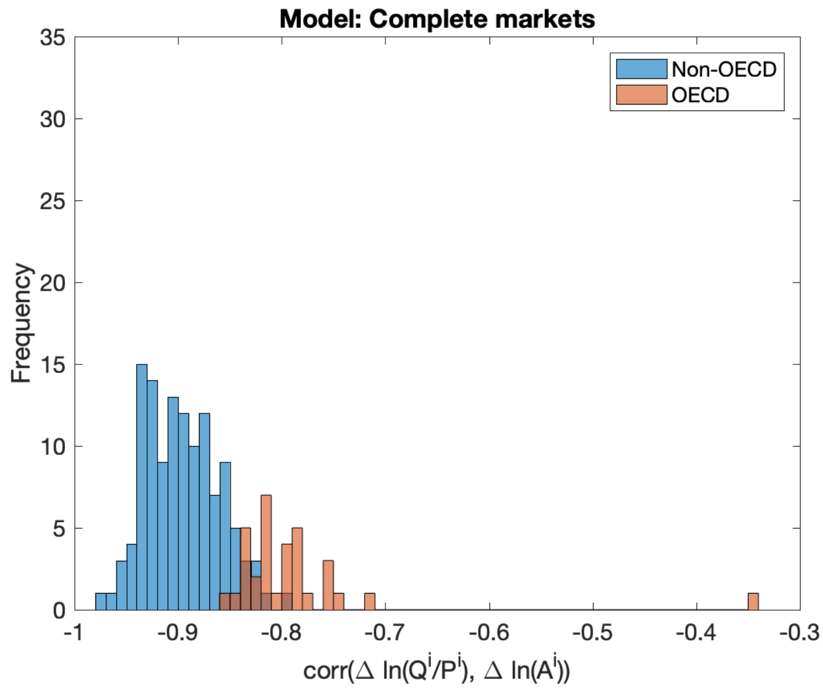

Figure 13: Correlation of growth in terms of trade & productivity growth: complete markets
Notes: Figure plots distribution of the correlation between the log change in terms of trade $(\Delta \ln \left(Q_t^i/P_t^i\right))$ and log change in productivity $(\Delta \ln A_t^i)$ in the model under complete markets. Sample includes the 156 countries (33 OECD, 123 non-OECD) for which data is available to construct productivity and trade costs in 2019. Appendix C lists countries included and OECD membership status.

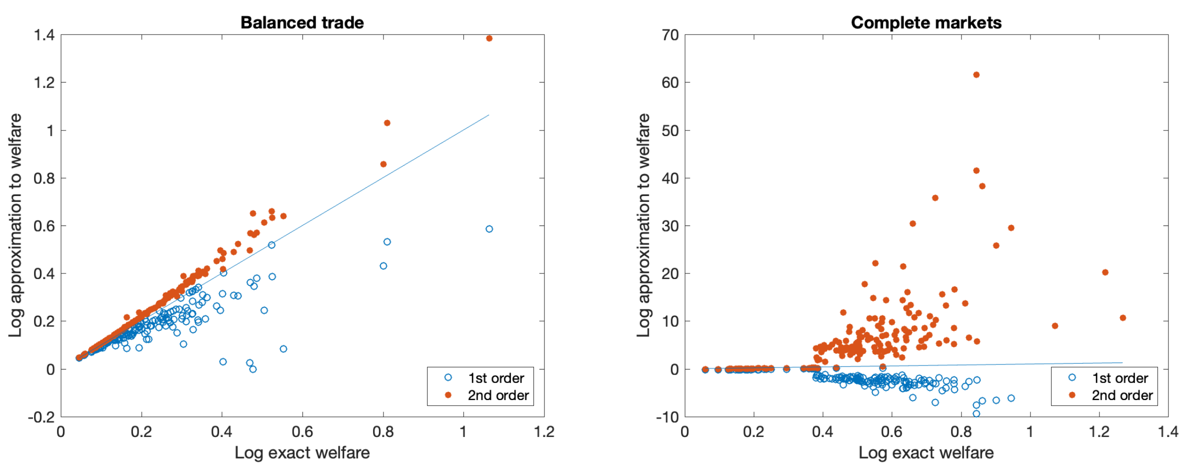

Figure 14: Exact welfare & 1st & 2nd order approximations
Notes: Left panel shows scatter plot of 1st and 2nd order approximations to welfare (y-axis) against exact welfare (x-axis) under balanced trade. Right panel shows scatter plot of 1st and 2nd order approximations to welfare (y-axis) against exact welfare (x-axis) under complete markets. 45^o line is plotted in both panels. Sample includes the 156 countries (33 OECD, 123 non-OECD) for which data is available to construct productivity and trade costs in 2019. Appendix C lists countries included and OECD membership status.

Table 13: Distribution of welfare cost of business cycles

	All	Rich	Poor
Mean	1.40	1.04	1.49
Med	1.42	1.04	1.46
Min	1.02	1.02	1.28
Max	2.30	1.06	2.30

Notes: Table reports summary statistics on the distribution of the welfare cost of business cycles. Cost of business cycles is calculated as ratio of ex-ante utility in autarky under no deviations of productivity growth from estimated trends to ex-ante utility in autarky under baseline productivity process. “Rich” refers to OECD countries plus the US. “Poor” refers to non-OECD countries. Sample includes the 156 countries (33 OECD, 123 non-OECD) for which data is available to construct productivity and trade costs in 2019. Appendix C lists countries included and OECD membership status.

Table 14: Parameters of productivity process for alternative productivity measures

	Baseline	Alternative I	Alternative II
g^{US}	0.0085	0.0085	0.0184
ψ^{US}	0.31	0.31	0.23
σ^{US}	0.012	0.012	0.020
γ^{O}	0.0103	0.0093	0.0074
ψ^{O}	0.11	0.15	0.26
$\psi^{O,US}$	0.38	0.32	0.20
σ^{O}	0.022	0.021	0.037
$\sigma^{O,US}$	0.008	0.008	0.012
γ^{N}	0.0016	0.0013	0.0001
ψ^{N}	0.00	0.02	0.14
$\psi^{N,US}$	0.15	0.06	-0.16
σ^{N}	0.066	0.057	0.080
$\sigma^{N,US}$	0.003	0.004	0.005

Notes: Estimation restricted to balanced panel 1970-2019, excluding Iraq. Appendix C lists included countries along with OECD membership. OECD and non-OECD coefficients estimated by maximum likelihood conditional on coefficients for US. Construction of the baseline productivity measure is described in the text. Construction of alternatives I and II is described in Appendix D.

Table 15: Distribution of gains from trade: Alternative productivity measures

	Static gains			3-D gains					
	2019			Balanced trade			Complete markets		
	All	Rich	Poor	All	Rich	Poor	All	Rich	Poor
	Alternative I								
Mean	1.18	1.21	1.17	1.24	1.22	1.25	1.53	1.26	1.61
Med	1.15	1.15	1.14	1.19	1.16	1.20	1.50	1.21	1.57
Min	1.04	1.05	1.04	1.05	1.05	1.05	1.06	1.06	1.33
Max	2.16	1.65	2.16	2.55	1.68	2.55	2.99	1.76	2.99
	Alternative II								
Mean	1.18	1.21	1.17	1.45	1.23	1.51	2.26	1.35	2.50
Med	1.15	1.15	1.14	1.36	1.17	1.40	2.14	1.30	2.28
Min	1.04	1.05	1.04	1.05	1.05	1.10	1.07	1.07	1.72
Max	2.22	1.65	2.22	3.96	1.73	3.96	7.85	1.91	7.85

Notes: Table reports summary statistics on the distribution of the 3-D gains from trade, constructed using the productivity processes estimated using Alternative measure I and Alternative measure II of productivity. Construction of these productivity measures is described in Appendix D. For Alternative I, initial productivity and trade costs are consistent with the productivity measure used to estimate the productivity process. For Alternative II, initial productivity and trade costs are consistent with the baseline mesure of productivity. "Rich" refers to OECD countries plus the US. "Poor" refers to non-OECD countries. Sample includes the 156 countries (33 OECD, 123 non-OECD) for which data is available to construct productivity and trade costs in 2019. Appendix C lists countries included and OECD membership status.

Table 16: Parameters of productivity process for alternative values of η

	Value of η				
Parameter	1.5	3	4	5	6
g^{US}	0.0085	0.0085	0.0085	0.0085	0.0085
ψ^{US}	0.31	0.31	0.31	0.31	0.31
σ^{US}	0.012	0.012	0.012	0.002	0.002
γ^{O}	0.0053	0.0090	0.0103	0.0109	0.0124
ψ^{O}	0.11	0.15	0.11	0.10	0.04
$\psi^{O,US}$	0.26	0.32	0.38	0.45	0.53
σ^{O}	0.025	0.021	0.022	0.026	0.032
$\sigma^{O,US}$	0.007	0.008	0.008	0.008	0.008
γ^{N}	0.0015	0.0014	0.0016	0.0016	0.0013
ψ^{N}	-0.03	0.01	0.00	-0.01	0.01
$\psi^{N,US}$	-0.06	0.06	0.15	0.20	0.16
σ^{N}	0.069	0.059	0.066	0.073	0.076
$\sigma^{N,US}$	0.004	0.003	0.003	0.001	-0.001

Notes: Estimation restricted to balanced panel 1970-2019, excluding Iraq. Appendix C lists included countries along with OECD membership. OECD and non-OECD coefficients estimated by maximum likelihood conditional on coefficients for US. Productivity is constructed as described in the text, but with alternative values of η.