Tariff Wars and Net Foreign Assets*

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Abstract

This paper examines whether and how international financial claims can be honored without default once a trade war erupts. Using a two-country two-good model, we study how unanticipated tariffs revalue international assets and how the outcome depends on ex-ante gross asset positions. When gross positions are strictly positive (each country has claims on the other), sufficiently high bilateral tariffs zero out any ex-ante net debt. Conversely, this net debt cannot be zeroed out when the debtor holds no claims on the creditor. When a country has liabilities denominated in the other country's goods (or prices), then a sufficiently large tariff levied by the second can immiserize the first, generating multiple equilibria if both sides have such liabilities. These results generalize to settings with an arbitrary number of goods, time, and uncertainty. In a dynamic version, under a severe trade war, domestic interest rates and the real exchange rate must decouple: interest rates align with their autarkic values, and international relative prices revalue net asset positions to zero. We calibrate a two-country model to the US net foreign asset position in 2023 and show that the welfare consequences of a tariff war that stops short of autarky depend critically on the US ex-ante gross portfolio composition.

1 Introduction

The two eras of globalization before the First World War and after the 1970s witnessed large declines in trade costs and substantial increases in cross-border asset positions. In the United States,

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both eras eventually ended in protectionism, exemplified in the US by the Smoot-Hawley tariffs in 1930 and the Trump tariffs in 2018-2025. This latter period, combining the implementation of substantial tariffs by the United States with unprecedented gross and net foreign asset positions accumulated during the prior decades of financial globalization, makes understanding the interaction between trade wars and international balance sheets a crucial question in international macroeconomics.

In this paper, we study whether and how the stock of international financial claims accumulated during a period of relatively free trade can be settled once a tariff war breaks out. Specifically, we analyze whether equilibria exist in which these claims are honored without explicit default, how international prices adjust to facilitate such settlement, and what welfare consequences follow for creditor and debtor nations.

Our key insight is that when tariffs restrict trade, changes in international relative prices revalue international assets. We show that the impact of this revaluation, and therefore the set of equilibria that can obtain in a tariff war, depends on the nature of the ex-ante gross positions. In this way, different ex-ante gross positions which may be consistent with identical net debt under free trade can give rise to very different outcomes in the event of a tariff war. For some ex-ante gross positions, a severe enough tariff war may wipe out net debt, for others, valuation effects cannot eliminate net debt.

We first develop these ideas in a two country (Home and Foreign) two-good static exchange economy, where Home is a net debtor at free trade prices. When Home and Foreign hold claims on each other's comparative advantage good for delivery in the country of the borrower, a situation we refer to as simple "positive claims," autarky is the unique equilibrium for high enough tariffs. In this zero-trade equilibrium, international relative prices adjust to revalue ex-ante claims in such a way that Home's net debt is zeroed out without default. While this price clears the financial market, high enough tariffs ensure there is no scope for goods arbitrage. At more moderate tariff levels, a trade war may induce a revaluation of the gross asset positions that supports a balanced-trade equilibrium with positive two-way trade flows.

In the case where Home (the net debtor) holds no claims on Foreign, we show that autarky fails to be an equilibrium, and Home must repay its debt by exporting, even for arbitrarily high tariffs. This highlights that gross positions are crucial to our first result.

By contrast, when Home's liabilities are incurred in the Foreign market, mirroring emerging markets' foreign-currency debt, if Foreign's tariff is high enough, there exists an equilibrium where Home must export both goods, regardless of its free-trade net position. The mechanism is self-fulfilling: trade flowing to Foreign raises the value of Home's liabilities (payable in Foreign) via the tariff wedge, increasing the real debt burden and forcing Home to export more to satisfy its budget constraint. In this scenario, Home is immiserized at the expense of Foreign. Crucially, if both Home and Foreign have liabilities for settlement in the other market, these conditions can apply to both countries simultaneously, generating a potential multiplicity of equilibria. The equilibrium then depends on self-fulfilling expectations about the direction of trade, reminiscent of the "debt dilemma" faced by emerging markets confronting a sharp dollar appreciation.

We extend our results to a two-country multiple-good static exchange economy with traded and non-traded goods, where households hold gross claims (positive or negative) on foreign goods. In an autarky allocation, domestic relative prices, determined by endowments, must value foreign claims. If both countries' external gross claims have the same sign when valued at these domestic prices (a generalization of the simple "positive claims" case), there exists a unique international relative price that zeros out these claims, and autarky is an equilibrium for high enough tariffs, without default on any obligations. The result is robust to the inclusion of flexible general trade costs.

Because this result depends only on the "same-sign" condition at autarkic prices, the logic extends naturally to dynamic settings with time and uncertainty. In a severe tariff war, domestic interest rates and the real exchange rate decouple: interest rates align with autarkic values to aggregate domestic claims, while the real exchange rate is pinned down by the condition that zeros out cross-border assets. This provides a potential rationalization for the fact that the recent tariff war affected the dollar exchange rate but had no major discernible impact on interest rates.

We also quantify the valuation, terms of trade, and welfare effects of a tariff war and show how these effects depend on the nature of the gross portfolio underlying a given net foreign asset position. To do so, we use a two-country (US and Rest of the World) dynamic stochastic gravity model based on Fitzgerald (2025). We calibrate to the US net foreign asset position and net exports in 2023.

We consider several portfolios consistent with the same free trade net debt, including one that closely matches the data, and a net-only portfolio where US liabilities must be repaid in the Rest of the World. The impact of a tariff war on net foreign assets, the terms of trade, and welfare varies substantially across portfolios. With the portfolio that matches the data, a tariff war may reduce US net debt, but for this to happen, the Rest of the World must retaliate. US welfare losses from this retaliation are modest. In contrast, when US liabilities must be repaid in the Rest of the World, Rest of the World retaliation increases US net debt and leads to large US welfare losses.

Literature review. Our questions and the associated answers echo the Transfer Problem famously debated by Keynes (1929) and Ohlin (1929) in the aftermath of World War I. That debate turned on whether a transfer by one country to another affected the terms-of-trade. With identical homothetic preferences, and no tariffs or trade costs, the distribution of wealth does not influence relative prices, but this is not true when the marginal dollar is spent differently. Our analysis is closest to the treatment in Samuelson (1952, 1954) in an environment in which preferences are identical and homothetic, but where there are trade frictions. Samuelson discusses how, when tariffs are present, the required transfer may have an adverse impact on the payer's terms-of-trade. Our question is the mirror image – if, under free trade, one country needs to make a net transfer to the other, can the disruption to trade from tariffs generate terms-of-trade changes that nullify the need to make the payment?[1]

Stockman and Dellas (1986) study an environment in which households can insure against tariff changes using financial assets that are contingent on tariff outcomes. They show that this insurance changes the welfare ranking of tariff policies.[2] Devereux and Lee (1999) show how ex-ante portfolio choices by households (in assets that are not contingent on tariffs) constrain future governments' incentives to manipulate the terms of trade through tariffs, demonstrating that trade in financial assets may weaken the incentives to tax trade in goods. This implies trade in assets is complementary to trade in goods, which provides a counterweight to the Cole and Obstfeld (1991) finding that the two are substitutes. They also find that the interaction between ex-ante financial trade and ex-post tariff policy can lead to multiple equilibria of the policy game, as households do not internalize the link between their portfolio choices and the subsequent optimal tariff rates. We also discuss how tariffs and asset positions can lead to indeterminacy, but our multiplicity arises for a constant set of tariffs and fixed asset positions. More generally, our main question (how large gross foreign asset positions are settled in a tariff war) is different.

The role of valuation effects in understanding the behavior of net foreign asset positions dates back to the pioneering work of Lane and Milesi-Ferretti (2001), Tille (2003), and Gourinchas and Rey (2007). This initial work focused in large part on the behavior of the dollar exchange rate to account for changes in asset valuations. Recent work by Atkeson, Heathcote, and Perri (2025) highlights that even if the depreciation of the US dollar accounts for the surprising stability of the US NFA position in the early 2000s in the face of large trade deficits, the spectacular performance of the US equities relative to foreign equity markets accounts for the US NFA deterioration after 2008. We argue that valuation effects through endogenous changes in international relative prices are also important in the context of a tariff war, providing conditions on portfolios for such price adjustments to rebalance net foreign asset positions during trade disruptions.

Prompted by recent policy events, several papers explore the interactions between tariffs and trade deficits.[3] Pujolas and Rossbach (2024) study the welfare effects of trade wars in an Arm-

[1]Epifani and Gancia (2017) study global imbalances in a trade model and discuss how the price indexes react differently in a trade model with increasing returns a la Krugman.

[2]For a modern treatment that quantifies this force, see Caliendo, Kortum, and Parro (2025).

[3]There is previous work on global imbalances, trade deficits and trade frictions. See for example, Razin and Svensson (1983), Dekle, Eaton, and Kortum (2007), Fitzgerald (2012), Reyes-Heroles (2017), Kehoe, Ruhl, and Steinberg (2018), Alessandria, Bai, and Woo (2024), among others. There is also a large body of work on gross asset positions and diversification, see for example Stockman and Dellas (1989), Baxter and Jermann (1997), Heathcote

ington model with trade imbalances, where countries have exogenous international net (but not gross) positions. Ignatenko, Lashkaripour, Macedoni, and Simonovska (2025) quantify the welfare effects of tariffs in a model with trade imbalances where net foreign asset positions are given but trade deficits endogenously respond to policy. Costinot and Werning (2025) analyze a dynamic model with fixed terms-of-trade and study the effects of a permanent increase in tariffs on the trade deficit by affecting the incentives of domestic households to save and consume. Obstfeld (2025) surveys the arguments for the determination of the U.S. trade deficit and argues that trade policies are not the main driver. In his discussion of this paper, Perri (2025) uses an international macro model to analyze the effects of tariff shocks, and their persistence, on the trade deficit, emphasizing also that the response of the economy to other shocks may be hampered by the presence of tariffs. Unlike these papers, our focus is on valuation effects and their role in determining equilibrium outcomes in tariff wars.

In a related and contemporaneous paper, Itskhoki and Mukhin (2025) study the optimal unilateral tariff in the presence of valuation effects in an Armington model.[4] In contrast, we do not explore optimal policy. We consider instead arbitrary tariff policies that may or may not be optimal (although we do discuss welfare implications). Our focus is on characterizing the set of equilibria that arise, including conditions under which autarky is an equilibrium, and where multiple equilibria coexist. We do not impose the Armington assumption, which allows us to consider situations where the pattern of trade becomes one-directional under high tariffs. This proved crucial for identifying equilibrium multiplicities in which the direction of trade is self-fulfilling, but it is not necessary: in Appendix C we show that the same economic forces generate multiplicity in an Eaton and Kortum (2002) (or equivalently, Armington) model.[5] There is also an important antecedent in Alvarez, Atkeson, and Kehoe (2009), a connection we discuss at the end of Section 4.

Paper structure. Section 2 sets the stage by presenting three facts on international portfolios. Section 3 analyzes how claims accumulated under free trade can be settled in a tariff war in the two (traded) goods case. Section 4 extends the analysis to a more general environment with multiple goods, time and uncertainty. Section 5 provides a quantification of valuation effects, terms

and Perri, 2013, Tille (2003), Broner, Didier, Erce, and Schmukler (2013), Lee (2024), among many others. An explicit connection between trade policy and intertemporal trade is found in Costinot, Lorenzoni, and Werning (2014), and more recently in Dávila, Rodríguez-Clare, Schaab, and Tan (2025).

[4] There is a classic and very general theory on optimal tariffs emphasizing the terms-of-trade effects, but it does not consider valuation effects. See Dixit and Norman (1980) and Dixit (1985) for comprehensive summaries.

[5] Related to the focus on optimal policy responses, Bianchi and Coulibaly (2025) analyze the optimal monetary policy to an increase in tariffs. See also the recent contributions of Werning, Lorenzoni, and Guerrieri (2025) and Auclert, Rognlie, and Straub (2025). For previous work on the monetary response to tariffs, we refer the reader to Erceg, Prestipino, and Raffo (2023) and Bergin and Corsetti (2023).

Table 1: Gross and net foreign assets in OECD and non-OECD countries

	$\frac{\text{Assets} + \lvert\text{Liabilities}\rvert}{GDP}$		$\frac{\lvert NFA\rvert}{\text{Assets} + \lvert\text{Liabilities}\rvert}$		$\frac{\lvert NFA\rvert}{GDP}$	
	OECD	Non-OECD	OECD	Non-OECD	OECD	Non-OECD
25th percentile	2.35	0.91	0.06	0.13	0.21	0.18
50th percentile	4.00	1.27	0.09	0.34	0.43	0.39
75th percentile	6.74	2.20	0.22	0.49	0.54	0.58

Notes: Data from the External Wealth of Nations database. There are 31 non-tax-haven OECD countries with GDP greater than 0.1% of US GDP in 2023. There are 70 non-tax-haven non-OECD countries with GDP greater than 0.1% of US GDP in 2023.

of trade effects, and welfare in a trade war which falls short of imposing autarky. Section 6 concludes. All proofs are collected in Appendix A. Extensions to allow for trade costs are presented in Appendix B. The extension to the Eaton-Kortum model with multiple goods, is provided in Appendix C.

2 International portfolios

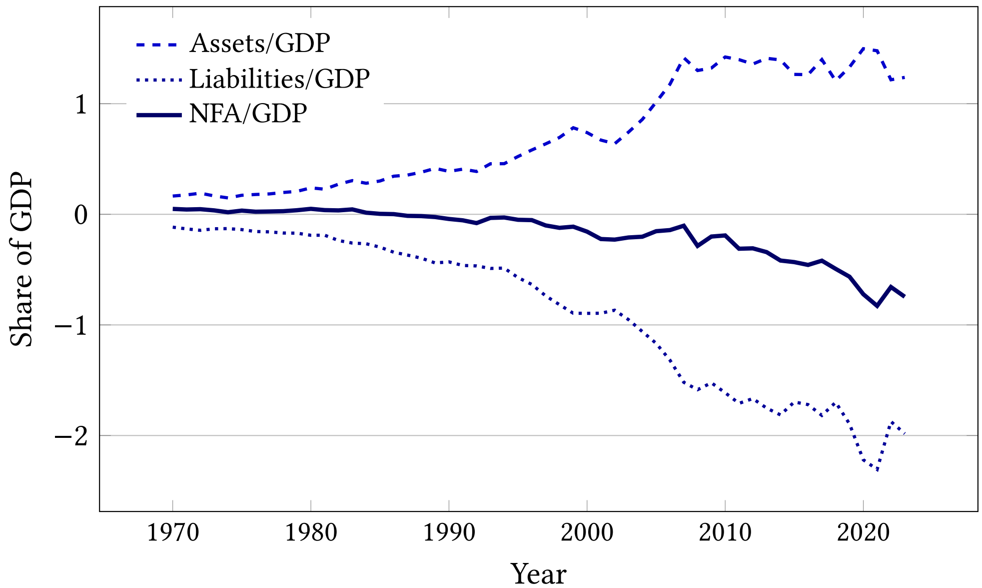
Before showing that ex-ante international asset positions play a role in determining outcomes in a tariff war, we set the stage by documenting three facts about these portfolios in the data.

Fact 1: Gross positions are much bigger than net positions. Since the collapse of the Bretton Woods system in the 1970s, international asset markets have become increasingly integrated. For the world as a whole, cross-border liabilities accounted for just over 20% of world GDP in 1970, but grew to more than 200% of world GDP in 2023.[6] Financial integration has increasingly taken the form of two-way asset trade. Summing net foreign assets across all countries for which they are negative gives an indication of the minimum cross-border asset holdings necessary to account for observed net foreign asset positions (what is a liability for one country is an asset for another). In 1970, this accounted for 28% of all cross-border asset holdings. By 2023, this had fallen to just 15% of cross-border asset holdings, consistent with faster growth of gross than net positions.

For developed countries, gross foreign asset positions are particularly large, whether expressed relative to GDP, or relative to net foreign assets. Gross positions are smaller for developing countries, but still exceed net positions. This is illustrated in Table 1, which reports the 25th, 50th and 75th percentile of gross foreign asset positions (assets + the absolute value

[6] Source: The External Wealth of Nations database; Lane and Milesi-Ferretti (2018).

Figure 1: US gross and net foreign assets relative to GDP, 1970-2023



Notes: Data from the External Wealth of Nations database.

of liabilities) relative to GDP in 2023 for non-tax-haven OECD and non-OECD countries whose 2023 GDP is greater than 0.1% of US GDP. It also reports the 25th, 50th and 75th percentile of the absolute value of net foreign assets relative to gross foreign assets for these countries.

Similar to other developed economies, the gross foreign asset position of the US is large relative to its GDP, and relative to its net foreign assets. Figure 1 plots foreign assets, foreign liabilities, and net foreign assets for the US from 1970-2023. The US has been a net debtor since 1987, but in 2023, the sum of US foreign assets and the absolute value of US foreign liabilities was 316% GDP, while US net foreign liabilities were just 75% of GDP.

Fact 2: Equity-like claims account for the majority of international liabilities. For valuation effects, it matters not just that gross asset positions differ from net positions, and are large: the nature of claims also matters. The types of claims traded internationally can be divided broadly into debt-like instruments (including bank loans) which may be denominated in home or foreign currency, and equity-like claims to shares of profits (this includes FDI). Throughout the period 1970-2014, debt accounted for a greater share of international liabilities than equity, but the equity share has trended upwards since the 1980s. In 2023, equity accounted for 53% of international liabilities, compared to 41% for debt (financial derivatives accounted for the remainder).

Since it accounts for nearly one quarter of international liabilities, and close to one sixth of international assets, the international portfolio of the United States is of particular interest. Table 2 reports the composition of the US portfolio in 2023, expressed as shares of 2023 US GDP.

Although more than half of US assets and liabilities are in the form of equity, two thirds of the US negative net foreign asset position is due to its negative net debt position. Strikingly, US debt assets and debt liabilities are both largely denominated in dollars.

Table 2: US International Portfolio in 2023

	Assets	Liabilities	Balance
Equity & FDI	0.78	1.07	-0.28
Debt (USD)	0.26	0.76	-0.50
Debt (non-USD)	0.07	0.05	0.02
Other	0.09	0.08	0.01
Total	1.20	1.96	-0.75

Notes: Data from the External Wealth of Nations database and US Treasury (currency composition of debt). Other category includes financial derivatives and foreign exchange reserves.

Fact 3: Debt may be issued in the lender's currency. The flip side of these dollar-denominated debt assets on the US balance sheet is that the counterparty debtor countries have substantial liabilities denominated in foreign currency. In this case, debts must be settled by the delivery of real quantities indexed to the lender country, rather than the borrower country. As we will see, this may expose borrowers to adverse valuation effects in the event of a trade war.

3 A Two-Good Environment

We first use a two-country two-good endowment model to study whether and how international financial claims accumulated under free trade can be settled in a tariff war. In a tariff war, international relative price movements revalue claims. We characterize equilibria in three salient cases for initial portfolios, each of which may be consistent with the same initial net debt. Revaluation affects net debt very differently in each of these three cases. These results provide the basis for similar results in more complicated models, including in a multi-good production environment based on Eaton and Kortum (2002) presented in Appendix C, in a dynamic extension in Section 4, and in the quantitative model of Section 5.

3.1 Environment

The environment is static and consists of two countries, Home and Foreign, and two goods, A and B. Using a star to denote Foreign, let endowments of A and B be denoted $Y = \{Y_A, Y_B\}$ and

$Y^* = \{Y_A^*, Y_B^*\}$, with global endowments denoted by $\overline{Y}_A \equiv Y_A + Y_A^*$ and $\overline{Y}_B \equiv Y_B + Y_B^*$. Without loss, we define good A as Home's relatively abundant good:

$$\infty > \frac{Y_A}{Y_B} > \frac{\overline{Y}_A}{\overline{Y}_B} > \frac{Y_A^*}{Y_B^*} > 0.$$

We bound the respective endowments of both goods away from zero to ensure that relative prices are well-defined in autarky.

Preferences are assumed to be identical for Home and Foreign and characterized by a strictly increasing, strictly concave, differentiable, and homothetic utility function $u(c_A, c_B)$ over consumption of the two goods. For $x = c_B/c_A$, and letting $u_i \equiv \partial u/\partial c_i$, we define the marginal rate of substitution (MRS) between B and A as:

$$g(x) \equiv \frac{u_B(1,x)}{u_A(1,x)} = \frac{u_B(c_A, c_B)}{u_A(c_A, c_B)},$$

where we use homotheticity to express the MRS as a function of the ratio of the two goods. The standard assumptions on preferences imply that $g'(x) < 0$ and that the inverse $h \equiv g^{-1}$ is well defined.

Let p_i, p_i^* denote the Home and Foreign prices of good $i \in \{A, B\}$ in units of a world numeraire currency. Tariffs are set uniformly across all imported goods, and we let $\tau \geq 0$ and $\tau^* \geq 0$ denote the Home and Foreign tariffs.[7] We restrict attention to tariffs as instruments of trade policy; export taxes are set equal to zero. International goods arbitrage requires that for all traded goods, $i \in \{A, B\}$, we have:

$$\frac{p_i}{p_i^*} = \begin{cases} (1+\tau) & \text{for Home imports: } c_i > Y_i \\ 1/(1+\tau^*) & \text{for Foreign imports: } c_i < Y_i \end{cases}$$
$$\text{with } p_i/p_i^* \in [1/(1+\tau^*), (1+\tau)] \text{ if not traded: } c_i = Y_i.$$

That is, if a good is imported into Home, its price is greater in Home than Foreign by the tariff factor, and vice versa if Foreign imports the good. For nontraded goods, the price differential is bounded by the two tariffs.

Households in each country start the period with gross claims on each other. Since relative prices can move around across goods and countries, it is important to be precise about which goods these are claims to, and *where* the claims are settled; that is, whether claims are paid before or after import tariffs are levied. More precisely, by location of "settlement" or "delivery", we mean that the value of an asset payout depends on the local price level in that location. Given two goods

[7] We introduce resource costs of trade in Appendix B.2.

Table 3: Taxonomy of simple assets

Asset	Definition
a_A	Home asset (liability if negative), denominated in A, for delivery in Foreign
a_B	Home asset (liability if negative), denominated in B, for delivery in Foreign
a_A^*	Foreign asset (liability if negative), denominated in A, for delivery in Home
a_B^*	Foreign asset (liability if negative), denominated in B, for delivery in Home

and two locations, many assets can be spanned by a simple structure. Let a_i denote Home's claim on Foreign denominated in good $i \in \{A, B\}$. Note that a_i may be positive or negative. If $a_i < 0$ this indicates that Home owes Foreign an amount of good i to be delivered in Foreign. This claim has value $p_i^* a_i$ in the numeraire currency, reflecting that payments are made in Foreign and thus are valued at Foreign prices. Symmetrically, let a_i^* denote Foreign's claim on Home in good i (again, a_i^* may be positive or negative), which in turn has value $p_i a_i^*$, for $i \in \{A, B\}$. To the extent that the law of one price fails due to tariffs, i.e. $p_i \neq p_i^*$, a claim denominated in a particular good will have a different payout depending on where it is settled. Table 3 summarizes the four assets.

From these simple assets we can construct more familiar assets. For example, an equity-like claim to a fraction γ of Foreign's total output has value $\gamma\left(p_A^* Y_A^* + p_B^* Y_B^*\right)$, which is equivalent to $a_A = \gamma Y_A^*$ and $a_B = \gamma Y_B^*$. Similarly, consider a bond that pays out in terms of a consumption basket in foreign with fixed weights: δ_A^* and $\delta_B^* = 1 - \delta_A^*$. Ownership of a of these bonds is equivalent to a portfolio with $a_i = \delta_i^* a$, $i \in \{A, B\}$. In a nominal model, if a country's monetary authority were to hold this consumer price index (CPI) constant, this portfolio would represent a nominal bond.[8]

With our asset notation, the budget constraints faced by Home and Foreign households are:

$$
\begin{aligned}
p_A(c_A + a_A^*) + p_B(c_B + a_B^*) &= p_A Y_A + p_A^* a_A + p_B Y_B + p_B^* a_B + T \\
p_A^*(c_A^* + a_A) + p_B^*(c_B^* + a_B) &= p_A^* Y_A^* + p_A a_A^* + p_B^* Y_B^* + p_B a_B^* + T^*,
\end{aligned}
$$

where T and T^* are government transfers in Home and Foreign, respectively. Note the presence of foreign prices in Home's budget constraint, and vice versa for the Foreign budget constraint.

[8]If the CPI has varying weights (such as the "ideal price index"), then a bond that pays the CPI cannot be directly spanned by our simple assets, as the portfolio is implicitly indexed to tariff policy. We explore such an asset in Appendix C.

This reflects the asset structure discussed above.[9] The governments' budget constraints are

$$T = \tau \sum_{i \in \{A,B\}} p_i^* \max\{c_i - Y_i, 0\} \quad \text{and} \quad T^* = \tau^* \sum_{i \in \{A,B\}} p_i \max\{c_i^* - Y_i^*, 0\}.$$

Finally, the resource constraints are $c_i + c_i^* = \overline{Y}_i$, $i \in \{A, B\}$.

The definition of equilibrium is standard:

Definition 1. *Given a pair of tariffs,* $\{\tau, \tau^*\}$, *we define an* equilibrium *as an allocation* $\{c_A, c_B, c_A^*, c_B^*\}$ *and prices* $\{p_A, p_B, p_A^*, p_B^*\}$ *such that: (i) households optimize subject to their budget constraints; (ii) prices satisfy goods arbitrage; (iii) the government budget constraints hold; and (iv) the aggregate resource constraints are satisfied.*

3.2 Free Trade Benchmark

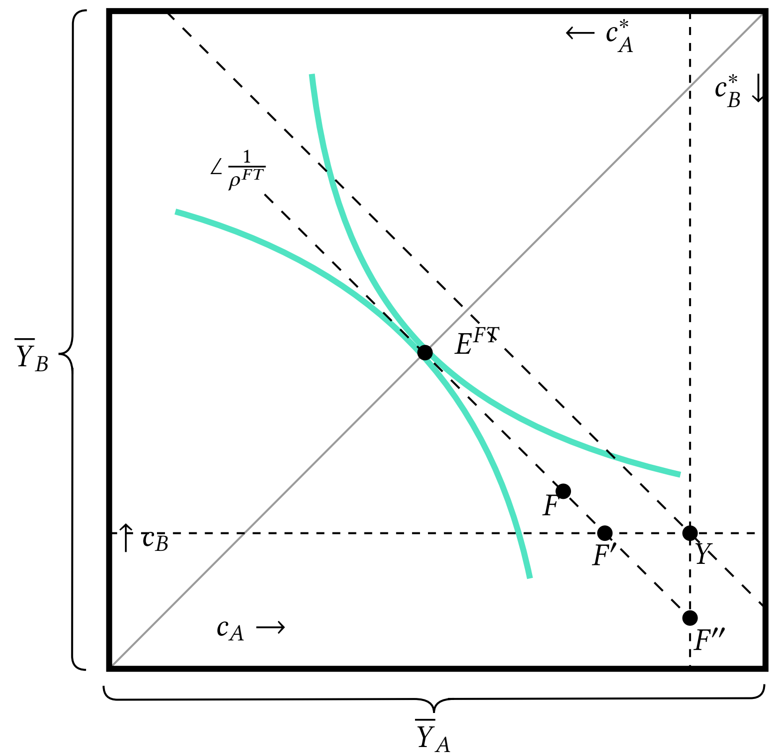
Before turning to trade wars, it is useful to establish the free-trade benchmark in which $\tau = \tau^* = 0$. This immediately implies prices for each good are the same in both countries, and hence the marginal rates of substitution are equalized across Home and Foreign. To provide a diagrammatic intuition, we use the Edgeworth box depicted in Figure 2. The dimensions of the box are $\overline{Y}_A \times \overline{Y}_B$. Let $Y \equiv (Y_A, Y_B)$ denote Home's endowment point. Home's consumption of A and B is depicted from the southwest corner, with A on the horizontal axis and B on the vertical axis. Foreign consumption is the mirror image, with its origin in the northeast corner. Given identical, homothetic preferences, the indifference curves are tangent along the diagonal, which thus outlines the efficient contract curve. The endowment point is depicted by the point Y. The point $F \equiv (Y_A - a_A^* + a_A, Y_B - a_B^* + a_B)$ depicts the endowment adjusted by initial asset positions.

As depicted, we have $a_A^* > a_A$ and $a_B^* < a_B$, which means that Home is short good A and long good B. This places F to the northwest of the endowment point Y. We define the international relative prices (the terms-of-trade in the case depicted) as

$$\rho^{FT} \equiv \frac{p_B^{FT}}{p_A^{FT}} = g\left(\frac{\overline{Y}_B}{\overline{Y}_A}\right),$$

[9] In standard trade models where $a_i = 0$ and $a_i^* = 0$, the levels of p_i and p_i^* do not matter for equilibrium determination; only relative prices across goods within the locations are necessary. This is Lerner symmetry: there is an equivalence in the choices of export and import taxes. In our environment with nonzero assets, p_i and p_i^* appear in the budget constraints, and Lerner symmetry fails. This failure of Lerner symmetry in the presence of cross-country asset positions appears in Costinot and Werning (2019), Barbiero, Farhi, Gopinath, and Itskhoki (2019), and, more recently, in Itskhoki and Mukhin (2025). (See also Farhi, Gopinath, and Itskhoki (2014).)

Figure 2: Free Trade Equilibrium



where the last equality uses the fact that under free trade and our preference assumptions the consumers in each country equate their MRS and hence the equilibrium lies along the diagonal. Dividing Home's budget constraint by p_A, and using the fact that under free trade the law of one price holds and tariff revenue is zero, we obtain:

$$c_A + \rho^{FT} c_B = (Y_A + a_A - a_A^*) + \rho^{FT}(Y_B + a_B - a_B^*).$$

This budget constraint is represented by the line through point F with slope $-1/\rho^{FT}$. From market clearing, the Foreign budget set can be read along the same line but from the opposite origin. The free-trade equilibrium consumption allocation is where this budget line intersects the diagonal, E^{FT}. As depicted, E^{FT} is to the left and above Y, indicating that Home exports good A and imports good B. The net foreign asset position of Home expressed in units of good A is given by $a_A - a_A^* + \rho^{FT}(a_B - a_B^*)$. This is the horizontal distance between the budget line and the line with the same slope which passes through the endowment point Y.

Note that the free trade equilibrium consumption allocation can be characterized *as if* the endowment point were shifted by the initial net asset positions (for each good); that is, treating F as a translated endowment point. However equilibrium trade flows depend on the position of the endowment point Y, not the financial position F. Since tariffs apply to import flows, this characterization of equilibrium by a simple translation of the endowment point does not extend in general to the case with tariffs.

In addition, note that the free trade equilibrium consumption allocation E^{FT} is consistent with *any* financial position along the free trade budget line. In particular, in addition to being consistent with financial position F, it is also consistent with the financial position that coincides with E^{FT}. It is consistent with financial position F', where $a_B^{*'} - a'_B = 0$. It is consistent with financial position F'', where $a_A^{*''} - a''_A = 0$. All financial positions along this line imply the same net foreign assets expressed in units of good A. In addition, each financial position along this line is consistent with multiple different portfolios of underlying assets. For example, financial position F is defined by $a_A - a_A^*$ and $a_B - a_B^*$, but is consistent with many different combinations of a_A, a_A^*, a_B and a_B^*.

We now focus on three different classes of ex-ante portfolios ("Positive claims," "Net-only portfolios," "Debt traps"), characterize outcomes in the event of a tariff war for each class, and show that these outcomes are qualitatively different even though the portfolios may be consistent with identical net debt and consumption under free trade.

It is simplest expositionally to build from a concrete free trade scenario. Specifically:

Assumption 1 (Free Trade Benchmark)**.** *In the free-trade equilibrium, we have:*

(i) Two way trade: $c_A^{FT} < Y_A$ *and* $c_B^{FT} > Y_B$*;*

(ii) Home is a net debtor: $c_A^{FT} + \rho^{FT} c_B^{FT} < Y_A + \rho^{FT} Y_B$*.*

Part (i) rules out an initial asset position so indebted that it forces one country to export both goods. Having two-way trade as the baseline allows for an interesting and well defined terms-of-trade as reference. Part (ii) simply defines the Home country as the net debtor at free-trade prices (as in Figure 2), which is essentially a normalization.

3.3 Tariff Wars with Simple "Positive Claims"

We first consider the case in which each country owns a positive claim on the other country's abundant good:

Assumption 2 (Simple Positive Claims)**.** *Each country has liabilities that pay its abundant good domestically. That is, (i)* $a_B > 0$ *and* $a_A^* > 0$*; and (ii) there are no other assets or liabilities:* $a_A = a_B^* = 0$.

Assumption 2 implies that liabilities are settled in the borrower country, not in the lender country, and that gross positions are bigger than net positions.[10] As Section 2 documents, these conditions are broadly satisfied by the international portfolios of advanced economies, including the United States.

To analyze the tariff equilibrium, we make use of the following result that characterizes any equilibrium under the stated assumptions:

Lemma 1. *Suppose Assumptions 1 and 2 hold. Then any equilibrium is characterized by* $\{c_A, c_B, c_A^*, c_B^*, \rho\}$ *such that:*

(i) Home (weakly) exports good A and (weakly) imports good B: $c_A \leq Y_A$ *and* $c_B \geq Y_B$*;*

(ii) Budget sets and resource constraints are satisfied:

$$\begin{aligned} c_A + \rho c_B &= Y_A - a_A^* + \rho(Y_B + a_B) \\ c_A^* + \rho c_B^* &= Y_A^* + a_A^* + \rho(Y_B^* - a_B), \end{aligned} \tag{1}$$

and $c_A + c_A^* = \overline{Y}_A$, $c_B + c_B^* = \overline{Y}_B$*;*

(iii) Households optimize and goods arbitrage holds:

$$g(c_B/c_A) = \alpha\rho \quad \text{and} \quad g(c_B^*/c_A^*) = \alpha^*\rho,$$

where $\alpha, \alpha^* \in [1/(1+\tau^*), 1+\tau]$ *and* $\alpha = 1+\tau$ *if* $c_B > Y_B$ *and* $\alpha^* = 1/(1+\tau^*)$ *if* $c_A < Y_A$.

The corresponding equilibrium prices are any $\{p_A, p_B, p_A^*, p_B^*\}$ *such that*

$$\rho \equiv p_B^*/p_A, \quad p_B = \alpha p_B^*, \quad \text{and} \quad p_A^* = p_A/\alpha^*.$$

Proof: See Appendix A.1.

Part (i) of the lemma states that tariffs cannot reverse the pattern of trade, which is intuitive given that tariffs are levied on whichever good is imported. If trade were reversed, then import tariffs would fall on the relatively abundant good, reducing the incentive to import it compared to the free trade equilibrium. In Section 3.5 we show that without Assumption 2, the pattern of trade may reverse. Part (i) also relies on no export taxes.

Part (ii) of the lemma indicates that the international terms of trade $\rho \equiv p_B^*/p_A$ is sufficient

[10] In the appendix, we impose a weaker assumption, Assumption 2′, which restricts $a_A + \rho^{FT} a_B > 0$ and $a_A^* + \rho^{FT} a_B^* > 0$. We make the stronger Assumption 2 in the text for expositional clarity.

to characterize the country budget sets (1). These are obtained by substituting T and T^* into the respective household budget sets using the governments' budget constraints. Again this relies on the simple asset structure of Assumption 2.

Part (iii) states that when trade is strictly positive in both goods, we have $p_B/p_A = (1+\tau)\rho$ and $p_B^*/p_A^* = \rho/(1+\tau^*)$. This implies that the households' first-order conditions are:

$$\left(\frac{1}{1+\tau}\right) g\left(\frac{c_B}{c_A}\right) = \rho = (1+\tau^*) g\left(\frac{\overline{Y}_B - c_B}{\overline{Y}_A - c_A}\right). \tag{2}$$

For this case, the single relative price ρ is sufficient for both optimality and the budget sets, as was the case under free trade. However, if a good is not actively traded, the respective MRS and equilibrium prices can take values within the specified intervals spanned by the tariff wedges.

We are now ready to state our main result for the case of a tariff war with simple positive claims:

Proposition 1. *Suppose that Assumptions 1 and 2 hold. There exist finite $\underline{\tau}, \underline{\tau}^*$ such that for any $\tau \geq \underline{\tau}$ and $\tau^* \geq \underline{\tau}^*$, the autarkic allocation $c_j = Y_j, c_j^* = Y_j^*$, $j \in \{A, B\}$ is the* unique equilibrium.

Proof: See Appendix A.2.

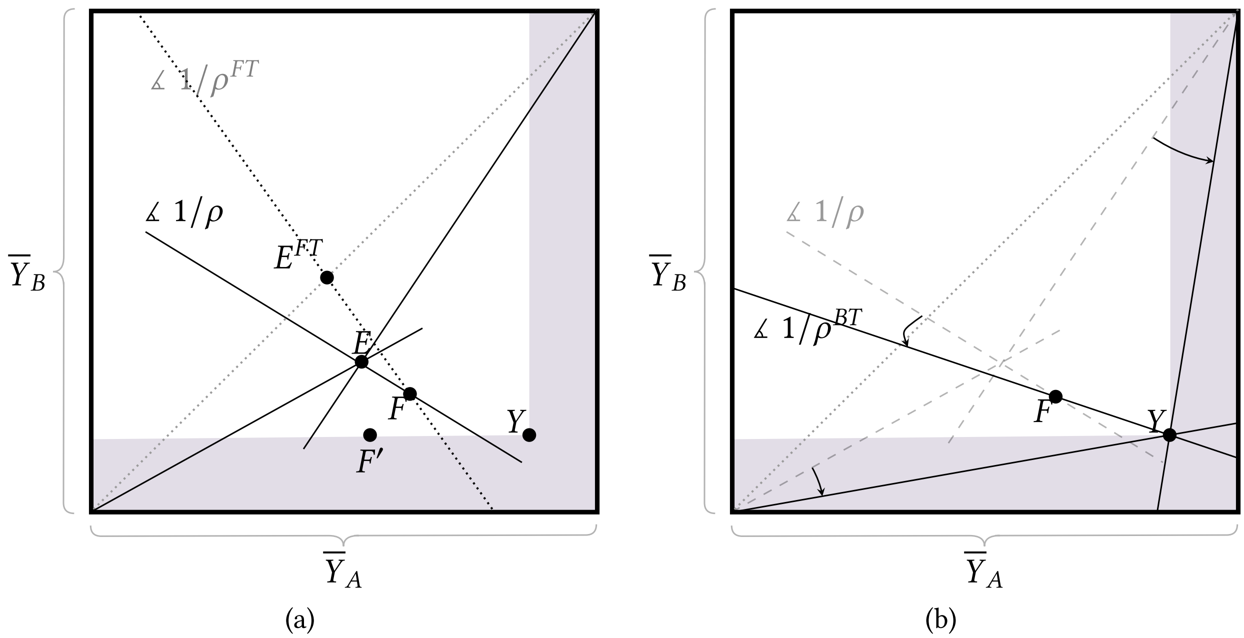
The key message of this proposition is that with an initial portfolio that satisfies Assumption 2, for high enough tariffs, autarky is the unique equilibrium, and this *does not involve default* on financial claims. The proof of existence is by construction. Uniqueness relies on part (i) of Lemma 1.

Given Assumption 2 we can give a graphic intuition using the Edgeworth box depicted in Figure 3. As before, $Y = (Y_A, Y_B)$ is the endowment point and $F = (Y_A - a_A^*, Y_B + a_B)$ is the endowment translated by the asset positions. Assumption 2 states that F is not in the shaded region.

First consider an equilibrium with positive tariffs $\{\tau > 0, \tau^* > 0\}$ and strictly positive trade in both goods. This is reflected in panel (a). Part (i) of Lemma 1 stating that the presence of tariffs cannot change the pattern of trade implies that equilibrium cannot be in the interior of the shaded region of Figure 3. It also implies that we can focus on the ratio of $p_B^*/p_A = \rho$ as the international terms of trade in any equilibrium. In addition, the MRS in each country are related to each other and to ρ by equation (2).

The ray from Home's origin traces out $c_B/c_A = h\left((1+\tau)\rho\right)$, the first equality in equation (2), where recall that h is the inverse of the MRS function g. Similarly, the ray from Foreign's

Figure 3: Tariff War Equilibrium



origin contains the locus $c_B^*/c_A^* = h(\rho/(1+\tau^*))$, the second equality of (2). Consistent with tariffs distorting Home's consumption toward good A and Foreign toward B, we depict the rays below the free-trade diagonal.

Equation (1) states that feasible consumption allocations for Home are those along the line with slope $-1/\rho$ passing through the point F. Using the resource constraint, Foreign's allocations viewed from the opposite origin lie along the same line.

The equilibrium ρ is such that these rays intersect along the budget line, depicted as point E. This intersection is the solution to three equations (the two optimality conditions in equation (2) and the Home budget constraint (1)) in three unknowns (c_A, c_B, ρ). The Foreign budget constraint is satisfied by Walras' Law. As depicted, the equilibrium terms-of-trade is such that Home is a net debtor, as evidenced by the fact that the endowment point Y lies beyond (to the right) of Home's budget line when evaluated at the equilibrium relative prices ρ. Rearranging the budget constraints as follows we can see that in any equilibrium the terms of trade, $\rho = p_B^*/p_A$, plays the dual role of being the international relative price of the goods, as well as being the relative price of the international claims:

$$
\begin{aligned}
c_A + \rho c_B &= Y_A + \rho Y_B - a_A^* + \rho a_B \\
c_A^* + \rho c_B^* &= Y_A^* + \rho Y_B^* + a_A^* - \rho a_B,
\end{aligned} \tag{3}
$$

Now suppose that we have an equilibrium as depicted in Figure 3 and we increase tariffs in both countries. This is now shown in panel (b). For a given ρ, from (2), the dashed rays from the origin rotate away from the diagonal, as each country's imports become more expensive. Intuitively, tariffs reduce trade and move the equilibrium closer to autarky, although some care must be taken with this intuition, as alternative equilibria arise once we relax Assumption 2. With that caveat, and given Assumption 2, increasing both tariffs pushes the equilibrium towards autarky. Since we have assumed positive endowments of both goods for both countries, marginal rates of substitution are bounded in autarky, and hence so too are the tariffs that implement autarky.

As noted above, the fact that imports become prohibitively expensive reduces the incentives to trade. However, this begs the question of how are financial claims settled without any trade (and hence, no net trade). We can rewrite Home's budget constraint as

$$c_A - Y_A + \rho(c_B - Y_B) = -a_A^* + \rho a_B = 0,$$

where the last equality follows from the fact that $c_A = Y_A$ and $c_B = Y_B$ in Autarky. Thus, Autarky is consistent with financial claims when

$$\rho = \rho^{BT} \equiv \frac{a_A^*}{a_B},$$

where the superscript BT denotes "balanced trade." As long as both a_B and a_A^* are strictly positive, this is a valid terms-of-trade. That is, the equilibrium terms of trade is pinned down by the asset positions. The alert reader may notice that this would also be true if both a_B and a_A^* were strictly *negative*, but in this case, uniqueness would not be guaranteed (unless tariffs are even higher), as we address in Sections 3.5 and 3.6.

To see what is occurring to the country's net worth, recall that in Figure 3 at equilibrium E, Home is a net debtor at the relative price ρ. That is $\rho a_B < a_A^*$. As tariffs increase, ρ increases to the point that $\rho = \rho^{BT}$ such that $\rho^{BT} a_B = a_A^*$. Thus, the relative value of Home's liability *declines* to ensure that financial claims can be settled with zero net trade.

Balanced Trade with Positive Trade In Appendix B.1 we show that as long as welfare in autarky is below welfare in the free trade equilibrium, for intermediate home and foreign tariffs, there exists a locus of tariffs (τ, τ^*) that supports a balanced trade equilibrium with positive trade. Notably, a Home tariff alone is not sufficient to achieve balanced trade: Foreign must also impose a sufficiently high tariff. That is, *the debtor country cannot unilaterally close its net foreign asset position through tariffs alone.*

Welfare. Briefly, it is worth saying something about the welfare consequences of a tariff war. There are three effects on each country's welfare: (i) losses from the gains to trade, which are always negative for both countries; (ii) terms-of-trade effects; and (iii) revaluation of gross asset positions. In the examples of Figure 3, Foreign's terms of trade improve as ρ increases, but this is offset by the decline in the net value of its claims on Home and by the loss of gains from trade. For Home, the first two effects are reversed: its terms of trade deteriorate, but its net debt burden falls. In panel (a), the movement from E^{FT} to E is an unambiguous loss for Home, since its free-trade indifference curve lies to the right of the free-trade budget line. Whether Foreign gains or loses depends on the relative magnitude of the three effects. In panel (b), the movement to autarky is unambiguously bad for Foreign, but ambiguous for Home, as the benefit of debt revaluation may or may not outweigh the combined cost of worse terms of trade and foregone gains from trade.

3.4 Net vs Gross Foreign Asset Positions

Section 2 shows that gross international asset positions are bigger than net positions. But since it is standard in much of the open economy macroeconomics literature to assume that only a single financial asset is traded, it is worth asking what happens in a tariff war when this is the case. In this section, we assume $a_A = a_B = a_B^* = 0$ but $a_A^* > 0$. This is a natural case: Home has a single liability in units of its abundant good. We depict this as point F' in Figure 3 panel (a). In this subsection we analyze outcomes in a tariff war under this scenario, and find they contrast sharply with those under strictly positive positions on both sides, even if those positions are arbitrarily small.

Let us guess and verify (for high enough tariffs) that an equilibrium allocation $E' = F'$ exits. That is, Home simply exports good A in an amount of a_A^* to Foreign and consumes its endowment of B. By construction, this will satisfy their budget sets at any prices.

The only thing to verify is optimality and goods arbitrage. Given that Home exports good A, $p_A^* = (1+\tau^*)p_A$. Since good B is not traded, we have that

$$\frac{p_B}{p_B^*} \in \left[\frac{1}{1+\tau^*}, 1+\tau\right]. \tag{4}$$

Let our candidate $\rho = p_B^*/p_A$ be defined by

$$\begin{aligned}\rho = \frac{p_B^*}{p_A} = \frac{(1+\tau^*)p_B^*}{p_A^*} &= (1+\tau^*)g\left(\frac{c_B^*}{c_A^*}\right) = (1+\tau^*)g\left(\frac{Y_B^*}{Y_A^* + a_A^*}\right) \\ \Leftrightarrow p_B^* &= p_A(1+\tau^*)g\left(\frac{Y_B^*}{Y_A^* + a_A^*}\right)\end{aligned}$$

which satisfies Foreign's optimality condition by definition. We just need to check Home's optimality and that the goods market arbitrage (4) holds. Home's optimality is:

$$g\left(\frac{Y_B}{Y_A - a_A^*}\right) = \frac{p_B}{p_A} \Leftrightarrow p_B = p_A g\left(\frac{Y_B}{Y_A - a_A^*}\right)$$

Good B's arbitrage condition can be combined with optimality to become:

$$\frac{1}{1+\tau^*} \leq \frac{p_B}{p_B^*} = \frac{1}{1+\tau^*}\frac{g\left(\frac{Y_B}{Y_A - a_A^*}\right)}{g\left(\frac{Y_B^*}{Y_A^* + a_A^*}\right)} \leq 1+\tau \Leftrightarrow 1 \leq \frac{g\left(\frac{Y_B}{Y_A - a_A^*}\right)}{g\left(\frac{Y_B^*}{Y_A^* + a_A^*}\right)} \leq (1+\tau)(1+\tau^*) \tag{5}$$

So as long as

$$\frac{Y_B}{Y_A - a_A^*} < \frac{Y_B^*}{Y_A^* + a_A^*}, \tag{6}$$

the inequalities in (5) will be satisfied for high enough τ, τ^*. A sufficient condition for (6) is that under free trade Home imports B and exports A; that is, Part (i) of Assumption 1. If this holds, our conjectured equilibrium is verified. We summarize this in the following Lemma:

Lemma 2. *Suppose Assumption 1 holds. Suppose that $a_A = a_B = a_B^* = 0$ and $Y_A > a_A^* > 0$. Then the allocation $(c_A = Y_A - a_A^*, c_B = Y_B, c_A^* = Y_A^* + a_A^*, c_B^* = Y_B^*)$ constitute an equilibrium for a home tariff τ high enough. In such an equilibrium, the terms-of-trade, ρ, is given by:*

$$\rho = (1+\tau^*)g(c_B^*/c_A^*).$$

Proof: See Appendix A.3.

It is instructive to contrast this result with that of Proposition 1. In particular, take the case of gross positions, $a_B > 0$ and $a_A^* > 0$, and let a_B be arbitrarily small. As in the case of Lemma 2, suppose we attempt to construct an equilibrium in which Home exports to pay a_A^*. Home's net wealth is given by

$$\rho a_B - a_A^* = (1+\tau^*)g\left(\frac{c_B^*}{c_A^*}\right)a_B - a_A^*.$$

As τ^* becomes large, given that $a_B > 0$, Home becomes arbitrarily wealthy. The reason is that if Home is exporting to Foreign, a higher tariff in Foreign makes Home *wealthier*, as it appreciates its claim on Foreign goods. At some wealth level, Home no longer is a net debtor and no longer needs to export to Foreign. This example shows that a net asset position composed of competing gross positions behaves fundamentally differently in a trade war than if we had a single financial asset, even if the non-zero gross positions are arbitrarily small.

3.5 Debt Traps

In many cases, particularly in emerging markets, a country is indebted in a foreign currency, as noted in Section 2. One way to study foreign currency liabilities in our setup is to consider the case where a borrower incurs liabilities for delivery in the lender country, a situation that is ruled out by Assumption 2. Relaxing this requirement opens the door to equilibria we refer to as "debt traps" when tariffs are high enough.

Assume that evaluated at free trade prices, Home has a net liability for delivery in Foreign, i.e. $a_A + \rho^{FT} a_B < 0$. In this case, when Foreign tariffs are high enough, an equilibrium emerges where Home must export *both* goods A and B to pay is liabilities. In this equilibrium, the terms of trade are such that the real value of Home's liability increases relative to free trade and it becomes immiserized by Foreign tariffs. We first state the result and then explain the economics:

Proposition 2 (Home's Debt Trap)**.** *Suppose Assumption 1 holds. If* $a_A + \rho^{FT} a_B < 0$ *and* τ^* *satisfies*

$$0 < \underline{\tau}^* < \tau^* < \overline{\tau}^* < \infty$$

where $\underline{\tau}^*, \overline{\tau}^*$ *are characterized relative to prices in the free trade equilibrium:*

$$\underline{\tau}^* \equiv \frac{(c_A^{FT} + \rho^{FT} c_B^{FT}) - (1/h(\rho^{FT}) + \rho^{FT}) Y_B}{-(a_A + \rho^{FT} a_B)}$$

$$\overline{\tau}^* \equiv \frac{c_A^{FT} + \rho^{FT} c_B^{FT}}{-(a_A + \rho^{FT} a_B)}.$$

Then there exists a tariff equilibrium where Home exports both goods: $c_A < Y_A$, $c_B < Y_B$*; relative prices are the same as under free trade:* $g(c_B/c_A) = \rho^{FT}$*; and Home's budget set relative to free trade is:*

$$c_A - c_A^{FT} + \rho^{FT}(c_B - c_B^{FT}) = \tau^*(a_A + \rho^{FT} a_B).$$

Proof: See Appendix A.4.

To see the intuition behind this result, consider the simple case in which $a_B < 0$ and $a_A = 0$. Let us guess and verify an equilibrium in which Home exports both goods. Goods arbitrage when $c_A < Y_A$ and $c_B < Y_B$ implies that $p_A^* = (1 + \tau^*) p_A$ and $p_B^* = (1 + \tau^*) p_B$. Hence, Foreign and Home households face the same relative prices, and these prices equate their marginal rates of substitution. That is, their indifference curves are tangent and hence the conjectured equilibrium allocation lies along the diagonal of the Edgeworth box. We depict our candidate equilibrium as point E_1 in Figure 4. Note that this implies that

$$g(c_B/c_A) = g(c_B^*/c_A^*) = g(\overline{Y}_B/\overline{Y}_A) = \rho^{FT},$$

where the last equality follows from the definition of ρ^{FT}. Thus, households face the free trade relative price between goods A and B. But their budget constraints differ from the ones under free trade. Home's budget constraint in this case is:

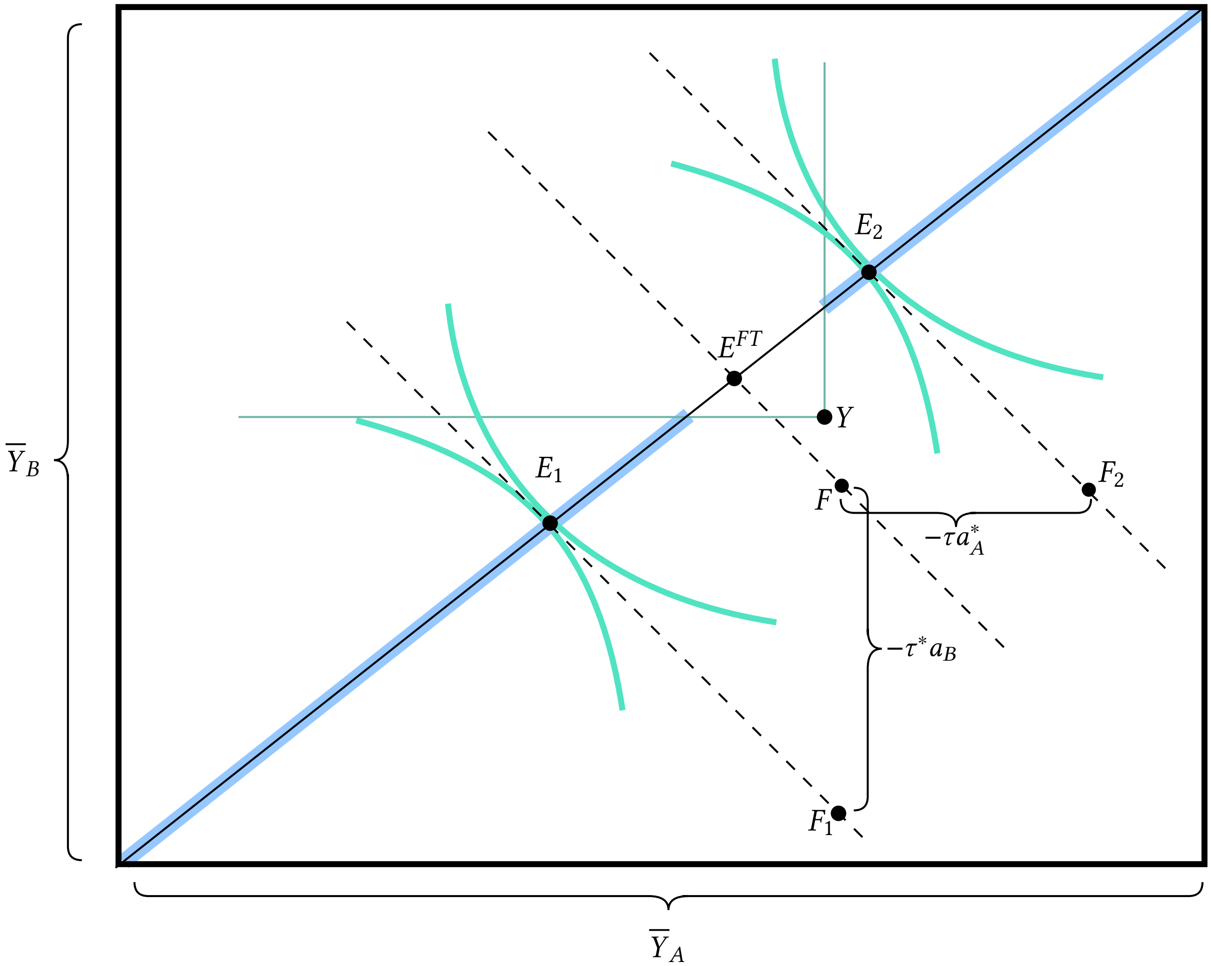
$$\begin{aligned} p_A c_A + p_B c_B &= p_A(Y_A - a_A^*) + p_B(Y_B + (1+\tau^*)a_B) \\ \Rightarrow c_A + \rho^{FT} c_B &= Y_A - a_A^* + \rho^{FT}(Y_B + (1+\tau^*)a_B) \\ \Rightarrow c_A + \rho^{FT} c_B &= Y_A - a_A^* + \rho^{FT}(Y_B + a_B) + \rho^{FT}\tau^* a_B. \end{aligned}$$

This is the same as the free-trade budget line but translated down by $\tau^* a_B < 0$. Define $F_1 \equiv (Y_A - a_A^*, Y_B + (1+\tau^*)a_B)$ as the translated financial position of Home in our conjectured equilibrium. Point E_1 in the figure must be at the intersection of the line with slope $-1/\rho^{FT}$ from F_1 and the diagonal.

The last thing to check is that E_1 is indeed in the region such that $0 < c_A < Y_A$ and $0 < c_B < Y_B$, as required for equilibrium. As we have moved along the diagonal from free trade to the left, we automatically have $c_A < c_A^{FT} < Y_A$. The lower bound on τ^* in the proposition ensures that F_1 is far enough down of E^{FT} such that $c_B < Y_B$ at E_1. The upper bound on τ^* ensures that it is still feasible for Home to pay the additional $\tau^*\rho^{FT}a_B$ relative to free trade; that is, that consumption is not forced to be negative to satisfy the budget set.

As E_1 is on the diagonal and strictly below the free trade equilibrium, it is unambiguously the case that Home suffers in welfare terms from the foreign tariff relative to free trade and Foreign gains. As $a_B < 0$, as τ^* increases, the burden of delivering good B to Foreign (in Foreign) increases for Home. It is "as if" Home must pay a liability in an appreciated foreign currency due to the foreign tariff. This immiserization forces Home to export both goods in order to service its debt. This outcome is reminiscent of a country (other than the US) that owes external debt in US dollars. As the United States raises tariffs, the dollar appreciates making the burden of debt greater when evaluated at the debtors' prices (or world prices). The higher tariffs in the United States thus paradoxically force the debtor to export more not less, in contrast to the case with positive assets in Proposition 1.

Figure 4: Tariff War: Multiplicity with Short Selling



Note: Multiplicity of equilibria when $a_B < 0$ and $a_A^* < 0$. E_1 and E_2 represent two tariff equilibria for the same tariff rates (τ, τ^*). The shaded segments in the contract curve represent allocations that can now be attained as an equilibrium outcome for some tariff combinations of (τ, τ^*).

Symmetry and Multiplicity Note that Proposition 2 does not rely on particular values for Home's tariffs or Foreign's liability. In fact, a symmetric result holds for Foreign:

Corollary 1 (Foreign's Debt Trap). *Suppose that Assumption 1 holds. If $a_A^* + \rho^{FT} a_B^* < 0$ and τ satisfies*

$$0 < \frac{(c_A^{*FT} + \rho^{FT} c_B^{*FT}) - (1 + \rho^{FT} h(\rho^{FT}))Y_A^*}{-(a_A^* + \rho^{FT} a_B^*)} < \tau < \frac{c_A^{*FT} + \rho^{FT} c_B^{*FT}}{-(a_A^* + \rho^{FT} a_B^*)},$$

then there exists a tariff equilibrium where $c_A^* < Y_A^*$, $c_B^* < Y_B^*$, $g(c_B^*/c_A^*) = \rho^{FT}$ *and*

$$c_A^* - c_A^{*FT} + \rho^{FT}(c_B^* - c_B^{*FT}) = \tau(a_A^* + \rho^{FT} a_B^*).$$

Proof: See Appendix A.4.

Foreign's debt trap is depicted by point E_2 in Figure 4. The point F_2 represents the position $(Y_A - (1+\tau)a_A^*, Y_B + a_B)$, which is the free trade position translated to the right by $-\tau a_A^*$. As Home's tariff increases, Foreign's liability in good A (delivered in Home) becomes more of a burden, and it must turn to exporting both goods. This pushes the equilibrium toward Foreign's origin, reducing Foreign's welfare and increasing Home's.

The fact that τ plays no role in the condition for E_1 and τ^* plays no role in the condition for E_2 reflects that both equilibria feature only one country with imports, and the other country's tariff is not relevant. This implies that conditions for both equilibria can be simultaneously satisfied. That is, either scenario can be an equilibrium *for a fixed set of tariffs* τ *and* τ^*.

It is worth emphasizing that these equilibria are dramatically different. In one of them, Home is poor, and consumes strictly less than its endowment of both goods. In the other one, Foreign is poor, and consumes strictly less than its endowment of both goods. Which country has a debt trap is thus completely dependent on the coordination of beliefs (i.e., a sunspot). The multiplicity arises from the revaluation of net foreign asset positions due to the direction of trade. Under Assumption 2, these equilibria do not emerge. The reason is that in such cases, higher tariffs in Foreign make the Home wealthier (rather than poorer) when trade flows from Home to Foreign, invalidating the equilibrium conjecture that Home is poor enough. However, when a country is indebted in a "foreign currency" (i.e., a liability to be settled in the other country) it is vulnerable to a self-fulfilling welfare loss if tariffs are appropriately high.

The Home debt trap equilibrium cannot be sustained when τ^* is too large, as the equilibrium becomes infeasible (and similarly in Foreign's case). This suggests that when tariffs are sufficiently high, the only equilibrium is autarky. This leads us to a more general uniqueness result than Proposition 1. In the course of proving this result, we identify an additional type of equilibria that emerges as τ or τ^* become large. This type retains some features of the debt trap outcome but without full immiserization. Condition (7) in Proposition 3 below rules these cases out. (See also Appendix C, Proposition C4.)

3.6 A More General Uniqueness Result

The next proposition confirms this, extending the uniqueness result of Proposition 1.

Proposition 3 (Uniqueness under Same-Sign Gross Assets)**.** *Suppose that the asset valuations satisfy:*

$$a_A + \rho^* a_B \neq 0 \text{ for all } \rho^* \in [\rho^*_{aut}, \rho^{FT}] \text{ , and } a^*_A + \rho\, a^*_B \neq 0 \text{ for all } \rho \in [\rho^{FT}, \rho_{aut}], \quad (7)$$

where $\rho_{aut} \equiv g(Y_B/Y_A)$, $\rho^*_{aut} \equiv g(Y^*_B/Y^*_A)$ *are the autarkic relative prices. Suppose also that the gross asset positions satisfy the same-sign condition:*

$$(a^*_A + \rho_{aut} a^*_B) \times (a_A + \rho^*_{aut} a_B) > 0. \quad (8)$$

Then there exist finite $\bar{\tau}, \bar{\tau}^*$ *such that for all* $\tau \geq \bar{\tau}$ *and* $\tau^* \geq \bar{\tau}^*$*, the autarky allocation is the* unique *equilibrium.*

Proof. See Appendix A.5. □

The same sign condition requires that when, evaluated at autarkic prices, both countries' gross claims are on the same side of the balance sheet. Condition (7) rules out a case where the asset positions would admit an additional non-autarkic equilibrium of the kind studied in Appendix C (see Proposition C4). Note that if all assets holdings are of the same sign and non-zero, both conditions (7) and (8) are automatically satisfied.

3.7 An Eaton Kortum Extension

In Appendix C we redo the analysis using the Eaton and Kortum (2002) model, specialized to two countries. This is a multi-good production model of trade to which we add two assets in each country (one asset is a claim on production; the other is a claim on consumption). The results generalize to this case in the following way:

- When assets holdings have the same sign (when evaluated at the autarkic prices), all equilibria converge to autarky as both tariffs increase (See Proposition C5.)
- There exists a locus of tariffs (τ, τ^*) that supports an equilibrium with zero net foreign asset positions. As before, the debtor country cannot unilaterally close its net foreign asset position through tariffs alone (See Section C.7.)
- When there is a single financial asset traded, then in the limit as tariffs increase, the debtor country is strictly worse off than autarky, and has to export a share of its output to Foreign earning nothing in return (See Section C.6.)

- When both countries hold negative claims on each other, then multiple equilibria can arise for sufficiently high (but not too high) tariffs (See Proposition C4.)

4 A General Model

We now introduce a general environment to analyze the role of trade wars in a broader context than the two-good model of the preceding section. The generality of the model is useful as it extends easily to a dynamic model with uncertainty, which is the most appropriate setting to analyze current account imbalances. The environment tracks textbook trade models closely (e.g., Dixit and Norman, 1980) although the existence of gross asset positions changes the analysis in interesting ways. Within the context of this environment, we generalize Proposition 1. It turns out that despite the presence of an arbitrary number of goods (both tradable and nontradable), the key equilibrating force is always a single relative price, which can be interpreted as the exchange rate at the time a trade war is initiated.

4.1 Environment

We start by extending the preceding section's static exchange economy with two goods to an arbitrary number of traded and non-traded goods. Specifically, suppose there are $N \geq 1$ traded goods, and M and M^* non-traded goods in Home and Foreign, respectively. Let $\mathcal{I} \equiv \{1, \ldots, N+M\}$ and $\mathcal{I}^* \equiv \{1, \ldots, N+M^*\}$ denote the set of goods in each of the two countries. Home endowments are denoted by $\boldsymbol{Y} = \{Y_1, \ldots, Y_N, Y_{N+1}, \ldots, Y_{N+M}\} = \{Y_i\}_{i\in\mathcal{I}}$. Foreign endowments are $\boldsymbol{Y}^* \equiv \{Y_i^*\}_{i\in\mathcal{I}^*}$. It is straightforward to extend the environment to allow for resource costs of trade for traded goods, and we do so in Appendix B.2.

The utility functions of the corresponding representative households in Home and Foreign are $u(\boldsymbol{c})$ and $u^*(\boldsymbol{c}^*)$, respectively, where $\boldsymbol{c} \equiv \{c_i\}_{i\in\mathcal{I}}$ and $\boldsymbol{c}^* \equiv \{c_i^*\}_{i\in\mathcal{I}^*}$ are the consumption vectors. Let $\boldsymbol{p} \equiv \{p_i\}_{i\in\mathcal{I}}$ and $\boldsymbol{p}^* \equiv \{p_i^*\}_{i\in\mathcal{I}^*}$ denote the domestic price vectors in Home and Foreign, all quoted in a numeraire unit.

As before, the representative household in each country starts the period with gross claims on the other country to be settled at the respective issuers' prices. For non-traded goods, we assume that the payouts are delivered in the prices of their respective countries. Specifically, let $\boldsymbol{a} \equiv \{a_i\}_{i\in\mathcal{I}^*}$ denote Home ownership of assets (or liabilities, if negative) for delivery in Foreign, denominated in each of the Foreign goods $i \in \mathcal{I}^*$ (potentially including claims indexed on non-traded goods). Similarly, we denote by $\boldsymbol{a}^* \equiv \{a_i^*\}_{i\in\mathcal{I}}$ Foreign ownership of assets denominated in goods for delivery at Home.

Home households maximize utility subject to their budget constraint:

$$\max_{\boldsymbol{c}} u(\boldsymbol{c}) \quad \text{subject to} \quad \boldsymbol{p} \cdot \boldsymbol{c} \leq \boldsymbol{p} \cdot \boldsymbol{Y} + \boldsymbol{p}^* \cdot \boldsymbol{a} - \boldsymbol{p} \cdot \boldsymbol{a}^* + T.$$

where T is Home's tariff revenue, and the operator $\cdot$ denotes the dot product. Similarly, the Foreign households' problem is:

$$\max_{\boldsymbol{c}^*} u^*(\boldsymbol{c}^*) \quad \text{subject to} \quad \boldsymbol{p}^* \cdot \boldsymbol{c}^* \leq \boldsymbol{p}^* \cdot \boldsymbol{Y}^* + \boldsymbol{p} \cdot \boldsymbol{a}^* - \boldsymbol{p}^* \cdot \boldsymbol{a} + T^*.$$

We allow for u and u^* to be different and potentially non-homothetic. We only impose that they are concave and differentiable functions with finite derivatives at the autarkic allocations:

Assumption 3. *The utility functions u and u^* are strictly increasing, concave, differentiable, and have finite marginal utilities evaluated at the endowment: $u_i(\boldsymbol{Y}) < \infty$ and $u_j^*(\boldsymbol{Y}^*) < \infty$ for $i \in \mathcal{I}$ and $j \in \mathcal{I}^*$.*

The requirement of finite derivatives is satisfied if the endowment of each good is strictly positive in both countries (as we assumed in the preceding section), or if the marginal utilities are finite when consumption is zero (for example, if the utility delivers a demand with a choke price). As before, the key is that domestic relative prices for all goods are well defined in autarky.

International goods arbitrage requires that for all traded goods, $i \in \{1, \ldots, N\}$, we have:

$$\frac{p_i}{p_i^*} = \begin{cases} (1+\tau) & \text{for Home imports: } c_i > Y_i \\ 1/(1+\tau^*) & \text{for Foreign imports: } c_i < Y_i \end{cases}$$
$$\text{with } p_i/p_i^* \in [1/(1+\tau^*), (1+\tau)] \text{ if } c_i = Y_i.$$

The governments' budget constraints are given by equating transfers to the tariffs raised on the goods actively imported:

$$T = \tau \sum_{i=1}^{N} p_i^* \max\{c_i - Y_i, 0\}, \quad \text{and} \quad T^* = \tau^* \sum_{i=1}^{N} p_i \max\{c_i^* - Y_i^*, 0\}.$$

Finally, the resource constraints are:

$$\begin{aligned} c_i + c_i^* &= Y_i + Y_i^* && \text{for all } i \in \{1, \ldots, N\} \\ c_i &= Y_i && \text{for all } i \in \{N+1, \ldots, N+M\} \\ c_i^* &= Y_i^* && \text{for all } i \in \{N+1, \ldots, N+M^*\}, \end{aligned}$$

where the last two constraints follow from the non-traded nature of these goods.

We now define a competitive equilibrium for a given set of tariff rates, $\{\tau, \tau^*\}$, which is standard:

Definition 2. *Given a pair of tariff rates, (τ, τ^*), an equilibrium is a pair of consumption allocations for Home and Foreign, $\boldsymbol{c}$ and $\boldsymbol{c}^*$, and domestic prices in Home and Foreign, $\boldsymbol{p}$ and $\boldsymbol{p}^*$, such that: (i) households optimize their consumption levels subject to their budget constraints; (ii) prices are consistent with international goods arbitrage; (iii) the government budget constraints hold; and (iv) the aggregate resource constraints hold.*

Free Trade As in the two-good case, it is useful to establish the free trade equilibrium as a benchmark. In a free trade equilibrium, prices are equalized across countries for all traded goods:

$$p_i = p_i^* \text{ for all } i \in \{1, \ldots, N\}.$$

Prices and consumption levels are related through the optimality condition of the respective households. For the traded goods, it must be that

$$\frac{p_i}{p_1} = \frac{u_i(\boldsymbol{c}_{FT})}{u_1(\boldsymbol{c}_{FT})} = \frac{u_i^*(\boldsymbol{c}_{FT}^*)}{u_1^*(\boldsymbol{c}_{FT}^*)} \text{ for all } i \in \{1, \ldots, N\}.$$

For the non-traded goods in each country we have that

$$\frac{p_i}{p_1} = \frac{u_i(\boldsymbol{c}_{FT})}{u_1(\boldsymbol{c}_{FT})} \text{ for } i \in \{N+1, \ldots, M\}, \quad \text{and} \quad \frac{p_i^*}{p_1^*} = \frac{u_i^*(\boldsymbol{c}_{FT}^*)}{u_1^*(\boldsymbol{c}_{FT}^*)} \text{ for } i \in \{N+1, \ldots, M^*\}.$$

Note that if $\boldsymbol{p}_{FT}^* \cdot \boldsymbol{a} - \boldsymbol{p}_{FT} \cdot \boldsymbol{a}^* < 0$, then Home is a net debtor under free trade prices. Home is a net creditor for the reverse inequality.

Autarky. We define the *autarkic allocation* to be the one where each country's consumption corresponds to their *physical* endowment. That is, $\boldsymbol{c} = \boldsymbol{Y}$ and $\boldsymbol{c}^* = \boldsymbol{Y}^*$. We proceed next to extend Proposition 1 to this more general environment by providing conditions on assets and tariffs such that the autarkic allocation is an equilibrium.

Towards this, denote by $\boldsymbol{\rho}_{aut}$ and $\boldsymbol{\rho}^*_{aut}$ the vectors of relative prices with respect to good 1 consistent with autarky for both Home and Foreign. That is, $\boldsymbol{\rho}_{aut} = \{1, \rho_{aut,2}, \ldots, \rho_{aut,N+M}\}$ and $\boldsymbol{\rho}^*_{aut} = \{1, \rho^*_{aut,2}, \ldots, \rho^*_{aut,N+M^*}\}$ such that:

$$\rho_{aut,i} \equiv \frac{u_i(\boldsymbol{Y})}{u_1(\boldsymbol{Y})} \text{ for all } i \in \mathcal{I}, \qquad \text{and} \qquad \rho^*_{aut,j} \equiv \frac{u^*_j(\boldsymbol{Y}^*)}{u^*_1(\boldsymbol{Y}^*)} \text{ for all } j \in \mathcal{I}^*.$$

Note that these relative prices are completely determined by endowments and preferences.

Then, we have the following result which generalizes Proposition 1 from the two-good environment:

Proposition 4. *Suppose Assumption 3 holds, and that* $\boldsymbol{a}$ *and* $\boldsymbol{a}^*$ *are such that*

$$(\boldsymbol{\rho}_{aut} \cdot \boldsymbol{a}^*) \times (\boldsymbol{\rho}^*_{aut} \cdot \boldsymbol{a}) > 0. \tag{9}$$

Then there exist $\underline{\tau}$ *and* $\underline{\tau}^*$ *such that for all tariffs such that* $\tau \geq \underline{\tau}$ *and* $\tau^* \geq \underline{\tau}^*$*, the autarky allocation* $\boldsymbol{c} = \boldsymbol{Y}$ *and* $\boldsymbol{c}^* = \boldsymbol{Y}^*$ *is an equilibrium.*

The proof is by construction, and it follows below.

Condition (9) may hold independently of whether Home is a net debtor or a net creditor under free trade. It generalizes and relaxes Assumption 2 introduced in the two-good environment. What is important is that the gross positions of each country are non-zero and of the same sign when evaluated at the autarkic relative prices. This nests both the case of simple "positive claims." and the case of symmetric negative positions from the two-good model. A simple case where this will occur is when all of the elements of $\boldsymbol{a}$ and $\boldsymbol{a}^*$ are of the same sign, with at least one of each non-zero.

Given that consumption allocations must equal endowments, we know that equilibrium Home and Foreign consumption levels can be supported by domestic relative prices that equal the ratios of the marginal utilities in autarky:

$$\boldsymbol{p} = p_1 \times \boldsymbol{\rho}_{aut}, \qquad \text{and} \qquad \boldsymbol{p}^* = p^*_1 \times \boldsymbol{\rho}^*_{aut}.$$

for some finite non-zero scalars p_1 and p^*_1. Assumption 3 guarantees that these prices are well defined. Since endowments pin down relative prices within each region, the ratio p^*_1/p_1 determines the ratio of the price *level* in Foreign relative to Home. Hence, p^*_1/p_1 will be proportional to the real exchange rate. For convenience, we therefore define

$$e \equiv p_1^*/p_1$$

to be our notion of a real exchange rate. This will be our counterpart to the relative price ρ that featured in the analysis of Section 3.

In an autarkic equilibrium, there is no trade; and no tariff revenue, $T = T^* = 0$. Hence, given that the budget constraints of both Home and Foreign hold with equality for $\boldsymbol{c} = \boldsymbol{Y}$ and $\boldsymbol{c}^* = \boldsymbol{Y}^*$, it must be that $\boldsymbol{p} \cdot \boldsymbol{a}^* - \boldsymbol{p}^* \cdot \boldsymbol{a} = 0$. For the net foreign asset positions to zero out we require:

$$\boldsymbol{p} \cdot \boldsymbol{a}^* = \boldsymbol{p}^* \cdot \boldsymbol{a} \quad \Rightarrow \quad p_1(\boldsymbol{\rho}_{Aut} \cdot a^*) = p_1^*(\boldsymbol{\rho}_{Aut}^* \cdot \boldsymbol{a}), \tag{10}$$

which will hold for a particular level of the real exchange rate:

$$e^{BT} \equiv \frac{\boldsymbol{\rho}_{Aut} \cdot \boldsymbol{a}^*}{\boldsymbol{\rho}_{Aut}^* \cdot \boldsymbol{a}} > 0,$$

where the inequality follows from the condition in the proposition. It is in this way that *asset positions pin down the real exchange rate* when tariffs are sufficiently high. This is the generalization of ρ^{BT} from Proposition 1.

To verify the rest of the equilibrium conditions, concavity of the utility functions guarantee that, given that the optimality condition holds by the construction of $\boldsymbol{p}$ and $\boldsymbol{p}^*$, the autarkic consumption bundles solve the households' problem at Home and Foreign. Moreover, the autarky allocations satisfy the resource constraints by construction. The only equilibrium condition left to check is the international goods arbitrage. For that, we need to ensure that the relative prices for each traded good across countries are consistent with no trade:

$$\frac{p_i}{p_i^*} = \frac{p_1}{p_1^*}\frac{u_i(\boldsymbol{Y})}{u_1(\boldsymbol{Y})}\frac{u_1^*(\boldsymbol{Y}^*)}{u_i^*(\boldsymbol{Y}^*)} \in \left[\frac{1}{1+\tau^*}, 1+\tau\right] \text{ for all } i \in \{1, \ldots, N\}.$$

The following (finite) lower bounds on tariffs ensure this is true for all goods:

$$\begin{aligned} \tau \geq \underline{\tau} &= \left(\frac{\boldsymbol{\rho}_{Aut}^* \cdot \boldsymbol{a}}{\boldsymbol{\rho}_{Aut} \cdot \boldsymbol{a}^*}\right) \max_{i\in\{1,\ldots,N\}} \left\{\frac{u_i(\boldsymbol{Y})}{u_1(\boldsymbol{Y})}\frac{u_1^*(\boldsymbol{Y}^*)}{u_i^*(\boldsymbol{Y}^*)}\right\} - 1, \\ \tau^* \geq \underline{\tau}^* &= \left(\frac{\boldsymbol{\rho}_{Aut} \cdot \boldsymbol{a}^*}{\boldsymbol{\rho}_{Aut}^* \cdot \boldsymbol{a}}\right) \max_{i\in\{1,\ldots,N\}} \left\{\frac{u_i^*(\boldsymbol{Y}^*)}{u_1^*(\boldsymbol{Y}^*)}\frac{u_1(\boldsymbol{Y})}{u_i(\boldsymbol{Y})}\right\} - 1. \end{aligned} \tag{11}$$

If tariffs satisfy these bounds, then the autarky allocation is an equilibrium.

It is instructive to consider a case where Assumption 3 fails, such as an Armington model with fully specialized endowments. If the marginal utility of the imported good approaches infinity at zero consumption, Assumption 3 is violated, rendering autarky unattainable via finite tariffs. However, introducing a choke price (implying finite marginal utility at zero) restores the validity of Assumption 3.[11]

4.2 Time and uncertainty

Trade imbalances are inherently a reflection of inter-temporal trade, or trade across states of the world. In this subsection we extend our analysis to a dynamic, stochastic economy. As Dixit and Norman (1980) highlight, if the analysis contains an arbitrary number of goods, it is straightforward to introduce intertemporal trade (and uncertainty). This just requires re-interpreting different goods as different times or states, and intra-temporal prices as inter-temporal ones, as in the classic Arrow-Debreu tradition.[12] The key outcome is to show how interest rates and exchange rates adjust so that desired inter-temporal risk-sharing becomes compatible with autarky.

Viewed in this way, Proposition 4 tells us that, in a dynamic environment, a permanent increase in bilateral tariffs that is high enough makes autarky an equilibrium as long as the total value of the asset positions of each country have the same sign *when evaluated at the respective autarky prices*. More explicitly, consider a deterministic dynamic economy starting at $t = 0$. For simplicity, we abstract from uncertainty, but the addition of one more index indicating the state of nature is straightforward. Suppose households' utilities are given by:

$$\sum_{t=0}^{\infty} \beta^t u(C_t), \quad \text{and} \quad \sum_{t=0}^{\infty} \beta^{*t} u^*(C_t^*),$$

where C_t and C_t^* are the consumption aggregates at date t. That is,

$$C_t = C(c_1(t), c_2(t), \cdots, c_{N+M}(t)), \quad \text{and} \quad C_t^* = C^*(c_1^*(t), c_2^*(t), \cdots, c_{N+M^*}^*(t)),$$

where $c_i(t)$ is consumption of good i at time t in Home, $c_j^*(t)$ is consumption of good j at t in Foreign, and C and C^* are the respective aggregator functions. Let $Y_i(t)$ and $Y_j^*(t)$ be the endowments of good $i \in \mathcal{I}$ in Home and $j \in \mathcal{I}^*$ in Foreign at time t.

Let $p_i(t)$ be the time-zero price of good $i \in \mathcal{I}$ in Home at time t, and let $p_j^*(t)$, $j \in \mathcal{I}^*$ be the price of good $j \in \mathcal{I}^*$ in Foreign. That is, $p_i(t)$ is the period-zero Arrow-Debreu price of a unit of

[11] A related question is whether the autarkic prices $\boldsymbol{p}$ and $\boldsymbol{p}^*$ are unique under Assumption 3. Provided all autarkic consumption levels are strictly positive, the differentiability of the utility functions ensures uniqueness. However, if consumption of some goods is zero and marginal utilities remain bounded, the supporting prices are not unique; any price exceeding the marginal valuation at zero would support autarky.

[12] Given that we now have an infinity of goods, it is necessary to assume that the values of the endowments and the claims are finite in each country when evaluated at autarkic prices.

good i delivered (at Home) in period t. All prices are expressed in terms of a common numeraire. Autarky relative prices are pinned down by endowments. In particular, relative prices in autarky at Home satisfy

$$\frac{p_i(t)}{p_j(t')} = \beta^{t-t'} \frac{u_i(C_t)}{u_j(C_{t'})},$$

where $u_i(C_t)$ is the marginal utility of good i in period t evaluated at the autarkic allocation. A similar condition determines relative prices in Foreign.

Let $a_j(t)$ denote Home's initial claim to good $j \in \mathcal{I}^*$ in period t in Foreign. Let $a_i^*(t)$ denote Foreign's initial claim to good $i \in \mathcal{I}$ in Home. The condition corresponding to (10), i.e. the condition which zeros out net foreign assets, is:

$$\sum_{i\in\mathcal{I}} \sum_t p_i(t) a_i^*(t) = \sum_{j\in\mathcal{I}^*} \sum_t p_j^*(t) a_j(t). \tag{12}$$

As relative prices in each region are determined by endowments, equation (12) pins down the relative value of a numeraire in each country. It will be useful to consider as our numeraire the value of a basket consisting of weighted sum of the goods in period zero in each location:

$$\overline{p}_0 \equiv \sum_{i\in\mathcal{I}} \gamma_i p_i(0) = p_1(0) \sum_{i\in\mathcal{I}} \gamma_i \frac{p_i(0)}{p_1(0)}$$
$$\overline{p}_0^* \equiv \sum_{j\in\mathcal{I}^*} \gamma_j^* p_j^*(0) = p_1^*(0) \sum_{j\in\mathcal{I}^*} \gamma_j^* \frac{p_j^*(0)}{p_1^*(0)},$$

where $\gamma_i, \gamma_j^* > 0$ and $\sum_i \gamma_i = \sum_j \gamma_j^* = 1$ are the weights for the numeraire price index. The second equality in each line above establishes that, as in the benchmark, there is only one degree of freedom given that relative prices are given by endowments. Let $q_0 \equiv \overline{p}_0^*/\overline{p}_0$ be the initial exchange rate (i.e., relative value of the numeraire baskets) that zeros out initial asset positions in (12).

Proposition 4 states that q_0 defined in this way supports an autarkic equilibrium for high enough (permanent) tariffs. The initial exchange rate is such that at period zero each country finds that the value of their initial assets equals their initial liabilities. From this period on, the financial positions can be netted out and there is no further need for additional trades.

We can shed more light on the dynamic behavior of the exchange rate by analyzing the corresponding sequence formulation of the autarkic equilibrium. Consider the two one-period risk-free bonds that promise at time t to pay one unit of the numeraire baseket at $t+1$ in each country. The time-zero value of a such a claim in Home is $\overline{p}_t \equiv \sum_{i\in\mathcal{I}} \gamma_i p_i(t)$, while in Foreign it is: $\overline{p}_t^* \equiv \sum_{j\in\mathcal{I}^*} \gamma_j^* p_j^*(t)$. The one-period interest rate at time t for each bond can be defined as the relative value of the promised basket for period $t+1$ relative to t:

$$R_t \equiv \frac{\overline{p}_t}{\overline{p}_{t+1}} \quad \text{and} \quad R_t^* \equiv \frac{\overline{p}_t^*}{\overline{p}_{t+1}^*}.$$

The interest rate on each bond ensures the residents in the respective countries do not wish to borrow or save in their own bonds. Defining the sequential exchange rate at time t as $q_t \equiv \frac{\overline{p}_t^*}{\overline{p}_t}$, we have

$$R_t = R_t^* \frac{q_{t+1}}{q_t},$$

which is the usual parity condition relating the differences in rates of return across countries to the expected depreciation of the exchange rate. More generally, interest rates and exchange rates are such that the risk-sharing condition of Backus and Smith (1993) holds, despite no trade occurring. That is, the exchange rate and interest rates are such that neither country wishes to borrow or save in the other country's bonds. Hence, the autarkic interest rates and the changes in the exchange rate ensure zero asset net positions after the initial period, while the *the level* of the initial exchange rate q_0 ensures that budgets balance in autarky.[13]

This result may provide a rationalization of why the recent trade war affected the dollar exchange rate, but had no discernible impact on interest rates. In a severe enough tariff war, the exchange rate and the interest rates are not connected but rather balance distinct equilibrium forces.

5 Quantifying valuation effects in a trade war

Our theoretical results apply mainly to the extreme case of full autarky. We now quantify the valuation, terms of trade, and welfare effects of a trade war which falls short of imposing autarky, and show how these effects depend on the nature of the gross portfolio underlying a given net foreign asset position. To do so, we use a dynamic stochastic gravity model based on Fitzgerald (2025). The model has many features typical of the quantitative trade literature (such as trade costs and roundabout production) which allow it to match the pattern of gross trade. We allow productivity to be stochastic, and assume that financial markets are complete and frictionless

[13] There is a clear antecedent in Alvarez, Atkeson, and Kehoe (2009). There, the authors analyze a two-country model where households consume only a non-traded good in each country, so by construction there cannot be international trade in goods. International financial markets are open, and thus rate of return equalization determines the required changes in exchange rates, just as here. The absence of trade in goods implies that the initial level of the exchange rate must be such that the initial net foreign asset position of each country is zero. Our result is related, with some distinctions. Our trade friction arises from the imposition of tariffs, and thus the absence of trade is not rule out by feasibility, but rather it is an equilibrium outcome. In addition, whether autarky is the unique equilibrium that emerges in the presence of high enough tariffs requires additional restrictions on initial asset positions.

with respect to productivity shocks. We calibrate the model to the US and a rest of the world (ROW) aggregate in 2023 (including net foreign assets) under the assumption that no trade war is expected. We then use the calibrated model to calculate the impact of an unexpected tariff war for four different asset portfolios, each of which is consistent with the same (data-driven) initial net foreign asset position.

5.1 Model

There are two countries, Home (the US) and Foreign (the rest of the world). Foreign variables are denoted by stars. In each period t, the world economy experiences one event, $s_t \in S$. Denote by s^t the history of events from date 0 to date t, $\{s_0, \ldots, s_t\}$. The probability at date 0 that history s^t will be realized at date t is given by $\pi\left(s^t\right)$, with $\sum_{s^t} \pi\left(s^t\right) = 1$. Since s_0 is known at date 0, $\pi\left(s_0\right) = 1$.

There is a representative agent in each country with CRRA expected utility. Date-0 utility in Home is given by:

$$U_0 = \frac{1}{1-\frac{1}{\sigma}}\left[\theta_0 C\left(s_0\right)^{1-\frac{1}{\sigma}} + \sum_{t=1}^{\infty}\sum_{s^t}\beta^t \pi\left(s^t\right) C\left(s^t\right)^{1-\frac{1}{\sigma}}\right]$$

where $C\left(s^t\right)$ is Home consumption of an aggregate good at date t given s^t. The inter-temporal elasticity of substitution (and inverse risk aversion) is σ. The parameter β is the discount factor. The parameter θ_0 is a preference shifter for the initial period, which we use to target the initial period allocation in our calibration. Preferences in Foreign are similar, with $C^*\left(s^t\right)$ denoting Foreign consumption, and θ_0^* the Foreign preference shifter at date 0 (all other parameters are identical in Home and Foreign).

The representative agent in Home supplies exogenous labor L in all dates t and histories s^t. Labor supply in Foreign is L^*. At date t given s^t, Home labor is used to produce $y\left(s^t\right)$ units of intermediate good 1:

$$y\left(s^t\right) = z\left(s^t\right)(L)^{1-\mu} M\left(s^t\right)^{\mu}$$

where $z\left(s^t\right)$ is productivity, $M\left(s^t\right)$ is material input, and μ is the materials share in production. Foreign labor is used to produce intermediate good 2 using a similar production function.

The intermediate goods are aggregated into a non-traded final good, X, which is used for consumption and materials:

$$X\left(s^t\right) = \left(x_1\left(s^t\right)^{\frac{\eta-1}{\eta}} + x_2\left(s^t\right)^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}} = C\left(s^t\right) + M\left(s^t\right)$$

where $x_i\left(s^t\right)$ is absorption of intermediate i in Home at s^t. The aggregator function in Foreign is identical. The Armington elasticity of substitution between the intermediates (η) determines the static trade elasticity ($\eta - 1$).

There are resource costs of trading intermediates across countries, assumed to be fixed. In order for one unit of Home's intermediate (good 1) to arrive in Foreign, $d^* \geq 1$ units must be shipped from Home. In order for one unit of Foreign's intermediate (good 2) to arrive in Home, $d \geq 1$ units must be shipped from Foreign. The resource constraints for intermediate goods must take account of resource costs of trade:

$$y\left(s^t\right) = x_1\left(s^t\right) + d^* x_1^*\left(s^t\right)$$

$$y^*\left(s^t\right) = d x_2\left(s^t\right) + x_2^*\left(s^t\right)$$

Markets for labor and intermediate goods are perfectly competitive, so wages are equal to the marginal product of labor, and intermediate goods prices in the country of production are equal to marginal cost. Let $p_t\left(s^t\right)$ be the price of 1 unit of the the Home intermediate (good 1) in Home at date t given s^t in terms of date-t "dollars" (freely traded). Let $1+\tau$ be the constant (gross) ad valorem tariff imposed by Home on imports from Foreign, and let $1+\tau^*$ be the constant tariff imposed by Foreign on imports from Home. The price in terms of date-t dollars of 1 unit of the Home intermediate in Foreign at date t given s^t inclusive of tariffs is $\left(1+\tau^*\right) d^* p_t\left(s^t\right)$. The price of 1 unit of the Foreign intermediate in Home is $\left(1+\tau\right) d p_t^*\left(s^t\right)$. Denote by $P_t\left(s^t\right)$ the price in date-t dollars of 1 unit of the final good in Home at date t given s^t. Since the market for final goods is perfectly competitive, $P_t\left(s^t\right)$ is given by:

$$P_t\left(s^t\right) = \left(p_t\left(s^t\right)^{1-\eta} + \left(\left(1+\tau\right) d p_t^*\left(s^t\right)\right)^{1-\eta}\right)^{\frac{1}{1-\eta}}$$

with a corresponding relationship between intermediate goods prices and the price of the final good in Foreign.

Tariff revenue in each country is rebated lump sum to agents in that country. Total Home tariff revenue in terms of date-t dollars at date t given s^t is:

$$T_t\left(s^t\right) = \tau d p_t^*\left(s^t\right) x_2\left(s^t\right)$$

with a similar expression for Foreign tariff revenue.

International Financial Positions. At date 0 agents can trade the full menu of Arrow-Debreu securities, which are claims to each of the intermediate goods, indexed by country of production and country of delivery. All agents face the same prices for these securities. Let $p_0(s^t)$ be the date-0 price of 1 unit of the Home intermediate good (good 1) in Home at date t given s^t, and let $p_0^*(s^t)$ be the price of the Foreign intermediate (good 2) in Foreign. Goods market arbitrage implies that the date-0 price of 1 unit of the Home intermediate in Foreign is $(1+\tau^*)\,d^*p_0(s^t)$, while the date-0 price of 1 unit of the Foreign intermediate in Home is $(1+\tau)\,dp_0^*(s^t)$. We denote Home tariff revenue in terms of date-0 dollars at date t given s^t by $T_0(s^t)$, while Foreign tariff revenue is $T_0^*(s^t)$.

The two countries may enter into date 0 with a portfolio of assets. Let $\alpha_0(s^t)$ denote Home's date-0 claim to the Foreign intermediate (good 2) delivered in Foreign at date t given s^t, expressed as a share of $y^*(s^t)$. In the notation used in the previous sections of the paper, $\alpha_0(s^t) = a_0(s^t)/y^*(s^t)$. Let $\alpha_0^*(s^t)$ denote Foreign's corresponding claim to a share of the Home intermediate (good 1) delivered in Home. Home's net foreign asset position at date 0 expressed in terms of date-0 dollars is then:

$$b_0 = \sum_{t=0}^{\infty}\sum_{s^t} p_0^*(s^t)\,\alpha_0(s^t)\,y^*(s^t) - \sum_{t=0}^{\infty}\sum_{s^t} p_0(s^t)\,\alpha_0^*(s^t)\,y(s^t) \tag{13}$$

which implies that the world net financial position at date 0 is zero:

$$b_0(s_0) + b_0^*(s_0) = 0$$

The date-0 budget constraint for Home is then:

$$\sum_{t=0}^{\infty}\sum_{s^t}\left(p_0(s^t)\,x_1(s^t) + (1+\tau)dp_0^*(s^t)\,x_2(s^t)\right) \le b_0(s_0) + \sum_{t=0}^{\infty}\sum_{s^t}\left(p_0(s^t)\,y(s^t) + T_0(s^t)\right)$$

Equilibrium. A competitive equilibrium is given by sequences of date-0 prices, intermediate goods absorption, materials and consumption such that:

(i) Given prices, tariffs and trade costs, country i chooses absorption of each of the intermediate goods, materials, and consumption for each date and state, to maximize welfare, subject to the country-i final good resource constraint, and the country-i ex-ante budget constraint.

(ii) Firm input demand is optimal given wages and intermediate prices.

(iii) All intermediate goods resource constraints are satisfied, and markets clear.

In the theoretical section, we focused on tariff wars that implement autarky, and thus required that autarky prices be well defined. In this case, the Armington assumption (complete specialization) together with the CES aggregator function imply that autarky prices are not well-defined, but this will not be an issue, as our interest is in tariff wars that do not lead to a complete trade shutdown.

5.2 Calibration

We treat the year 2023 as our period zero and the US and a composite of the rest of the world (ROW) as our two countries. The approach to calibrating preferences, the Armington parameters, and the productivity processes is similar to Fitzgerald (2025), and is detailed in Appendix D.

To calibrate initial net foreign asset positions, we use 2023 asset data from the External Wealth of Nations dataset (Lane and Milesi-Ferretti, 2002). We compute the net positions of our two regions by summing net positions in equity and foreign direct investment (FDI) and net debt positions. Making 2023 nominal US GDP the numeraire, we define

$$b_0 \equiv \frac{\text{(US FDI + equity assets - liabilities) + (US debt assets - liabilities)}}{GDP^{US}_{2023}} = -0.76.$$

This corresponds to the BEA net foreign asset position for the US, excluding financial derivatives and gold (which we ignore in the analysis). By market clearing, $b_0^* = -b_0$.

We calibrate to ensure that the initial period allocation in the model matches targeted macro aggregates observed in the 2023 data. Specifically, given b_0 as well as the structural parameters for preferences, production, and the productivity process estimated in Appendix D, we choose values for: period-0 relative productivity in ROW, z_0^* (normalizing $z_0 = 1$); the two trade costs, $\{d, d^*\}$; and the initial preference shifter for the ROW, θ_0^* (setting $\theta_0 = 1$); in order to match: the ratio of ROW nominal GDP to US GDP in 2023; US and ROW 2023 nominal exports as a fraction of US GDP; and the relative price levels in the two regions as measured by 2023 PPPs. Details of the computational algorithm are contained in Appendix D. The calibrated parameters are also reported there.

The choice of θ_0^* is used to capture the fact that the US runs a large trade deficit in 2023 despite its negative net foreign asset position. In effect, a $\theta_0^* < 1$ induces a transitory "savings glut" in the rest of the world that raises relative absorption in the US in the initial period. Setting $\theta_0 = 1$ is not without loss, as alternative values of this parameter would generate alternative inter-temporal prices, and hence shift the inter-temporal budget sets given observed initial asset positions.

5.3 Selecting the initial portfolios

As described above, we compute the benchmark equilibrium using the observed net foreign asset positions in 2023 expressed as a share of 2023 US GDP. In order to perform counterfactual exercises, we need to take a stand on the exact portfolio that underlies this net financial position.

To illustrate the role of gross asset positions in determining the impact of a given set of tariff policies, we propose four different candidate portfolios, each of which gives rise to the same b_0 in the free trade equilibrium.

Benchmark Portfolio. The first, benchmark, portfolio assumes that initial assets are claims to a constant share of the other region's output such that the value of US foreign assets relative to US GDP and the value of US foreign liabilities relative to US GDP equal their values in the data. This is reminiscent of the simple "positive claims" of Assumption 2. Hence, we find $\left\{\alpha_0^{bench}, \alpha_0^{*,bench}\right\}$ such that US foreign assets $a_0 > 0$ and ROW foreign assets $a_0^* > 0$ are given by:

$$a_0 = \alpha_0^{bench}\left(\sum_{t=0}^{T}\sum_{s^t} p_0^*\left(s^t\right)^{FT} y^*\left(s^t\right)^{FT}\right)$$

$$a_0^* = \alpha_0^{*,bench}\left(\sum_{t=0}^{T}\sum_{s^t} p_0\left(s^t\right)^{FT} y\left(s^t\right)^{FT}\right)$$

where the left-hand-side are the data described above, and the right-hand-side prices and quantities are those from the free trade equilibrium.[14]

No Gross Positions. The second portfolio assumes that the rest of the world holds a claim to a constant share of US output, but the US holds no claim to rest of the world output, i.e. a net-only portfolio expressed in terms of the US intermediate. This is akin to the one-asset case discussed in Section 3.4. Hence we find $\alpha_0^{*,netus} > 0$ such that:

$$-b_0 = b_0^* = \alpha_0^{*,netus}\left(\sum_{t=0}^{T}\sum_{s^t} p_0\left(s^t\right)^{FT} y\left(s^t\right)^{FT}\right)$$

while $\alpha_0^{netus} = 0$. Again, the left-hand-side is given by the data described above, while the right-hand-side prices and quantities are those from the free trade equilibrium. Now $a_0 = 0$ and $a_0^* = b_0^*$.

Debt-Trap Portfolio. The third portfolio assumes that the US owes a constant share of rest of the world output, but the rest of the world holds no claim to US output, i.e. a net-only portfolio

[14] We also construct a portfolio with equity and US and ROW bonds of different maturities, to match the actual portfolios from 2023 more closely. Results with this portfolio are very similar to the benchmark equity-only case, and are reported in Appendix D.

expressed in terms of the rest of the world intermediate. Hence we find $\alpha_0^{netrow} < 0$ such that:

$$b_0 = -b_0^* = \alpha_0^{netrow} \left(\sum_{t=0}^{T} \sum_{s^t} p_0^* \left(s^t\right)^{FT} y^* \left(s^t\right)^{FT} \right)$$

while $\alpha_0^{*,netrow} = 0$. This implies $a_0 = b_0 < 0$ while $a_0^* = 0$. This is reminiscent to the debt trap portfolio discussed in Section 3.5.

Contract Curve Portfolio. The final portfolio (the "contract curve" portfolio) exactly delivers equilibrium absorption at free trade prices. That is, we choose the $\left\{\alpha_0\left(s^t\right), \alpha_0^*\left(s^t\right)\right\}$ in equation (13) such that:

$$dx_2 \left(s^t\right)^{FT} = \alpha_0 \left(s^t\right)^{cc} y^* \left(s^t\right)^{FT}$$

$$d^* x_1^* \left(s^t\right)^{FT} = \alpha_0^* \left(s^t\right)^{cc} y \left(s^t\right)^{FT}$$

and the two terms on the right-hand-side of equation (13) give $a_0 > 0$ and $a_0^* > 0$. Like the benchmark portfolio, this portfolio satisfies the simple "positive claims" of Assumption 2.

5.4 Trade War Counterfactual

We assume that before any trade takes place at date 0, the US unexpectedly announces a tariff τ on all imports, current and future, while ROW announces a tariff τ^* on all imports, present and future. We assume that the process for productivity is unaffected by the trade war, and all claims contingent on the history of realizations of productivity are honored despite the fact that tariffs have unexpectedly changed.

For our primary trade war exercise, we set tariffs equal to 25% in each country ($\tau = \tau^* = 0.25$). In the Appendix, we also examine the impact of a unilateral 25% tariff in the US, and a unilateral 25% tariff in ROW. For each portfolio, Table 4 reports the 2023 US foreign assets and liabilities, net foreign assets, and net exports as percentages of 2023 US GDP. It also reports the percent change in the 2023 US terms of trade and consumption real exchange rate relative to free trade, and the percent change in 2023 US and ROW welfare (in consumption-equivalent units) relative to free trade.

The first panel reports results for the the contract curve portfolio; the second panel is the baseline portfolio; the third panel is the no gross positions portfolio; and the final panel is the debt-trap portfolio. Additional results for both the baseline tariffs and for different values of the tariffs are reported in Appendix D.

As hinted by our theoretical analysis, the impact of a tariff war on net foreign assets, the terms of trade, and welfare varies across ex-ante portfolios. When the US holds substantial assets

Table 4: Trade War Scenarios

τ	τ^*	a	a^*	$a - a^*$	NX	$\Delta(p/p^*)$	$\Delta(P/P^*)$	ΔU	ΔU^*
				Contract Curve Portfolio					
0	0	314%	391%	-77%	-2.9%	0	0	0	0
25%	25%	319%	380%	-61%	-2.6%	-0.3%	0.5%	-0.52%	−0.32 %
				Baseline Portfolio					
0	0	81%	157%	-76%	-2.9%	0	0	0	0
25%	25%	83%	154%	-71%	-2.2%	-1.2%	-0.3%	-0.99%	−0.18 %
				No Gross Positions					
0	0	0	76%	-76%	-2.9%	0	0	0	0
25%	25%	0	75%	-75%	-2.1%	-1.5%	-0.6%	-1.18%	−0.12 %
				Debt-Trap Portfolio					
0	0	-76%	0	-76%	-2.9%	0	0	0	0
25%	25%	-79%	0	-79%	-1.9%	-1.9%	-1.0%	-1.38%	−0.06 %

Notes: a, a^* and NX are date-0 values of US claims to ROW, ROW claims to US, and US net exports respectively, expressed as percentages of date-0 US GDP. $\Delta(p/p^*)$ and $\Delta(P/P^*)$ are percentage deviations of date-0 US terms-of-trade and real exchange rate from their levels at date 0 under free trade. ΔU and ΔU^* are percentage deviations of date-0 US and ROW welfare (in consumption-equivalent units) from their levels at date 0 under free trade.

in ROW, as well as ROW holding substantial assets in the US, the movement of international relative prices in the tariff war reduces US net debt. This is true in the baseline portfolio, and is particularly true in the contract curve portfolio. As shown in the Appendix, reducing the value of US net debt requires retaliation on the part of ROW: with US tariffs only, the value of US net debt increases.

In our calibration, a trade war with symmetric 25% tariffs always reduces welfare for both parties. Consistent with valuation effects exacerbating standard losses in a trade war in one case and mitigating them in the other, the welfare cost for the US is greatest under the debt-trap portfolio, and smallest under the contract curve portfolio.

Broadly speaking, these results confirm our theoretical analysis, for levels of tariffs which do not lead to autarky. In Appendix D, we also report the Nash equilibrium and unilaterally optimal tariffs for each portfolio, as well as their implications for assets, net exports, international relative prices, and welfare. Valuation effects under "positive claims" portfolios (baseline and contract curve) depress tariffs relative to the case of no gross positions, while in the case of the debt-trap portfolio, if unconstrained by US default, the ROW would choose to immiserize the US.

6 Conclusion

Using a series of two-country models, we study whether and how a net debt accumulated under free trade can be settled in the event of a tariff war. We show the answer depends on the composition of ex-ante international portfolios. Our key insight is that unanticipated tariffs revalue international assets, and that the impact of these valuation effects depends on ex-ante gross asset positions, not just net debt.

We consider three broad cases for international portfolios: strictly positive claims, claims on one side only, and negative claims on at least one side. The international asset portfolios of advanced economies broadly satisfy the strictly positive criterion, while emerging markets borrowing in foreign currency have some negative claims on their balance sheets.

When ex-ante gross positions are strictly positive, for high enough tariffs on both sides, valuation effects can wipe out an ex-ante net debt. This requires a real depreciation of the net debtor. However if claims are on one side only, tariff-induced valuation effects cannot wipe out an ex-ante debt. In the case where the debtor's liabilities must be honored in the creditor country, the creditor can immiserize the debtor by levying tariffs which raise the value of the debt relative to resources for repayment. This mechanism can lead to multiple equilibria if both countries have liabilities which must be honored in the lending country.

We provide a quantitative exploration of how the impact of a tariff war depends on the nature of ex-ante gross portfolios in a model calibrated to the US net foreign asset position in 2023. At the US gross portfolio in 2023, a sufficiently severe tariff war can reduce the value of US net debt, but it requires that the rest of the world to retaliate by levying tariffs on imports from the US.

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A Proofs

A.1 Proof of Lemma 1

Proof Part (i)

We shall prove Part (i) of the lemma for a relaxed version of Assumption 2:

Assumption 2′. *Asset positions satisfy:*

$$a_A + \rho^{FT} a_B > 0$$
$$a_A^* + \rho^{FT} a_B^* > 0.$$

We shall use this relaxation in the proof of Lemma 1.

Proof. Part (i) states that Home weakly exports good A and weakly imports good B. We prove this by ruling out alternative patterns for trade via contradiction:

(a) Suppose Home imports A and weakly exports B: $c_A > Y_A$ and $c_B \leq Y_B$:

This trade pattern implies that

$$\frac{c_B}{c_A} < \frac{Y_B}{Y_A} < \frac{\overline{Y}_B}{\overline{Y}_A} < \frac{Y_B^*}{Y_A^*} < \frac{c_B^*}{c_A^*}.$$

Household optimality then requires:

$$\frac{p_B}{p_A} = g(c_B/c_A) > g(c_B^*/c_A^*) = \frac{p_B^*}{p_A^*}.$$

Goods arbitrage implies that $p_A = (1+\tau)p_A^*$. Using this and the above, we have that

$$p_B > (1+\tau)p_B^*.$$

However, this violates goods arbitrage for good B, which requires $p_B \leq (1+\tau)p_B^*$, generating a contradiction.

(b) Home imports both goods: $c_A > Y_A$ and $c_B > Y_B$:

In this case, $p_A = (1+\tau)p_A^*$ and $p_B = (1+\tau)p_B^*$. The households optimality conditions then imply:

$$g(c_B/c_A) = g(c_B^*/c_A^*) = g(\overline{Y}_B/\overline{Y}_A) = \rho^{FT}.$$

Consider now the budget constraint for Foreign:

$$p_A^*(c_A^* + a_A) + p_B^*(c_B^* + a_B) = p_A^* y_A^* + p_B^* y_B^* + p_A a_A^* + p_B a_B^*,$$

where we used that $T^* = 0$ given that Foreign has no imports. Dividing by p_A^* and using $p_B^*/p_A^* = \rho^{FT}$ and $p_i = (1+\tau)p_i^*$ for $i \in \{A, B\}$, we obtain

$$\begin{aligned} c_A^* + \rho^{FT} c_B^* &= Y_A^* + \rho^{FT} Y_B^* + (a_A^* + \rho^{FT} a_B^*) - (a_A + \rho^{FT} a_B) + \tau(a_A^* + \rho^{FT} a_B^*) \\ &= c_A^{*,FT} + \rho^{FT} c_B^{*,FT} + \tau(a_A^* + \rho^{FT} a_B^*) \\ &\geq c_A^{*,FT} + \rho^{FT} c_B^{*,FT}, \end{aligned}$$

where the second line uses the fact that $Y_A^* + \rho^{FT} Y_B^* + (a_A^* + \rho^{FT} a_B^*) - (a_A + \rho^{FT} a_B)$ is the total wealth available for Foreign consumption under free trade and the final line uses the fact that $\tau(a_1^* + \rho^{FT} a_2^*) \geq 0$ by Assumption 2′. Given that Foreign households face the same prices as under free trade but have weakly more resources, they cannot choose a level of consumption of either good that is lower than under free trade. In particular, $c_A^* \geq c_A^{*,FT}$. However, Assumption 1 states that $c_A^{*,FT} > Y_A^*$, and hence $c_A^* > Y_A^*$, contradicting the premise that Foreign exports good A.

(c) Home exports both A and B: $c_A < Y_A$ and $c_B < Y_B$.

This case is symmetric to the previous one. In particular, $p_A^* = (1+\tau^*)p_A$ and $p_B^* = (1+\tau^*)p_B$. Hence $p_B/p_A = \rho^{FT}$. Retracing the same steps as in case (b), Home's budget set is:

$$\begin{aligned} c_A + \rho^{FT} c_B &= Y_A + \rho^{FT} Y_B + (a_A + \rho^{FT} a_B) - (a_A^* + \rho^{FT} a_B^*) + \tau^*(a_A + \rho^{FT} a_B) \\ &= c_A^{FT} + \rho^{FT} c_B^{FT} + \tau^*(a_A + \rho^{FT} a_B) \\ &\geq c_A^{FT} + \rho^{FT} c_B^{FT}, \end{aligned}$$

contradicting that $c_B < Y_B < c_B^{FT}$.

(d) Home does not trade good A and exports good B: $c_A = Y_A$ and $c_B < Y_B$:

This case is symmetric to case (a). Specifically, we have

$$\frac{c_B}{c_A} < \frac{Y_B}{Y_A} < \frac{\overline{Y}_B}{\overline{Y}_A} < \frac{Y_B^*}{Y_A^*} < \frac{c_B^*}{c_A^*}.$$

Household optimality then requires:

$$\frac{p_B}{p_A} = g(c_B/c_A) > g(c_B^*/c_A^*) = \frac{p_B^*}{p_A^*}.$$

Goods arbitrage implies that $p_B^* = (1 + \tau^*)p_B$. Using this and the above, we have that

$$p_A^* > (1 + \tau^*)p_A.$$

However, this violates goods arbitrage for good A.

Cases (a) and (b) imply that Home weakly exports good A. Case (c) and (d) further establishes that Home weakly imports good B.

□

Proof Part (ii)

Proof. Given Assumption 2, the budget set for Home is:

$$\begin{aligned} 0 &= p_A(Y_A - a_A^* - c_A) + p_B(Y_B - c_B) + p_B^* a_B + T \\ &= p_A(Y_A - a_A^* - c_A) + p_B(Y_B - c_B) + p_B^* a_B + \tau p_B^*(c_B - Y_B) \\ &= p_A(Y_A - a_A^* - c_A) + p_B^*(Y_B - c_B) + p_B^* a_B \end{aligned}$$

where the second line uses the definition of T and the fact from Part (i) that Home weakly imports good B and exports good A, and the third line uses that $p_B = (1 + \tau)p_B^*$ if $c_B > Y_B$ plus $c_B \geq Y_B$ and hence $p_B(Y_B - c_B) = (1 + \tau)p_B^*(Y_B - c_B)$. Dividing through by p_A and using $\rho \equiv p_B^*/p_A$ gives the first line of (1). The second line follows the same steps for the Foreign budget constraint, but with $T^* = \tau^* p_A(c_A^* - Y_A^*)$ and $p_A^*(Y_A^* - c_A^*) = (1 + \tau^*)p_A(Y_A^* - c_A^*)$.

□

Proof Part (iii)

Note that the proof of Part (iii) does not rely on Assumption 2.

Proof. Goods arbitrage requires $p_B^*/(1 + \tau^*) \leq p_B \leq (1 + \tau)p_B^*$, where the last inequality holds with equality when $c_B > Y_B$. Dividing through by p_A and using the Home optimality condition $g(c_B/c_A) = p_B/p_A$, we have

$$\frac{\rho}{1 + \tau^*} \leq g(c_B/c_A) \leq (1 + \tau)\rho.$$

Similar steps establish the condition for Foreign. □

We have shown that a tariff equilibrium must satisfy Conditions (i), (ii) and (iii). The reverse is straightforward: we can construct an equilibrium if Conditions (i), (ii), and (iii) are satisfied.

A.2 Proof of Proposition 1

We shall prove the proposition under the relaxed assumption 2′.

Proof. The proof of Proposition 4 below establishes that autarky is an equilibrium when the tariff rates are high enough in a general model that nests this case. To prove uniqueness, we rule out any non-autarkic equilibrium when tariffs are high enough. To do so, we exploit the fact that Part (i) of Lemma 1 applies and hence there is no trade reversal.

(a) Home exports good A and good B is not traded: $c_A < Y_A$ and $c_B = Y_B$:

Part (iii) of Lemma 1 implies

$$g(c_B^*/c_A^*) = \frac{\rho}{1+\tau^*}, \quad \text{and} \quad g(c_B/c_A) = \alpha\rho,$$

for $\alpha \in [1/(1+\tau^*), 1+\tau]$. Using $c_B = Y_B$ and $p_B/p_A = \alpha\rho$, the budget constraint for Home implies that:

$$\begin{aligned} c_A - Y_A &= (1+\tau^*)a_A - a_A^* + \rho(a_B - \alpha a_B^*) \\ &= (1+\tau^*)(a_A + g(c_B^*/c_A^*)a_B) - (a_A^* + g(c_B/c_A)a_B^*) \end{aligned}$$

Given $c_A < Y_A$ and $c_B = Y_B$, we have that $c_B^*/c_A^* < Y_B^*/Y_A^*$ and $c_B/c_A > Y_B/Y_A$, and thus

$$g(c_B^*/c_A^*) > g(Y_B^*/Y_A^*) \equiv \rho_{Aut,B}^* \quad \text{and} \quad g(c_B/c_A) < g(Y_B/Y_A) \equiv \rho_{Aut,B}.$$

Now from household's optimality we have that

$$g(c_B^*/c_A^*) = \frac{1}{\alpha(1+\tau^*)} g(c_B/c_A) \quad \Rightarrow g(c_B^*/c_A^*) \le g(c_B/c_A)$$

as $\alpha \in [1/(1+\tau^*), 1+\tau]$, and thus $\alpha(1+\tau^*) \ge 1$. This implies that

$$c_B^*/c_A^* \ge c_B/c_A \Rightarrow Y_B^*/c_A^* \ge Y_B/(\overline{Y}_A - c_A^*) \Rightarrow Y_B^*\overline{Y}_A - Y_B^*c_A^* \ge Y_B c_A^* \Rightarrow Y_B^*\overline{Y}_A \ge \overline{Y}_B c_A^*.$$

and thus given that $c_B = Y_B$,

$$c_B^*/c_A^* \ge \overline{Y}_B/\overline{Y}_A.$$

A similar argument then shows that

$$c_B/c_A \le \overline{Y}_B/\overline{Y}_A.$$

Taken together, we have then that

$$\rho_{Aut,B}^* = g(Y_B^*/Y_A^*) < g(c_B^*/c_A^*) \le g(\overline{Y}_B/\overline{Y}_A) = \rho^{FT} \le g(c_B/c_A) < g(Y_B/y_A) = \rho_{Aut,B}.$$

Now, if a_B is positive, we have that

$$a_A + g(c_B^*/c_A^*)a_B \geq a_A + \rho^*_{Aut,B}a_B.$$

If a_B is negative, then,

$$a_A + g(c_B^*/c_A^*)a_B \geq a_A + \rho_{FT}a_B.$$

Together, this means that

$$a_A + g(c_B^*/c_A^*)a_B \geq \min\{a_A + \rho_{FT}a_B, a_A + \rho^*_{Aut,B}a_B\} > 0.$$

where the inequality follows from Assumption 2′. Similarly, we can show that

$$a_A^* + g(c_B/c_A)a_B^* \leq \max\{a_A^* + \rho_{FT}a_B^*, a_A^* + \rho_{Aut,B}a_B^*\}.$$

Returning to the budget constraint, we have then that:

$$c_A - Y_A \geq (1+\tau^*)\underbrace{\min\{a_A + \rho_{FT}a_B, a_A + \rho^*_{Aut,B}a_B\}}_{>0} - \max\{a_A^* + \rho_{FT}a_B^*, a_A^* + \rho_{Aut,B}a_B^*\}.$$

It follows that $c_A > Y_A$ for τ^* large enough, generating a contradiction and ruling Case 1 out.

(b) Home imports good B and A is not traded: $c_A = Y_A$ and $c_B > Y_B$. This case can be ruled out with a symmetric argument to the one used for case (a), and showing that for τ sufficiently large, it cannot be that $c_B^* < Y_B^*$.

(c) Home exports good A and imports good B: $c_A < Y_A$ and $c_B > Y_B$:

From optimality and goods arbitrage we have that

$$g(c_B/c_A) = (1+\tau)\rho, \quad \text{and} \quad g(c_B^*/c_A^*) = \rho/(1+\tau^*).$$

It follows then that

$$g(c_B/c_A) = (1+\tau)(1+\tau^*)g(c_B^*/c_A^*).$$

Now, $c_A < Y_A$ and $c_B > Y_B$ imply that

$$c_B/c_A > Y_B/Y_A \Rightarrow g(c_B/c_A) < g(Y_B/Y_A),$$

and

$$c_B^*/c_A^* < Y_B^*/Y_A^* \Rightarrow g(c_B^*/c_A^*) > g(Y_B^*/Y_A^*).$$

Taken together we have that

$$g(Y_B/Y_A) > g(c_B/c_A) = (1+\tau)(1+\tau^*)g(c_B^*/c_A^*) > (1+\tau)(1+\tau^*)g(Y_B^*/Y_A^*).$$

If tariffs are sufficiently high so that

$$(1+\tau)(1+\tau^*) > g(Y_B/Y_A)/g(Y_B^*/Y_A^*),$$

then we have a contradiction of the comparative advantage assumption in 3.1.

According to Lemma 1, the only case left is autarky. □

A.3 Proof of Lemma 2

Proof. The proof follows from Lemma 1, which can be applied as $a_A = a_B = a_B^* = 0$ and $a_A^* \geq 0$. We just need to show that inequality (6) follows from Assumption 1. Note that under free trade, Home's budget constraint is:

$$c_A^{FT} + \rho^{FT} c_B^{FT} = Y_A + \rho^{FT} Y_B - a_A^*.$$

From the household first order condition under free trade:

$$g(c_B/c_A) = \rho^{FT} \Rightarrow c_B^{FT} = h(\rho^{FT})c_A^{FT}.$$

Using this in the budget constraint leads to:

$$(1/h(\rho^{FT}) + \rho^{FT})c_B^{FT} = Y_A - a_A^* + \rho^{FT} Y_B$$

Assumption 1 requires that $c_B^{FT} > Y_B$. Using this in the above equation, we get that:

$$(1/h(\rho^{FT}) + \rho^{FT})Y_B^{FT} < Y_A - a_A^* + \rho^{FT} Y_B \Rightarrow \frac{Y_B}{Y_A - a_A^*} < h(\rho^{FT}) = \frac{\overline{Y}_B}{\overline{Y}_A}.$$

A symmetric argument for Foreign shows that

$$h(\rho^{FT}) = \frac{\overline{Y}_B}{\overline{Y}_A} < \frac{Y_B^*}{Y_A^* + a_A^*}.$$

Taken together these last two imply condition (6). The rest of the proof follows from argument in the body of the text. □

A.4 Proof of Proposition 2 and Corollary 1

Proof. Suppose that τ^* satisfies the inequalities in the proposition. In the conjectured equilibrium, Home prices are $p_B/p_A = \rho^{FT}$ and Foreign prices are $p_B^*/p_A^* = (1+\tau^*)p_B/((1+\tau^*)p_B) = \rho^{FT}$. Note that with these prices, goods arbitrage holds. Let c_A and c_B be such that $g(c_B/c_A) = \rho^{FT}$ and Home budget constraint holds with equality. This guarantees that Home's household optimality conditions hold. Home's budget constraint can be rewritten as:

$$c_A + \rho^{FT} c_B = Y_A + \rho^{FT} Y_B + p_A^*/p_A(a_A + \rho^{FT} a_B) - (a_A^* + \rho^{FT} a_B^*).$$

Using the fact that $p_A^*/p_A = (1+\tau^*)$, we can rewrite this as

$$c_A + \rho^{FT} c_B = Y_A + \rho^{FT} Y_B + (1+\tau^*)(a_A + \rho^{FT} a_B) - (a_A^* + \rho^{FT} a_B^*).$$

And using the fact that $c_A^{FT} + \rho^{FT} c_B^{FT} = Y_A + \rho^{FT} Y_B + (a_A + \rho^{FT} a_B) - (a_A^* + \rho^{FT} a_B^*)$, we can rewrite this once more as

$$c_A + \rho^{FT} c_B = c_A^{FT} + \rho^{FT} c_B^{FT} + \tau^*(a_A + \rho^{FT} a_B). \tag{14}$$

Note that $\tau^* < (c_A^{FT} + \rho^{FT} c_B^{FT})/(-(a_A + \rho^{FT} a_B))$ by the condition in the Proposition. This implies that $c_A + \rho^{FT} c_B > 0$. Hence, expenditure is strictly positive and optimality implies that c_A and c_B are both non-negative. Next, we need to check that $c_A < Y_A$ and $c_B < Y_B$. Note that $c_A = c_B/h(\rho^{FT})$. Thus, from (14) we have that

$$c_B(1/h(\rho^{FT}) + \rho^{FT}) = c_A^{FT} + \rho^{FT} c_B^{FT} + \tau^*(a_A + \rho^{FT} a_B).$$

For $c_B < Y_B$, it suffices to show that:

$$Y_B(1/h(\rho^{FT}) + \rho^{FT}) > c_A^{FT} + \rho^{FT} c_B^{FT} + \tau^*(a_A + \rho^{FT} a_B).$$

But this is the condition in the proposition that imposes $\frac{(c_A^{FT}+\rho^{FT} c_B^{FT}) - (1/h(\rho^{FT}) + \rho^{FT})Y_B}{-(a_A + \rho^{FT} a_B)} < \tau^*$. For $c_A < Y_A$, recall from Assumption 1 that $c_A^{FT} < Y_A$ and $c_B^{FT} > Y_B$. Given that we have shown that $c_B < Y_B$, it follows that $c_B < c_B^{FT}$. Then, $c_A < c_A^{FT} < Y_A$, by homotheticity. Using the resource constraints, we can obtain c_A^* and c_B^*. Note that $c_A^* = \overline{Y}_A - c_A > 0$ and $c_B^* = \overline{Y}_B - c_B > 0$. Foreign optimality condition holds as $g(c_B^*/c_A^*) = g(c_B/c_A) = g(\overline{Y}_B/\overline{Y}_A) = \rho^{FT}$. We need to check that Foreign's budget constraint holds with equality. This follows from Walras law. We have thus constructed an equilibrium with the properties of the Proposition. A final thing to check is that $\underline{\tau}^* > 0$. Note that

$$Y_B(1/h(\rho^{FT}) + \rho^{FT}) < c_B^{FT}(1/h(\rho^{FT}) + \rho^{FT}) = c_A^{FT} + \rho^{FT} c_B^{FT}$$

where the first inequality follows by $c_B^{FT} > Y_B$ and the equality from $c_A^{FT} = c_B^{FT}/h(\rho^{FT})$. This implies that indeed

$$0 < \frac{(c_A^{FT} + \rho^{FT} c_B^{FT}) - (1/h(\rho^{FT}) + \rho^{FT}) Y_B}{-(a_A + \rho^{FT} a_B)} \equiv \underline{\tau}^*.$$

□

The proof of Corollary 1 is symmetric to the proof of the proposition.

A.5 Proof of Proposition 3

Proof. **Existence.** Follows from Proposition 4 restricted to two goods. The same-sign condition (8) is the two-good specialization of the general same-sign condition. For tariffs above the thresholds, the autarky allocation is supported as an equilibrium with exchange rate

$$e^{BT} = \frac{a_A^* + \rho_{aut} a_B^*}{a_A + \rho_{aut}^* a_B}.$$

Uniqueness. We rule out all non-autarkic equilibria for large enough tariffs. Any non-autarkic equilibrium must involve positive trade in at least one good. We consider each possible trade pattern:

Case 1: One country exports both goods. Suppose Home exports both goods. Foreign imports both, so $p_i^* = (1 + \tau^*) p_i$ for $i = A, B$ and $T = 0$. Since the tariff wedge $(1 + \tau^*)$ is common across goods, relative prices within each country are undistorted: $p_B/p_A = p_B^*/p_A^* = \rho^{FT}$. Normalizing $p_A = 1$, Home's budget constraint is:

$$c_A + \rho^{FT} c_B = Y_A + \rho^{FT} Y_B + (1 + \tau^*)(a_A + \rho^{FT} a_B) - (a_A^* + \rho^{FT} a_B^*).$$

If $a_A + \rho^{FT} a_B \neq 0$, the right-hand side diverges to $+\infty$ or $-\infty$ as $\tau^* \to \infty$, while $c_A + \rho^{FT} c_B \in [0, Y_A + \rho^{FT} Y_B]$—a contradiction. The symmetric argument bounds τ when Foreign exports both goods, using $a_A^* + \rho^{FT} a_B^* \neq 0$.

Case 2: Equilibria with two-way trade (Home exports one good, imports the other). Suppose there exist $\bar{\tau}_2, \bar{\tau}_2^*$ such that for all $\tau \geq \bar{\tau}_2$ and $\tau^* \geq \bar{\tau}_2^*$ there is an equilibrium with positive bilateral trade—say Home exports good A and imports good B. Goods arbitrage then requires:

$$\frac{p_A^*}{p_A} = 1 + \tau^*, \qquad \frac{p_B}{p_B^*} = 1 + \tau,$$

so the ratio of domestic relative prices satisfies

$$\frac{g(c_B/c_A)}{g(c_B^*/c_A^*)} = \frac{p_B/p_A}{p_B^*/p_A^*} = (1 + \tau)(1 + \tau^*). \tag{15}$$

Under this trade pattern, $c_B > Y_B$ (Home imports B) and $c_A^* > Y_A^*$ (Foreign imports A). For the numerator: $c_B/c_A \geq Y_B/\bar{Y}_A > 0$, so $g(c_B/c_A) \leq g(Y_B/\bar{Y}_A) < \infty$ (since g is decreasing). For the denominator: $c_B^*/c_A^* \leq \bar{Y}_B/Y_A^*$, so $g(c_B^*/c_A^*) \geq g(\bar{Y}_B/Y_A^*) > 0$. Hence the left-hand side of (15) is bounded above, while the right-hand side diverges—a contradiction.

The same argument rules out the reversed pattern (Home exports B, imports A) by symmetry.

Case 3: One-way trade in good A. Only good A is traded ($c_B = Y_B$, $c_B^* = Y_B^*$). Then c_A is the only free allocation variable, with $c_A^* = \bar{Y}_A - c_A$. Define the domestic relative prices as functions of c_A:

$$\rho(c_A) \equiv g\left(\frac{Y_B}{c_A}\right), \qquad \rho^*(c_A) \equiv g\left(\frac{Y_B^*}{\bar{Y}_A - c_A}\right).$$

At autarky ($c_A = Y_A$): $\rho(Y_A) = \rho_{aut}$ and $\rho^*(Y_A) = \rho_{aut}^*$.

Case 3a: $c_A < Y_A$ (Home exports A). The no-trade condition for good B requires that Foreign does not import B from Home:

$$\rho^*(c_A) \le \rho(c_A). \tag{16}$$

Since $c_A < Y_A$, we have $\rho(c_A) < \rho_{aut}$ and $\rho^*(c_A) > \rho_{aut}^*$. Moreover, ρ is increasing in c_A and ρ^* is decreasing, and they coincide at the free-trade allocation c_A^{FT} where $\rho(c_A^{FT}) = \rho^*(c_A^{FT}) = \rho^{FT}$. Since (16) requires $\rho^* \le \rho$, we must have $c_A \ge c_A^{FT}$, which gives $\rho(c_A) \ge \rho^{FT}$ and $\rho^*(c_A) \le \rho^{FT}$. Together:

$$\rho_{aut}^* \;\le\; \rho^*(c_A) \;\le\; \rho^{FT} \;\le\; \rho(c_A) \;\le\; \rho_{aut}. \tag{17}$$

Since Foreign imports A, goods arbitrage gives $p_A^* = (1+\tau^*)\, p_A$ and $T = 0$. Normalizing $p_A = 1$, Home's budget constraint is:

$$Y_A - c_A \;=\; \big(a_A^* + \rho(c_A)\, a_B^*\big) \;-\; (1+\tau^*)\big(a_A + \rho^*(c_A)\, a_B\big).$$

The left-hand side satisfies $0 < Y_A - c_A \le Y_A$. By (17), $\rho^*(c_A) \in [\rho_{aut}^*, \rho^{FT}]$. By condition (7), $a_A + \rho^*(c_A)\, a_B \ne 0$ for all feasible c_A, so $(1+\tau^*)(a_A + \rho^*(c_A)\, a_B)$ diverges uniformly as $\tau^* \to \infty$, while the left-hand side is bounded—a contradiction for τ^* large enough.

Case 3b: $c_A > Y_A$ (Home imports A). Since Home imports A, $p_A = (1+\tau)\, p_A^*$. For good B not to also flow from Foreign to Home, the tariff-inclusive cost of importing B must weakly exceed Home's domestic price: $(1+\tau)\, p_B^* \ge p_B$. Substituting $p_B = \rho\, p_A = \rho(1+\tau)\, p_A^*$ and $p_B^* = \rho^*\, p_A^*$ gives $\rho^*(c_A) \ge \rho(c_A)$. But $c_A > Y_A$ gives $\rho(c_A) > \rho_{aut}$, while $\rho^*(c_A) < \rho_{aut}^* \le \rho_{aut} < \rho(c_A)$, contradicting $\rho^* \ge \rho$. So this case is impossible as the comparative advantage assumption in 3.1 implies that $\rho_{aut}^* \le \rho_{aut}$.

Case 4: One-way trade in good B. Suppose only good B is traded ($c_A = Y_A$, $c_A^* = Y_A^*$). If $c_B > Y_B$ (Home imports B), the no-trade condition for A requires $\rho^* \le \rho$, and the same bounds (17) hold with $\rho(c_B) \in [\rho^{FT}, \rho_{aut}]$ and $\rho^*(c_B) \in [\rho_{aut}^*, \rho^{FT}]$. The budget constraint involves $(1+\tau)(a_A^* + \rho(c_B)\, a_B^*)$. By condition (7), $a_A^* + \rho\, a_B^* \ne 0$ for all $\rho \in [\rho^{FT}, \rho_{aut}]$, so the divergence argument gives a contradiction for τ large enough. If $c_B < Y_B$ (Home exports B), the no-trade condition for A requires $\rho^* \ge \rho$, but $\rho > \rho_{aut} \ge \rho_{aut}^* > \rho^*$—a contradiction, ruling out this case. □

A.6 Proof of Proposition 4

Proof contained in the text following the proposition.

B Model Extensions

In this appendix we introduce three extensions of the analysis: (i) Balanced-trade equilibria with positive trade, extending the results of Proposition 1; (ii) An example of the terms-of-trade change when Home is a net debtor but its terms-of-trade improve in a trade war; and (iii) Introducing trade costs into the model of Section 4.

B.1 Balanced trade equilibria with positive trade

Proposition 1 restricts attention to cases in which tariffs are high enough that autarky emerges as an equilibrium. We now show that for intermediate tariff levels, balanced trade is also attainable away from autarky.

We do so under Assumptions 1 and 2 and hence Lemma 1 applies. As we show in equation (3), with positive trade, the budget constraint for Home becomes:

$$c_A + \rho c_B = (Y_A - a_A^*) + \rho(Y_B + a_B)$$

where $\rho = p_B^*/p_A$ are the equilibrium terms-of-trade. Balanced trade occurs when $Y_A - c_A + \rho(Y_B - c_B) = 0$. From the budget constraint, this requires the net foreign assets positions to be zero:

$$\rho a_B = a_A^*.$$

Therefore, the terms-of-trade must be as in the autarky case, $\rho = \rho^{BT} \equiv a_A^*/a_B$.

Let us define

$$\underline{\theta} \equiv \frac{\overline{Y}_B - h(\rho^{BT})\left(Y_A - \rho^{BT} Y_B^*\right)}{h(\rho^{BT})\rho^{BT}\overline{Y}_A + Y_A^* - \rho^{BT} Y_B}.$$

As we show below, $\underline{\theta}$ reflects the Foreign consumption ratio, c_B^*/c_A^*, that satisfies balanced trade and the resource conditions when Home's tariff is zero. Any Foreign relative consumption ratio that is weakly greater than this and less than Y_B^*/Y_A^* can be implemented with an appropriate weakly positive Home tariff. We have the following result (proved at the end of this appendix):

Proposition A1 (Balanced Trade Equilibria). *Suppose that Assumptions 1 and 2 hold. Suppose also that* $g(Y_B^*/Y_A^*) < \rho^{BT} < g(Y_B/Y_A)$. *Let*

$$T^* \equiv [\mathbb{I}_{\{\underline{\theta}>0\}}(\rho^{BT}/g(\underline{\theta}) - 1), \rho^{BT}/g(Y_B^*/Y_A^*) - 1).$$

Then

(i) T^* *is non-empty.*

(ii) For any $\tau^* \in T^*$ *there exists a* $\tau \geq 0$ *such that there exists a tariff equilibrium with active trade such that trade is balanced:*

$$Y_A - c_A = -\rho^{BT}(Y_B - c_B),$$

and Home exports good A.

The construction of the set of tariffs consistent with balanced and nonzero trade proceeds as in the characterization of the equilibrium of Figure 3, but in reverse. Specifically, we know *a priori* the required terms-of-trade and seek the associated tariffs. Balanced trade is thus associated with the budget line through F with slope $-1/\rho^{BT}$, as shown in Figure 5. Recall that $F = (Y_A - a_A^*, Y_B + a_B)$; the fact that the slope of the budget line is $-a_B/a_A^*$ therefore implies it contains the endowment point $Y = (Y_A, Y_B)$, as well. That is, the value of the net foreign asset positions are zero along this line.

The requirement that $g(Y_B^*/Y_A^*) < \rho^{BT} < g(Y_B/Y_A)$ guarantees that in the absence of tariffs, when facing a hypothetical world price of ρ^{BT}, Home will be an exporter of good A, while Foreign will be an exporter of good B. A tariff equilibrium cannot reverse the pattern of trade, so the equilibrium must remain to the north-west of the endowment point.

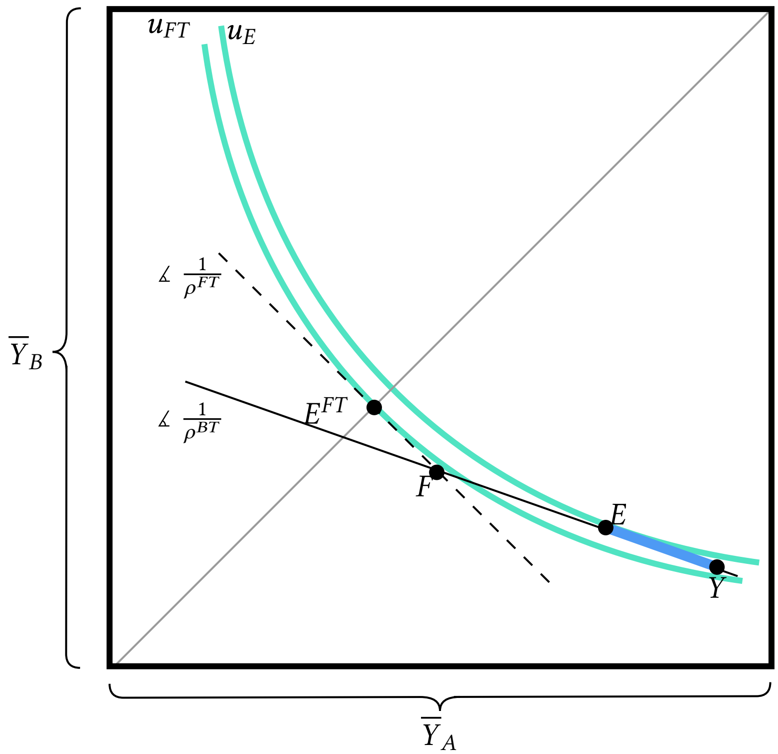
For a given foreign tariff rate τ^*, we obtain the slope of the ray from Foreign's origin:

$$\frac{c_B^*}{c_A^*} = h\left(\rho^{BT}/(1+\tau^*)\right).$$

Where this intersects the budget line is the balanced-trade equilibrium candidate associated with τ^*. Given the c_B/c_A associated with this intersection, we can recover the necessary Home tariff from Home's optimality condition:

$$g\left(\frac{c_B}{c_A}\right) = (1+\tau)\rho^{BT}.$$

Figure 5: Tariff War Equilibrium With Balanced Trade



Note: Home is a net debtor, and its terms-of-trade worsen.

The condition in Proposition that requires $\tau^* \geq \mathbb{1}_{\{\underline{\theta}>0\}}(\rho^{BT}/g(\underline{\theta}) - 1)$ ensures that this allocation is consistent with a non-negative Home tariff.

Proceeding in this way, for each candidate τ^* large enough, we obtain a unique τ that balances trade. The proof of Proposition A1 derives this formally. Note that as tariffs increase, the equilibrium allocation approaches the autarkic outcome along the budget line. The condition that $g(Y_B^*/Y_A^*) < \rho^{BT}$ guarantees that indeed the autarky limit is reached when $\tau^* = \rho^{BT}/g(Y_B^*/Y_A^*) - 1$.

Figure 5 shows a tariff war that achieves balanced trade. Recall that the dashed line is the budget line at free-trade prices, ρ^{FT}. The solid line represents the new budget line at prices $\rho^{BT} > \rho^{FT}$. Let point E represent the point on this new budget line associated with a zero tariff at Home (which will contain the ray from Home's origin with slope $h(\rho^{BT})$). At this point, an equilibrium with balanced trade is obtained when Foreign imposes the tariff that equates its marginal rate of substitution at point E's allocation to $\rho^{BT}/(1+\tau^*)$.

As we increase Home's and Foreign's tariffs, the associated rays from the respective origins rotate away from the diagonal, tracing out the set of balanced trade equilibria (depicted by the segment connecting E to Y). As both tariffs increase, we eventually converge to the autarkic allocation consistent with Proposition 1.

Proof of Appendix Proposition A1

Proof. Let $t_1 = \mathbb{I}\left(\underline{\theta} > 0\right)\left(\frac{\rho^{BT}}{g(\underline{\theta})} - 1\right)$ and $t_2 = \frac{\rho^{BT}}{g(Y_B^*/Y_A^*)} - 1$.

The first thing to check is that $t_1 < t_2$. Note that $t_2 > 0$ by the assumption that $g(Y_B^*/Y_A^*) < \rho^{BT}$. So if $\underline{\theta} \leq 0$, it follows that $t_1 < t_2$. For $\underline{\theta} > 0$, doing some algebra, we get that

$$\underline{\theta} < \frac{Y_B^*}{Y_A^*} \Leftrightarrow Y_B(Y_A^* + \rho^{BT} Y_B^*) < g^{-1}(\rho^{BT}) Y_A (Y_A^* + \rho^{BT} Y_B^*) \Leftrightarrow Y_B < g^{-1}(\rho^{BT}) Y_A$$

which holds by $g(Y_B/Y_A) > \rho^{BT}$.

Consider a $\tau^* \in [t_1, t_2)$, and let us construct a tariff equilibrium for some $\tau \geq 0$. Given that there is trade in equilibrium, $g(c_B^*/c_A^*) = \rho^{BT}/(1+\tau^*)$. Define

$$\theta^* \equiv g^{-1}\left(\frac{\rho^{BT}}{1+\tau^*}\right) = \frac{c_B^*}{c_A^*} < \frac{Y_B^*}{Y_A^*}, \tag{18}$$

where the last equality follows from condition (i): $g(Y_B^*/Y_A^*) < \rho^{BT}/(1+\tau^*)$. Using the fact that $a_A^* - \rho^{BT} a_B = 0$ by definition of ρ^{BT}, the Foreign budget constraint at world prices implies

$$c_A^* = \frac{Y_A^* + \rho^{BT} Y_B^*}{1 + \rho^{BT}\theta^*},$$

where we have used $c_B^* = \theta^* c_A^*$. The proposed equilibrium allocation for Foreign is thus:

$$(c_A^*, c_B^*) = \left(\frac{Y_A^* + \rho^{BT} Y_B^*}{1 + \rho^{BT}\theta^*}, \frac{\theta^*\left(Y_A^* + \rho^{BT} Y_B^*\right)}{1 + \rho^{BT}\theta^*}\right). \tag{19}$$

Note that $c_A^* > 0$ and $c_B^* > 0$. From the resource conditions, we can obtain the implied consumption allocation for Home:

$$\begin{aligned} c_A &= \overline{Y}_A - c_A^* = \frac{Y_A + \rho^{BT}(\theta^* \overline{Y}_A - Y_B^*)}{1 + \rho^{BT}\theta^*} = Y_A - \frac{\rho^{BT}(Y_B^* - \theta^* Y_A^*)}{1 + \rho^{BT}\theta^*} \\ c_B &= \overline{Y}_B - c_B^* = \frac{\overline{Y}_B - \theta^* Y_A^* + \rho^{BT}\theta^* Y_B}{1 + \rho^{BT}\theta^*} = Y_B + \frac{Y_B^* - \theta^* Y_A^*}{1 + \rho^{BT}\theta^*}. \end{aligned} \tag{20}$$

As $Y_B^* > \theta^* Y_A^*$ from (18), we have $c_B > Y_B > 0$ and $c_A < Y_A$ as required. That is, Home exports good A. To ensure that $c_A \geq 0$, we require

$$0 \leq Y_A + \rho^{BT}(\theta^* \overline{Y}_A - Y_B^*) \iff \frac{\rho^{BT} Y_B^* - Y_A}{\rho^{BT}\overline{Y}_A} \leq \theta^* = g^{-1}\left(\frac{\rho^{BT}}{1+\tau^*}\right).$$

If $\rho^{BT} Y_B^* \leq Y_A$, then this condition is satisfied immediately. Otherwise, the following condition ensures

$c_A \geq 0$:

$$\tau^* \geq \frac{\rho^{BT}}{g\left(\frac{\rho^{BT} Y_B^* - Y_A}{\rho^{BT} \overline{Y}_A}\right)} - 1. \tag{21}$$

Next, we need to construct a tariff τ such that the allocation in (20) is optimal for Home. Consider a candidate τ given by

$$\tau = \frac{g\left(c_B/c_A\right)}{\rho^{BT}} - 1,$$

where c_A and c_B are given by (20). With this tariff rate, the proposed allocation would be optimal for Home but we need to verify $\tau \geq 0$:

$$\tau \geq 0 \iff (1+\tau)\rho^{BT} \geq \rho^{BT} \iff \frac{c_B}{c_A} = g^{-1}((1+\tau)\rho^{BT}) \leq g^{-1}(\rho^{BT}).$$

Substituting for c_B/c_A using (20), we require

$$g^{-1}(\rho^{BT}) \geq \frac{\overline{Y}_B + \rho^{BT}\theta^* Y_B - \theta^* Y_A^*}{Y_A + \rho^{BT}(\theta^* \overline{Y}_A - Y_B^*)}$$

$$\iff$$

$$\left(g^{-1}(\rho^{BT})\rho^{BT}\overline{Y}_A + Y_A^* - \rho^{BT} Y_B\right)\theta^* \geq \overline{Y}_B - g^{-1}(\rho^{BT})\left(Y_A - \rho^{BT} Y_B^*\right). \tag{22}$$

Note that the left hand side coefficient is strictly positive:

$$g^{-1}(\rho^{BT})\rho^{BT}\overline{Y}_A + Y_A^* - \rho^{BT} Y_B > (Y_B/Y_A)\rho^{BT}\overline{Y}_A + Y_A^* - \rho^{BT} Y_B = \rho^{BT} Y_B(\overline{Y}_A/Y_A - 1) + Y_A^* > 0$$

where the first inequality follows from $g^{-1}(\rho^{BT}) > Y_B/Y_A$ (a premise in the lemma).
Dividing through (22) by this coefficient, we obtain the following condition as requirement for the Home tariff to be non-negative:

$$\theta^* \geq \frac{\overline{Y}_B - g^{-1}(\rho^{BT})\left(Y_A - \rho^{BT} Y_B^*\right)}{g^{-1}(\rho^{BT})\rho^{BT}\overline{Y}_A + Y_A^* - \rho^{BT} Y_B} \equiv \underline{\theta}. \tag{23}$$

If $\overline{Y}_B - g^{-1}(\rho^{BT})\left(Y_A - \rho^{BT} Y_B^*\right) \leq 0$, and thus $\underline{\theta} \leq 0$, then $\theta^* \geq \underline{\theta}$ and the condition holds.
Otherwise, if $\underline{\theta} > 0$, using $g(\theta^*) = \rho^{BT}/(1+\tau^*)$, the condition is equivalent to

$$\tau^* \geq \frac{\rho^{BT}}{g(\underline{\theta})} - 1. \tag{24}$$

And thus $\tau^* \in T^*$ implies that the Home tariff is non-negative.

Finally, let us come back to the restriction we needed for $c_A \geq 0$, (21). Note that,

$$\frac{\rho^{BT} Y_B^* - Y_A}{\rho^{BT} \overline{Y}_A} < \underline{\theta}.$$

Thus if $\underline{\theta} \leq 0$, we have that $\rho^{BT} Y_B^* - Y_A < 0$. And if $\underline{\theta} > 0$, we have that

$$\frac{\rho^{BT}}{g(\underline{\theta})} - 1 > \frac{\rho^{BT}}{g\left(\frac{\rho^{BT} Y_B^* - Y_A}{\rho^{BT} \overline{Y}_A}\right)} - 1$$

and (21) is implied by (24). And thus $\tau^* \in T^*$ is sufficient to guarantee $c_A \geq 0$.

Summarizing this: for any $\tau^* \in [t_1, t_2)$ condition (23) holds, and thus there exists a $\tau \geq 0$ such that the allocation in (19) and (20) is a balanced trade equilibrium. In this equilibrium, Home exports good A.

□

B.2 Trade costs

Modern quantitative trade theory emphasizes the role of trade costs in generating the "gravity" relation between bilateral expenditure shares, economic sizes, and distance. It is straightforward to incorporate such costs into the multi-good environment of Section 4. We focus now on an environment where all goods can potentially be traded (so $M = M^* = 0$). Let G be an "export" technology that takes as input a vector of goods in Home and generates a vector of goods in Foreign. Specifically, for $G(\boldsymbol{x})$ to reach Foreign, a vector of goods $\boldsymbol{x}$ needs to allocated. Let G^* be the analogous "export" technology for Foreign. These technologies capture all feasible methods of transporting goods across countries. We assume that G and G^* are weakly increasing, weakly concave, and differentiable functions in $\mathbb{R}^N_{\geq 0}$, both are constant returns to scale, and satisfy $\mathbf{0} = G(\mathbf{0}) = G^*(\mathbf{0})$. The case with no trade costs corresponds to $G(\boldsymbol{x}) = \boldsymbol{x}$ and $G^*(\boldsymbol{x}^*) = \boldsymbol{x}^*$. The standard case of good-specific iceberg trade costs corresponds to $G(\boldsymbol{x}) = \Lambda \boldsymbol{x}$ and $G^*(\boldsymbol{x}^*) = \Lambda^* \boldsymbol{x}^*$, where Λ and Λ^* are diagonal matrices with elements in $(0, 1]$, reflecting the survival rate of goods in transit (that is, the inverse of the iceberg trade costs).

The resource constraints are modified as follows:

$$\boldsymbol{Y} = \boldsymbol{c} + \boldsymbol{x} - G^*(\boldsymbol{x}^*), \quad \text{and} \quad \boldsymbol{Y}^* = \boldsymbol{c}^* + \boldsymbol{x}^* - G(\boldsymbol{x}),$$

where $\boldsymbol{x} \geq 0$ is the vector of Home goods used as inputs (or lost in transit for the iceberg case) to produce Foreign goods (exports), and $\boldsymbol{x}^* \geq 0$ is the corresponding vector for Foreign.

Competitive firms operate these technologies. Their final outputs are taxed with tariffs at

Home and in Foreign. The optimization problems of the representative exporting firms are:[15]

$$\max_{\{\boldsymbol{x}\}} \left\{ \frac{1}{1+\tau^*}\, \boldsymbol{p}^* \cdot G(\boldsymbol{x}) - \boldsymbol{p} \cdot \boldsymbol{x} \right\}, \quad \text{subject to: } \boldsymbol{x} \geq 0.$$
$$\max_{\{\boldsymbol{x}^*\}} \left\{ \frac{1}{1+\tau}\, \boldsymbol{p} \cdot G^*(\boldsymbol{x}^*) - \boldsymbol{p}^* \cdot \boldsymbol{x}^* \right\}, \quad \text{subject to: } \boldsymbol{x}^* \geq 0.$$

International goods arbitrage is encapsulated in these optimization problems. Constant returns to scale in the exporting technologies imply that exporting firms make zero profits in equilibrium, and thus the households' budget constraints remain as before. The governments' tariff revenues become:

$$T = \frac{\tau}{1+\tau}\, \boldsymbol{p} \cdot G^*(\boldsymbol{x}^*), \quad \text{and} \quad T^* = \frac{\tau^*}{1+\tau^*}\, \boldsymbol{p}^* \cdot G(\boldsymbol{x}).$$

The definition of equilibrium is analogous to the baseline definition, Definition 2, but includes $\boldsymbol{x}$ and $\boldsymbol{x}^*$ as equilibrium objects, have the updated resource constraints and government budget constraints, and requires that exporting firms optimize.

Consider the autarky allocation where $\boldsymbol{Y} = \boldsymbol{c}$, $\boldsymbol{Y}^* = \boldsymbol{c}^*$, and $\boldsymbol{x} = \boldsymbol{x}^* = \boldsymbol{0}$. Is this an equilibrium outcome for sufficiently high tariffs? This allocation satisfy the resource constraints. Domestic relative prices are determined by the respective autarkic allocations (endowments) exactly as in the frictionless case. Furthermore, the international relative price that zeros out the net foreign asset positions remains uniquely determined by the ratio of asset valuations at these autarkic prices, given that the households budget constraints are unchanged as there is no tariff revenue. This pins down $\boldsymbol{p}$ and $\boldsymbol{p}^*$.

The only remaining equilibrium condition to check is whether $\boldsymbol{x} = \boldsymbol{x}^* = \boldsymbol{0}$ is optimal for the exporting firms given these prices and tariffs. This holds if the marginal products of G and G^* are bounded above at $\boldsymbol{0}$, a natural restriction satisfied, for example, by linear iceberg costs. Under this restriction, a sufficiently high bilateral tariff pair (τ, τ^*) renders exporting unprofitable for any good, sustaining autarky as an equilibrium.[16]

[15] In our notation, domestic prices are measured inclusive of tariffs. As a result, the revenue of exporting firms must be scaled by $1/(1+\tau)$ and $1/(1+\tau^*)$ to reflect the pre-tariff revenue retained by firms.

[16] The exact condition, which follows from the concavity of the export technologies, is

$$\max_{i\in\{1,\dots,N\}} \sum_{j=1}^{N} \frac{p_j}{p_i^*} \left.\frac{\partial G_j^*(\boldsymbol{x}^*)}{\partial x_i^*}\right|_{\boldsymbol{x}^*=\boldsymbol{0}} - 1 = \underline{\tau} \leq \tau, \quad \text{and} \quad \max_{i\in\{1,\dots,N\}} \sum_{j=1}^{N} \frac{p_j^*}{p_i} \left.\frac{\partial G_j(\boldsymbol{x})}{\partial x_i}\right|_{\boldsymbol{x}=\boldsymbol{0}} - 1 = \underline{\tau}^* \leq \tau^*.$$

This collapses to (11) when $G(\boldsymbol{x}) = G^*(\boldsymbol{x}) = \boldsymbol{x}$, the case without trade frictions.

C An Eaton-Kortum (2003) Extension

In this appendix, we embed the asset structure of Section 4 into a production economy based on Eaton and Kortum (2002).

C.1 Model Setup

Two countries, Home (H) and Foreign (F), trade a continuum of goods $j \in [0,1]$. Preferences are CES over the continuum with elasticity of substitution $\sigma > 1$. Productivity for good j in each country is drawn i.i.d. from a Fréchet distribution with common scale $\mathcal{T}$ and shape $\theta \geq 1$. Assume the CES elasticity satisfies $\sigma \in (1, \theta + 1)$, and normalize the productivity scale to $\mathcal{T} = \Gamma\big(1 - (\sigma - 1)/\theta\big)^{\theta/(1-\sigma)}$ where $\Gamma(\cdot)$ denotes the gamma function. There are no iceberg trade costs ($d = d^* = 1$). Each country has one unit of labor supplied inelastically. Home wage is w; Foreign wage is normalized to $w^* = 1$. Home levies ad valorem tariff $\tau \geq 0$ on imports from Foreign; Foreign levies $\tau^* \geq 0$ on imports from Home.

Let π_{ik} denote the share of expenditure in country i on goods from country k. Standard EK derivations yield:

$$\pi_{HH} = \frac{(1+\tau)^\theta}{(1+\tau)^\theta + w^\theta}, \qquad \pi_{HF} = 1 - \pi_{HH} = \frac{w^\theta}{(1+\tau)^\theta + w^\theta}, \tag{25}$$

$$\pi_{FF} = \frac{(1+\tau^*)^\theta w^\theta}{(1+\tau^*)^\theta w^\theta + 1}, \qquad \pi_{FH} = 1 - \pi_{FF} = \frac{1}{(1+\tau^*)^\theta w^\theta + 1}. \tag{26}$$

Given our assumptions on technology and preferences, the ideal price index in each country is:

$$P_H = \left[w^{-\theta} + (1+\tau)^{-\theta}\right]^{-1/\theta}, \tag{27}$$

$$P_F = \left[1 + \big((1+\tau^*)w\big)^{-\theta}\right]^{-1/\theta}. \tag{28}$$

These can be written in terms of domestic shares as:

$$P_H = w\,\pi_{HH}^{1/\theta}, \qquad P_F = \pi_{FF}^{1/\theta}. \tag{29}$$

Countries hold two types of cross-border claims:

- **Production-indexed assets:** a denotes Home's claim on Foreign value added, i.e., Foreign labor; a^* denotes Foreign's claim on Home production (units of Home labor).

- **Consumption-indexed assets:** b denotes Home's claim on Foreign consumption baskets (units of Foreign CPI bundle); b^* denotes Foreign's claim on Home consumption baskets (units of Home CPI bundle).

Assets can be positive (claims) or negative (liabilities). The four parameters (a, a^*, b, b^*) are exogenous. The net transfer from Foreign to Home, valued in the Foreign wage numéraire, is:

$$\mathscr{A}(w) = \underbrace{(a - a^* w)}_{\text{production-indexed}} + \underbrace{(b\, P_F - b^*\, P_H)}_{\text{consumption-indexed}}$$

Differentiating:

$$\mathscr{A}'(w) = -a^* + \frac{b\, P_F\, \pi_{FH} - b^*\, P_H\, \pi_{HH}}{w}.$$

Total expenditure in each country equals labor income plus tariff revenue plus net asset income.

Lemma C3 (Expenditure)**.** *Total expenditures are:*

$$X_H = \frac{(1+\tau)(w + \mathscr{A})}{1 + \tau\, \pi_{HH}}, \tag{30}$$

$$X_F = \frac{(1+\tau^*)(1 - \mathscr{A})}{1 + \tau^*\, \pi_{FF}}. \tag{31}$$

Proof. Home's budget: $X_H = w + T_H + \mathscr{A}$ where tariff revenue is $T_H = \tau(1 - \pi_{HH})X_H/(1+\tau)$. Substituting and solving: $X_H[(1 + \tau\, \pi_{HH})/(1+\tau)] = w + \mathscr{A}$. Foreign follows symmetrically with income 1 and net transfer $-\mathscr{A}$. □

Home labor income equals domestic absorption of Home goods plus exports:

$$w = X_H\, \pi_{HH} + \frac{X_F\, \pi_{FH}}{1 + \tau^*}. \tag{32}$$

Proposition C2. *Define:*

$$\alpha \equiv (1+\tau)^{\theta+1}, \qquad \beta \equiv (1+\tau^*)^{\theta+1}.$$

Then the wage $w > 0$ satisfies:

$$w = \frac{\alpha\,(w + \mathscr{A}(w))}{\alpha + w^\theta} + \frac{1 - \mathscr{A}(w)}{1 + \beta\,w^\theta} \tag{33}$$

where $\mathscr{A}(w) = (a - a^* w) + (b\,P_F(w) - b^*\,P_H(w))$ with P_H, P_F given by (27)–(28).

Proof. Substitute (30) and (31) into (32). Using $\pi_{HH} = (1+\tau)^\theta/[(1+\tau)^\theta + w^\theta]$ and $1 + \tau\,\pi_{HH} = (\alpha + w^\theta)/[(1+\tau)^\theta + w^\theta]$:

$$X_H\,\pi_{HH} = \frac{(1+\tau)(w + \mathscr{A})}{1 + \tau\,\pi_{HH}} \cdot \pi_{HH} = (1+\tau)(w+\mathscr{A}) \cdot \frac{(1+\tau)^\theta}{\alpha + w^\theta} = \frac{\alpha(w + \mathscr{A})}{\alpha + w^\theta}.$$

Similarly, $X_F\,\pi_{FH}/(1+\tau^*) = (1 - \mathscr{A})/(1 + \beta w^\theta)$. □

Define $G(w)$ as the excess supply function:

$$G(w) \equiv w - \frac{\alpha\,(w + \mathscr{A}(w))}{\alpha + w^\theta} - \frac{1 - \mathscr{A}(w)}{1 + \beta\,w^\theta}. \tag{34}$$

Equilibrium requires $G(w) = 0$ together with feasibility.

Definition C3 (Equilibrium)**.** *An* ***equilibrium*** *is a wage $w > 0$ satisfying:*

(E1) $G(w) = 0$ *(labor market clearing),*

(E2) $w + \mathscr{A}(w) > 0$ *and* $1 - \mathscr{A}(w) > 0$ *(non-negative expenditures).*

Condition (E2) requires that both countries can afford positive consumption:

$$w + \mathscr{A} = (1 - a^*)w + a + b\,P_F - b^*\,P_H > 0,$$
$$1 - \mathscr{A} = 1 - a + a^* w - b\,P_F + b^*\,P_H > 0.$$

These sum to $w + 1 > 0$, which always holds.

Welfare in each country is real expenditure, $W_i = X_i/P_i$. Using (30)–(31) and (29):

$$W_H = \frac{X_H}{P_H} = \frac{(1+\tau)(w + \mathscr{A})}{(1 + \tau\,\pi_{HH})\,w\,\pi_{HH}^{1/\theta}}, \tag{35}$$

$$W_F = \frac{X_F}{P_F} = \frac{(1+\tau^*)(1 - \mathscr{A})}{(1 + \tau^*\,\pi_{FF})\,\pi_{FF}^{1/\theta}}. \tag{36}$$

In autarky, $W_H^{aut} = W_F^{aut} = 1$ since each country consumes its own output.

C.2 Free Trade Equilibrium

Condition (32) under free trade becomes

$$w = \frac{w+1}{1+w^\theta}.$$

Which can be written

$$w(1+w^\theta) = w+1 \quad \Longleftrightarrow \quad w = 1,$$

so the free-trade wage equals unity regardless of the asset position $\mathscr{A}$.

Free-trade welfare follows from (35)–(36) evaluated at $\tau = \tau^* = 0$:

$$W_H^{ft} = \frac{w+\mathscr{A}}{P_H} = (1+\mathscr{A})\, 2^{1/\theta},$$
$$W_F^{ft} = \frac{1-\mathscr{A}}{P_F} = (1-\mathscr{A})\, 2^{1/\theta}.$$

The common factor $2^{1/\theta}$ captures the gains from trade relative to autarky. With $\mathscr{A} = 0$, both countries gain equally: $W_H^{ft} = W_F^{ft} = 2^{1/\theta} > 1$. A positive net transfer $\mathscr{A} > 0$ shifts welfare toward Home at the expense of Foreign.

C.3 Uniqueness with Non-Negative Assets

When $\tau = \tau^* = 0$, the transfer terms cancel from the equilibrium equation and $G(w) = 0$ yields $w = 1$ regardless of asset positions. For the remainder of this subsection, assume $(\tau, \tau^*) > (0,0)$, so $\alpha\beta > 1$. Now we show that if both countries hold non-negative claims on each other's, then the equilibrium is unique.

Proposition C3 (Uniqueness)**.** *Suppose $a, a^* \geq 0$ and $b^* \geq 0$ (with b unrestricted). Then for any $(\tau, \tau^*) > (0,0)$, there is at most one feasible equilibrium wage $w > 0$.*

Proof. The idea of the proof is to show that two solutions to (33) necessary generate a contradiction. Multiplying (33) through by $(\alpha + w^\theta)(1+\beta w^\theta)$ and collecting terms we have that the following must hold:

$$(\alpha\beta - 1)\,\mathscr{A} = w + \beta\, w^{\theta+1} - 1 - \alpha\, w^{-\theta}.$$

where $\mathscr{A} = (a - a^* w) + bP_F - b^* P_H$.

The equilibrium equation can be rewritten as:

$$bP_F(w) - b^*P_H(w) = \frac{w + \beta\, w^{\theta+1} - 1 - \alpha\, w^{-\theta}}{\alpha\beta - 1} - a + a^* w \equiv \hat{\mathscr{A}}(w) \tag{37}$$

Step 1: $P_H(w)/P_F(w)$ **is strictly increasing.** Define $R(w) \equiv P_H(w)/P_F(w)$. Then:

$$\frac{d}{dw}\ln R(w) = \frac{\pi_{HH}(w) - \pi_{FH}(w)}{w} = \frac{\pi_{HH}(w) + \pi_{FF}(w) - 1}{w}.$$

From (25)–(26):

$$\pi_{HH}(w) + \pi_{FF}(w) - 1 = \frac{w^\theta\left[(1+\tau)^\theta(1+\tau^*)^\theta - 1\right]}{\left[(1+\tau)^\theta + w^\theta\right]\left[(1+\tau^*)^\theta w^\theta + 1\right]} > 0$$

since $(\tau, \tau^*) > (0, 0)$. Hence $R(w)$ is strictly increasing in w.

Step 2: $\hat{\mathscr{A}}(w)/P_F(w)$ **is strictly increasing.** Write $\pi_{FH}(w) = wP_F'(w)/P_F(w) \in (0, 1)$ and define the numerator function:

$$\hat{N}(w) \equiv (\alpha\beta - 1)\,\hat{\mathscr{A}}(w) = w + \beta\, w^{\theta+1} - 1 - \alpha\, w^{-\theta} + (\alpha\beta - 1)(-a + a^* w).$$

Since $\alpha\beta - 1 > 0$, to show that $\hat{\mathscr{A}}(w)/P_F(w)$ is strictly increasing, it suffices to show $\hat{N}(w)/P_F(w)$ is strictly increasing in w.

Now

$$\frac{d\frac{\hat{N}(w)}{P_F(w)}}{dw} = \frac{\hat{N}'(w)}{P_F(w)} - \frac{\hat{N}(w)}{P_F(w)^2}P_F'(w) = \frac{1}{P_F}\left[\hat{N}'(w) - \frac{\hat{N}(w)}{P_F(w)}P_F'(w)\right] = \frac{1}{wP_F}\left[\hat{N}'(w)w - \hat{N}(w)\pi_{FH}(w)\right]$$

Now, using the definition of $\hat{N}(w)$ we get,

$$\begin{aligned} & w\,\hat{N}'(w) - \pi_{FH}(w)\,\hat{N}(w) \\ &= \underbrace{w(1-\pi_{FH})}_{>0} + \underbrace{\beta\, w^{\theta+1}(\theta + 1 - \pi_{FH})}_{>0} + \underbrace{\alpha\, w^{-\theta}(\theta + \pi_{FH})}_{>0} + \underbrace{\pi_{FH}}_{>0} + (\alpha\beta - 1)\left[\underbrace{a^* w(1-\pi_{FH})}_{\geq 0} + \underbrace{\pi_{FH}\, a}_{\geq 0}\right]. \end{aligned}$$

The first four terms are strictly positive (using $0 < \pi_{FH} < 1$, $\theta > 0$, $\alpha, \beta \geq 1$); the last two are non-negative since $a, a^* \geq 0$.

Step 3: Contradiction from two roots. Suppose that we have two roots. That is, w_1 and w_2 such that $0 < w_1 < w_2$ and both satisfy (37). That is:

$$b - b^*\frac{P_H(w_1)}{P_F(w_1)} = \frac{\hat{\mathscr{A}}(w_1)}{P_F(w_1)} \quad \text{and} \quad b - b^*\frac{P_H(w_2)}{P_F(w_2)} = \frac{\hat{\mathscr{A}}(w_2)}{P_F(w_2)},$$

after dividing (37) by P_F.

Subtracting these two equations, we get that

$$b^*\underbrace{\left[\frac{P_H(w_2)}{P_F(w_2)}-\frac{P_H(w_1)}{P_F(w_1)}\right]}_{>0}=\underbrace{\frac{\hat{\mathscr{A}}(w_1)}{P_F(w_1)}-\frac{\hat{\mathscr{A}}(w_2)}{P_F(w_2)}}_{<0},$$

where the first inequality follows from Step 1 and the second from Step 2. Given that $b^* \geq 0$, we obtain a contradiction. □

The parameter b drops out in the cross-multiplication step, so the proof holds for arbitrary b. By the model's Home-Foreign symmetry ($w \longrightarrow 1/w$, $\tau \leftrightarrow \tau^*$, $a \leftrightarrow a^*$, $b \leftrightarrow b^*$), uniqueness also holds when $b \geq 0$ with b^* unrestricted. Combining: if $a, a^* \geq 0$ and at least one of b, b^* is non-negative, the equilibrium is unique.

C.4 Multiplicity with Symmetric Negative Assets

The uniqueness result of Proposition C3 requires $a, a^* \geq 0$. We now show that when both countries hold negative claims on each other's production, multiple equilibria can arise. We focus on the case $b = b^* = 0$ (pure production-indexed assets) and consider the fully symmetric case $a = a^* < 0$, $\tau = \tau^* \equiv \bar{\tau} > 0$, so $\alpha = \beta$, for simplicity.

The transfer is $\mathscr{A}(w) = a(1-w)$, and the equilibrium equation (34) becomes

$$G(w) = w - \frac{\alpha\big[(1-a)w + a\big]}{\alpha + w^{\theta}} - \frac{(1-a) + a\,w}{1 + \alpha\,w^{\theta}}.$$

Symmetric equilibrium. At $w = 1$: $\mathscr{A}(1) = 0$, and $G(1) = 1 - \alpha/(\alpha+1) - 1/(\alpha+1) = 0$. So $w = 1$ is always an equilibrium.

Slope at $w = 1$. Evaluating $G'(1)$:

$$G'(1) = \frac{1}{\alpha+1}\left[1 + a(\alpha - 1) + \frac{2\theta\alpha}{\alpha+1}\right]. \tag{38}$$

x The first and third terms in brackets are positive; the second is negative when $a < 0$. For $|a|$ large enough, $G'(1) < 0$ and at least two additional roots emerge.

Proposition C4 (Multiplicity)**.** *Let $a < 0$, $\bar{\tau} > 0$, and $\alpha = (1+\bar{\tau})^{\theta+1}$. Define:*

$$\bar{a} \equiv -\frac{(1+\alpha) + 2\theta\alpha}{(\alpha-1)(1+\alpha)}, \qquad \underline{a} \equiv -\frac{1}{\bar{\tau}}.$$

Then $\underline{a} < \overline{a} < 0$, and:

(i) *If $a > \overline{a}$, then $G'(1) > 0$.*

(ii) *If $\underline{a} < a < \overline{a}$, there exist at least two additional equilibria $w_\ell < 1 < w_h$ with $w_\ell \cdot w_h = 1$.*

Proof. **Ordering of thresholds.** From (38), $G'(1)$ is affine in a with positive slope $(\alpha-1)/(\alpha+1)$, and $\overline{a}$ is the unique zero. To show $\underline{a} < \overline{a}$, it suffices to verify $G'(1) < 0$ at $a = \underline{a} = -1/\bar{\tau}$. Let $\gamma = (1+\bar{\tau}) > 1$, $u = \ln\gamma > 0$, and $q = 1 + 2\theta > 1$. Substituting $\bar{a} = -1/(\gamma-1)$ into (38) and simplifying, $G'(1) < 0$ reduces to $\sinh(qu/2) > q\sinh(u/2)$. Since $f(x) = \sinh(x)$ is strictly convex on $(0,\infty)$ and $f(0) = 0$, we have $f(qv) > qf(v)$ for $q > 1$ and $v > 0$. Hence $\underline{a} < \overline{a}$.

Part (i). Immediate from (38).

Part (ii). The feasibility constraints $w + \mathscr{A} > 0$ and $1 - \mathscr{A} > 0$ define the feasible region $w \in (w_{\min}, w_{\max})$ where

$$w_{\min} = \frac{|a|}{1+|a|}, \qquad w_{\max} = \frac{1+|a|}{|a|} = \frac{1}{w_{\min}}.$$

In the symmetric case, G satisfies $G(w) + w\,G(1/w) = 0$ for all $w > 0$ (verified by direct substitution using $a = a^*$ and $\alpha = \beta$). At the lower boundary, $w + \mathscr{A} \to 0^+$, so $G(w_{\min}^+) < 0$ provided $\alpha\, w_{\min}^{\theta+1} < 1$, which is equivalent to $a > \underline{a}$. By the symmetry identity, $G(w_{\max}^-) > 0$. Since $a < \overline{a}$ implies $G'(1) < 0$, we have $G(1-\epsilon) > 0$ and $G(1+\epsilon) < 0$ for small $\epsilon > 0$. By the intermediate value theorem, there exist roots $w_\ell \in (w_{\min}, 1)$ and $w_h \in (1, w_{\max})$. The symmetry identity gives $w_h = 1/w_\ell$. □

Mechanism. The symmetric equilibrium $w = 1$ becomes "unstable" when symmetric debts exceed $|\overline{a}|$. Two asymmetric equilibria emerge: in one, Home is the low-wage debtor ($w_\ell < 1$); in the other, Foreign is ($w_h > 1$). The mechanism is self-fulfilling: a low wage raises the real burden of production-indexed debt, depressing Home income further. This is the counterpart, in the EK setting, of the debt traps identified in Section 3.5.

A similar result holds for the symmetric case with pure consumption-indexed assets: $a = a^* = 0$ and $b = b^* < 0$. The transfer becomes $\mathscr{A}(w) = b(P_F - P_H)$, which again vanishes at $w = 1$, so the symmetric equilibrium persists. The same destabilizing force operates through the price-index channel: a decline in w raises P_F relative to P_H, increasing the real burden of Home's consumption-indexed liabilities and reinforcing the initial wage decline.

C.5 Autarky Limit with Gross Assets of the Same Sign

We have shown above that if assets are non-negative, the equilibrium is unique, while multiplicity can arise for negative asset positions.

In our next result, we show that if both tariffs grow towards infinity, and the countries have gross position of the same signs, then the net foreign asset position must balance in any equilibrium and the countries must converge to autarky. Note that this is a stronger result than in the main text, as uniqueness is shown independenty of the sign of asset positions.

As tariffs $\tau, \tau^* \to \infty$, the price indices converge to $P_H \to w$ and $P_F \to 1$, so the transfer becomes

$$\mathscr{A} \to (a+b) - (a^* + b^*)w = \tilde{a} - \tilde{a}^* w,$$

where $\tilde{a} \equiv a+b$ and $\tilde{a}^* \equiv a^* + b^*$ are the *effective assets*. In this limit, the distinction between asset types vanishes and the equilibrium equation takes the same form as the pure production-indexed case with (a, a^*) replaced by $(\tilde{a}, \tilde{a}^*)$.

Proposition C5 (Autarky Wage). *Suppose $\tilde{a} \equiv a + b$ and $\tilde{a}^* \equiv a^* + b^*$ satisfy*

$$\tilde{a} \times \tilde{a}^* > 0.$$

and that

$$\varphi a + b \neq 0, \text{ and } \varphi a^* + b^* \neq 0,$$

for all $\varphi \geq 1$. Then as $\tau, \tau^* \to \infty$, every equilibrium wage converges to

$$w^{aut} = \frac{\tilde{a}}{\tilde{a}^*}.$$

Proof. Consider a sequence of (τ_t, τ_t^*) such that $\tau_t, \tau_t^* \to \infty$ as $t \to \infty$. Suppose that for some subsequence, $w_s \to 0$. Then

$$P_H = \left[w_s^{-\theta} + (1+\tau_s)^{-\theta}\right]^{-1/\theta} \to 0.$$

Then,

$$\mathscr{A}(w) = (a - a^* w) + (bP_F - b^* P_H) \to a + b \lim_{s\to\infty} P_F$$

From (E2), we require that

$$-w < A(w) < 1$$

and thus

$$0 \leq a + b \lim_{s\to\infty} P_F \leq 1 \tag{39}$$

From (34)

$$0 \equiv w - \frac{\alpha\,(w + a - a^{*}w + bP_F - b^{*}P_H)}{\alpha + w^{\theta}} - \frac{1 - (a - a^{*}w + bP_F - b^{*}P_H)}{1 + \beta\, w^{\theta}}.$$

Taking the limits as $s \longrightarrow \infty$:

$$0 \equiv \lim_{s\longrightarrow\infty} \frac{(\beta_s w_s^{\theta})(a + bP_{Fs}) + 1}{1 + \beta_s w_s^{\theta}}. \tag{40}$$

Now from (39), the limit of the numerator must be strictly positive. So the only possibility for (40) to hold is that $\beta_s w_s^{\theta} \longrightarrow \infty$. This requires that

$$a + b \lim_{s\longrightarrow\infty} P_{Fs} = 0 \Leftrightarrow -a = b \lim_{s\longrightarrow\infty} P_{Fs}.$$

where P_{Fs}

$$P_{Fs} = \left[1 + ((1+\tau_s^{*})w_s)^{-\theta}\right]^{-1/\theta}$$

Thus, we require that

$$a\left[1 + \lim_{s\longrightarrow\infty} \frac{1}{((1+\tau_s^{*})w_s)^{\theta}}\right]^{1/\theta} + b = 0$$

So as long as $a\varphi + b \neq 0$ for all $\varphi \geq 1$, w cannot go to zero.

A symmetric argument using that $\varphi a^{*} + b^{*} \neq 0$ for $\varphi \geq 1$ rules out subsequences where $w_s \longrightarrow \infty$. Hence the wage remains bounded and away from zero.

Given that the wage must remain bounded and away from zero, the equation (34) becomes in the limit

$$G(w) \longrightarrow w - (w + \mathscr{A}(w)) - 0 = -\mathscr{A}(w).$$

Since $P_H \longrightarrow w$ and $P_F \longrightarrow 1$ in this limit, $\mathscr{A}(w) \longrightarrow \tilde{a} - \tilde{a}^{*}\, w$. Setting $G = 0$ gives $\tilde{a} - \tilde{a}^{*}\, w = 0$, i.e., $w = \tilde{a}/\tilde{a}^{*}$. □

At the autarky wage, there is no trade and each country consumes its own output. The international relative price w^{aut} is pinned entirely by the requirement that net foreign assets value to zero, exactly as in Proposition 4. In this case, Home and Foreign welfare converge to their autarky levels. As $\tau, \tau^{*} \longrightarrow \infty$, we have $\pi_{HH} \longrightarrow 1$, $\pi_{FF} \longrightarrow 1$, $P_H \longrightarrow w$, $P_F \longrightarrow 1$, and $\mathscr{A} \longrightarrow 0$. Substituting into (35)–(36):

$$W_H \longrightarrow \frac{w}{w} = 1 = W_H^{aut},$$
$$W_F \longrightarrow \frac{1}{1} = 1 = W_F^{aut}.$$

The requirement that $\varphi a + b \neq 0$ and $\varphi a^{*} + b^{*} \neq 0$ for all $\varphi \geq 1$ plays the same role as condition (7) in Proposition 3. If this condition fails, then there are equilibria where the wage goes to zero or infinity as tariffs grow without bound, and the autarky limit is not the unique limit point of the equilibrium correspondence.[17]

C.6 Limit with Net Assets Only

Let us contrast the result of the previous section with the case where $a = b = b^* = 0$ and $a^* > 0$: Home is a pure debtor with no consumption-indexed assets. In this case, the net foreign asset position cannot be balanced to zero, since Home owes Foreign a positive amount regardless of the tariffs.

So $\mathscr{A}(w) = -a^* w$ and home has to give a share of its output to Foreign independently of the tariff levels. The equilibrium equation (34) becomes

$$1 \equiv \frac{\alpha\,(1 - a^*)}{\alpha + w^\theta} + \frac{1/w + a^*}{1 + \beta\, w^\theta}.$$

This can be rewritten as:

$$\frac{1/w + a^*}{1 + \beta w^\theta} = a^* + \frac{(1 - a^*) w^\theta}{\alpha + w^\theta}. \tag{41}$$

Note that (E2) requires that $1 > a^*$.

Let us now consider the limit as $\tau, \tau^* \longrightarrow \infty$, so $\alpha, \beta \longrightarrow \infty$. Suppose that for some sequence, w does not go to 0. Then the left hand side of (41) converges to 0, while the right hand side remains bounded away from zero since $1 > a^* > 0$. This is a contradiction, so Home's wage most go to zero. In particular, equation (41) further implies that

$$w(1 + \tau^*) \longrightarrow (a^*)^{-1/(1+\theta)}$$

[17] These other equilibria retain a flavor of the debt-traps of Proposition 2, as the wage collapses to zero (or explodes to infinity). However, feasibility does not break in the limit. For example, if there is a $\varphi > 1$ such that $\varphi a + b = 0$, then there is an equilibrium where $w \longrightarrow 0$ and $P_F \longrightarrow \varphi$ and Home welfare goes to

$$W_H = 1 - \left((-b/a)^\theta - 1\right)^{(\theta+1)/\theta},$$

which is strictly less than the autarky welfare $W_H^{aut} = 1$.

A reader may ask whether the existence of another equilibrium contradicts the uniqueness statement of Proposition C3 as b could be negative even with $a, a^*, b^* > 0$. Note however that the same-sign restriction of Proposition C5 requires that $a + b > 0$ in this case. Given $a > 0$, it follows $\varphi a + b > 0$ for all $\varphi \geq 1$, and thus the multiplicity underlying the need for condition C5 of Proposition C5 does not arise.

With this, we can obtain the welfare limits for Home and Foreign:

$$W_H = \frac{X_H}{P_H} = \frac{w(1-a^*)}{w\,\pi_{HH}^{1/\theta}} \longrightarrow 1 - a^* < 1 = W_H^{aut}$$
$$W_F = \frac{X_F}{P_F} = \frac{(1+\tau^*)(1+a^* w)}{(1+\tau^*\,\pi_{FF})\,\pi_{FF}^{1/\theta}} \longrightarrow \left[1 + (a^*)^{\theta/(\theta+1)}\right]^{(\theta+1)/\theta} > 1 = W_F^{aut}.$$

Thus, Home is strictly worse off than in autarky, while Foreign is strictly better off. This is a consequence of the fact that in the limit, Home has to give a share of its output to Foreign, and does not benefit from trade.

C.7 Balanced but Positive Trade

Finally, we can use the EK framework to construct examples of balanced but strictly positive trade.

Consider the case of pure production-indexed assets ($b = b^* = 0$) and suppose Home is a net debtor with positive assets: $0 < a < a^*$. The transfer is $\mathscr{A}(w) = a - a^* w$. So let us consider the locus of tariffs (τ, τ^*) such that $\mathscr{A}(w) = 0$ is an equilibrium.

Substituting into (33) we obtain that:

$$\beta = \frac{\alpha(w_{aut})^{-\theta} - w_{aut} + 1}{(w_{aut})^{\theta+1}} \qquad \text{with} \qquad w_{aut} = \frac{a}{a^*} < 1. \tag{42}$$

where recall that $\alpha = (1+\tau)^{\theta+1}$ and $\beta = (1+\tau^*)^{\theta+1}$. So this simple linear relationship between α and β determines the manifold of tariffs such that the net foreign asset positions balance to zero.

Lemma C4. *Suppose that $w_{aut} = a/a^* < 1$. Then for any $\alpha \geq 1$, there is a unique $\beta > 1$ satisfying (42).*

Proof. There is at most one $\beta \geq 0$ associated with every $\alpha \geq 0$. The numerator is strictly increasing in α and the denominator is positive, so β is strictly increasing in α. At $\alpha = 1$,

$$\beta = \frac{(w_{aut})^{-\theta} - w_{aut} + 1}{(w_{aut})^{\theta+1}} > 1$$

since for $w_{aut} < 1$ and $\theta > 0$: (i) $(w_{aut})^{-\theta} > 1$, (ii) $-w_{aut} + 1 > 0$, and (iii) $(w_{aut})^{\theta+1} < 1$, so the numerator exceeds 1 while the denominator is less than 1.

□

Note that this simple lemma implies that Home cannot engineer a balanced trade outcome by setting its tariff to a positive number without Foreign responding with a positive tariff of its own. The debtor country cannot close the net foreign asset position by simply raising its own tariff.

D Quantification appendix

D.1 Calibration of Armington parameters and productivity processes

The model has four parameters, $\{\beta, \mu, \eta, \rho\}$. We set $\beta = 0.96$ (discounting), $\mu = 0.5$ (materials share), $\eta = 4$ (Armington elasticity), and $\rho = 0.99$ (inverse of risk aversion, so utility close to log). We then need to take a stand on labor supply, trade costs, tariffs, productivity and foreign asset portfolios in date 0, as well as the process for labor supply, trade costs, tariffs and productivity going forward from date 0. We also need to calibrate the preference shifter θ_0^*.

We first recover historical productivity and trade costs consistent with the model for the period 1970-2023 using the approach of Fitzgerald (2025).[18] This requires data on labor supply (we use population), national income accounts and expenditure PPPs for the US and ROW aggregate. Given these data and $\{\mu, \eta\}$, model inversion yields time series for $\left\{z_t, z_t^*\right\}$ and $\left\{d_t, d_t^*\right\}$ for 1970-2023 such that the model exactly matches the data. This procedure is invariant to intertemporal preferences and the nature of the asset market. Details of data construction and the model inversion procedure are below.

Going forward from date 0, we assume that labor supply and iceberg trade costs are fixed at their levels in 2023 (i.e. $L = L_{2023}$, $L^* = L_{2023}^*$, $d = d_{2023}$, $d^* = d_{2023}^*$). In the baseline, we set tariffs permanently equal to zero (i.e. $\tau = \tau^* = 0$) consistent with how we recover productivity and trade costs. We use the 54-year panel of data on productivity for the US and ROW to estimate a process for productivity. We assume the US is the world productivity leader, and that $\Delta \ln z_t$ follows:

$$\Delta \ln z_t = g + \varepsilon_t$$

where

$$\varepsilon_t = \psi \varepsilon_{t-1} + \sigma \nu_t$$

and $\nu_t \sim N(0, 1)$. Meanwhile $\Delta \ln z_t^*$ follows:

$$\Delta \ln z_t^* = g + \gamma^* \left(\ln z_{t-1} - \ln z_{t-1}^*\right) + \varepsilon_t^*$$

[18] We assume that there are only resource costs of trade, and no tariffs throughout this period. Given data on total tariff revenues for the US, and ROW revenues from tariffs levied on imports from the US for the period 1970-2023, it would be straightforward to allow for tariffs.

where

$$\varepsilon_t^* = \psi^* \varepsilon_{t-1}^* + \chi^* \varepsilon_{t-1} + \sigma^* \nu_t^* + \omega^* \nu_t$$

and $\nu_t^* \sim N(0, 1)$. We estimate this process by maximum likelihood. Parameter estimates are reported in Table 5.

Table 5: Parameters of estimated productivity process

Parameter		Value	95% CI
US			
US long-run growth rate	g	0.026	[0.020, 0.034]
Autocorr. of deviations from trend, US	ψ	0.58	[0.30, 0.70]
SD of innovations to deviations from trend, US	σ	0.012	[0.009, 0.014]
ROW			
Rate of convergence to US	γ^*	0.0024	[−0.002, 0.008]
Autocorr. of deviations from trend	ψ^*	0.29	[−0.03, 0.44]
Corr. with lagged US deviations from trend	χ^*	0.28	[−0.04, 0.71]
SD of innovations to deviations from trend	σ^*	0.017	[0.012, 0.020]
Corr. with US innovations to deviations from trend	ω^*	0.009	[0.004, 0.015]

Notes: ROW coefficients are estimated by maximum likelihood conditional on coefficients for US. Bootstrapped 95% confidence intervals based on 100 draws with replacement.

Our estimates imply a similar long-run growth rate for the US and ROW (ROW convergence is very slow), but that productivity is more volatile in ROW than in the US. In making use of this estimated process, we condition on $\{\ln \mathbf{z}_0, \Delta \ln \mathbf{z}_0\} = \{\ln \mathbf{z}_{2023}, \Delta \ln \mathbf{z}_{2023}\}$.

D.2 Data used to recover historical productivity and trade costs

We use World Development Indicators (WDI) data on US population to proxy for US labor supply. We construct ROW labor supply by subtracting US population from world population from WDI. We take the value of US GDP expressed in current dollars from WDI. We construct ROW GDP by subtracting US GDP from world GDP. US imports from ROW are given by total US imports in current dollars from WDI, and ROW imports from the US are given by total US exports. We use Penn World Tables (PWT) data on expenditure PPPs to construct a ROW PPP by weighting expenditure PPPs for all countries other than the US by their share in ROW gross expenditure (using the WDI data to construct the weights). For this purpose, we augment the WDI national income accounts data using OECD national income accounts data, and national income accounts data from the Taiwanese statistical agency as described in Fitzgerald (2025).

D.3 Model inversion to recover historical productivity and trade costs

Gross output in US is (ROW is analogous):

$$OUT_t = \frac{GDP_t}{1-\mu}$$

Expenditure on domestic production for the US is (ROW is analogous):

$$IMS_t = OUT_t - EX_t$$

where IMS_t is expenditure on domestic production and EX_t is total exports. Gross expenditure (absorption) for the US is (ROW is analogous):

$$ABS_t = OUT_t - EX_t + IM_t$$

where IM_t is total imports. The expenditure PPP for ROW is constructed as a weighted average of PPPs in all countries other than the US:

$$PPP_t^* = \sum_{i=1, i\neq US}^{N} \frac{ABS_t^i}{ABS_t^{ROW}} \frac{PPP_t^i}{PPP_t^{US}}$$

where

$$ABS_t^* = \sum_{i=1, i\neq US}^{N} ABS_t^i$$

Then the model tells us:

$$\left(\frac{p_t}{p_t^*}\right)^{1-\eta} = \frac{IMS_t}{ABS_t} \frac{ABS_t^*}{IMS_t^*} \left(\frac{PPP_t}{PPP_t^*}\right)^{1-\eta}$$

and

$$(d_t)^{1-\eta} = \frac{IMP_t}{ABS_t} \frac{ABS_t^*}{IMS_t^*} \left(\frac{PPP_t}{PPP_t^*}\right)^{1-\eta}$$

where IMP_t is US imports from ROW, and

$$(d_t^*)^{1-\eta} = \frac{EXP_t}{ABS_t^*} \frac{ABS_t}{IMS_t} \left(\frac{PPP_t^*}{PPP_t}\right)^{1-\eta}$$

where EXP_t is US exports to ROW.

Time-series variation in the US output price is given by:

$$\frac{p_t}{p_{1970}} = \frac{OUT_t/Realout_t}{OUT_{1970}/Realout_{1970}}$$

Normalize $p_{1970} = 1$. The output prices used to calculate ROW productivity are:

$$\left(\hat{p}_t^*\right)^{1-\eta} = \left(\frac{p_t^*}{p_t}\frac{p_t}{p_{1970}}\right)^{1-\eta}$$

Expenditure prices used to calculate US productivity are (ROW is analogous):

$$\hat{P}_t = \left(\frac{\left(\hat{p}_t\right)^{1-\eta} + \left(d\hat{p}_t^*\right)^{1-\eta}}{\left(\hat{p}_{1970}\right)^{1-\eta} + \left(d\hat{p}_{1970}^*\right)^{1-\eta}}\right)^{\frac{1}{1-\eta}}$$

Real output in the US is then (ROW is analogous):

$$y_t = \frac{OUT_t}{\hat{p}_t}$$

Materials inputs to production in the US are (ROW is analogous):

$$M_t = \mu\frac{\hat{p}_t y_t}{\hat{P}_t}$$

and productivity in the US is (ROW is analogous):

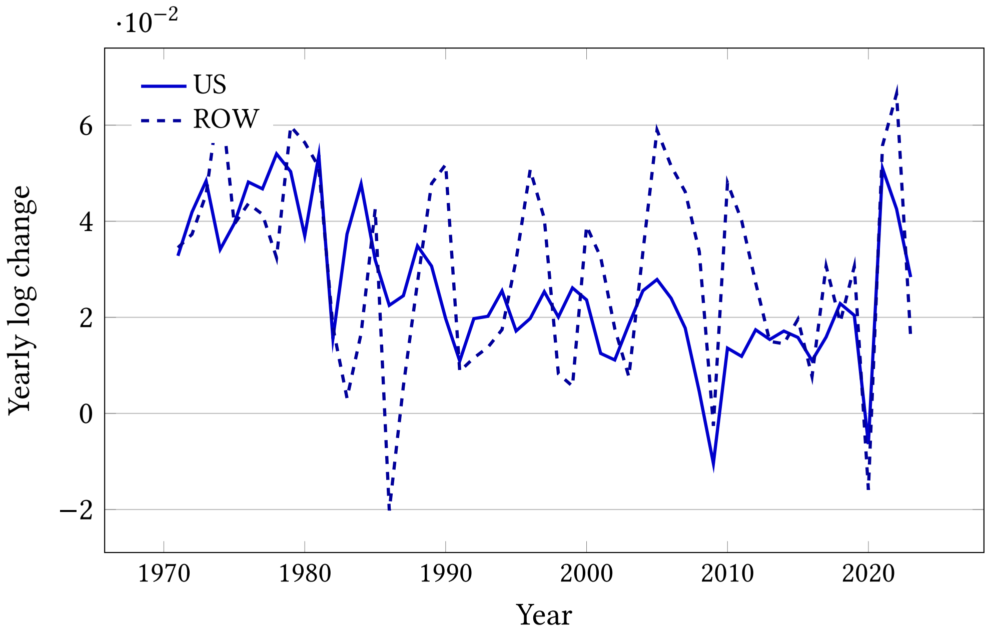
$$z_t = \frac{y_t}{\left(L_t\right)^{1-\mu}\left(M_t\right)^{\mu}}$$

Figure 6 plots the log change in the resulting productivity series for the US and ROW.

D.4 Construction of portfolio shares

US FDI & equity assets and liabilities relative to US GDP and US debt assets and liabilities relative to US GDP are taken from the External Wealth of Nations data. ROW assets are set equal to US liabilities, and ROW liabilities are set equal to US assets. The dollar share of US debt assets is taken from the US Treasury's International Capital (TIC) annual report on U.S. Portfolio Holdings of Foreign Securities (Table 14 in report dated December 29, 2023). The dollar share of US debt liabilities is from the US Treasury's International Capital (TIC) annual report on Foreign Portfolio Holdings of U.S. Securities (constructed from Table 12 in report dated June 30, 2023).

Figure 6: Log change in productivity



D.5 Computation of competitive equilibrium without tariffs

Set $\theta_0 = 1$. Fix a value for θ_0^*. Then guess a vector of output prices (i.e. $p_0\left(s^t\right)_0^{FT}$ and $p_0^*\left(s^t\right)_0^{FT}$ for all t, s^t), normalizing one value to 1. Calculate consumption prices using:

$$P_0\left(s^t\right)_0^{FT} = \left(\left(p_0\left(s^t\right)_0^{FT}\right)^{1-\eta} + \left(dp_0^*\left(s^t\right)_0^{FT}\right)^{1-\eta}\right)^{\frac{1}{1-\eta}}$$

(and similarly for $P_0^*\left(s^t\right)_0^{FT}$). Then output is

$$y\left(s^t\right)_0^{FT} = L\left(z\left(s^t\right)\right)^{\frac{1}{1-\mu}}\left(\frac{\mu p_0\left(s^t\right)_0^{FT}}{P_0\left(s^t\right)_0^{FT}}\right)^{\frac{\mu}{1-\mu}}$$

and net foreign assets are

$$b_0\left(s_0\right)_0^{FT} = \left(\chi_0 - \chi_0^*\right)\left(1-\mu\right)p_0\left(s_0\right)_0^{FT} y\left(s_0\right)_0^{FT}$$

Define

$$\Pi_0\left(s_0\right)_0^{FT} =$$

$$\left(\frac{1-\beta}{\theta_0\left(1-\beta\right)+\beta\left(1-\beta^T\right)}\right)^{\frac{\rho}{1-\rho}}\left[\begin{array}{c}\left(\theta_0\right)^\rho\left(P_0\left(s_0\right)_0^{FT}\right)^{1-\rho}+\\ \sum_{t=1}^T\sum_{s^t}\left(\beta^t\pi\left(s^t\right)\right)^\rho\left(P_0\left(s^t\right)_0^{FT}\right)^{1-\rho}\end{array}\right]^{\frac{1}{1-\rho}}$$

Then

$$M\left(s^t\right)_0^{FT}=\mu^{\frac{1}{1-\mu}}\left(\frac{p_0\left(s^t\right)_0^{FT}z\left(s^t\right)}{P_0\left(s^t\right)_0^{FT}}\right)^{\frac{1}{1-\mu}}L^i$$

and

$$C\left(s_0\right)_0^{FT}=$$
$$\left(\frac{\Pi_0\left(s_0\right)_0^{FT}\theta_0}{P_0\left(s_0\right)_0^{FT}}\frac{1-\beta}{\theta_0\left(1-\beta\right)+\beta\left(1-\beta^T\right)}\right)^\rho\left(\frac{b_0\left(s_0\right)_0^{FT}+\sum_{t=0}^T\sum_{s^t}\left(\left(1-\mu\right)p_0\left(s^t\right)_0^{FT}y\left(s^t\right)_0^{FT}\right)}{\Pi_0\left(s_0\right)_0^{FT}}\right)$$

and for $t\geq 1$

$$C\left(s^t\right)_0^{FT}=$$
$$\left(\frac{\Pi_0\left(s_0\right)_0^{FT}\beta^t\pi_{0,t}\left(s^t\right)}{P_0\left(s^t\right)_0^{FT}}\frac{1-\beta}{\theta_0\left(1-\beta\right)+\beta\left(1-\beta^T\right)}\right)^\rho\left(\frac{b_0\left(s_0\right)_0^{FT}+\sum_{t=0}^T\sum_{s^t}\left(\left(1-\mu\right)p_0\left(s^t\right)_0^{FT}y\left(s^t\right)_0^{FT}\right)}{\Pi_0\left(s_0\right)_0^{FT}}\right)$$

so

$$X\left(s^t\right)_0^{FT}=C\left(s^t\right)_0^{FT}+M\left(s^t\right)_0^{FT}$$
$$x_1\left(s^t\right)_0^{FT}=\left(\frac{P_0\left(s^t\right)_0^{FT}}{p_0\left(s^t\right)_0^{FT}}\right)^\eta X\left(s^t\right)_0^{FT}$$

and

$$x_2\left(s^t\right)_0^{FT}=\left(\frac{P_0\left(s^t\right)_0^{FT}}{dp_0^*\left(s^t\right)_0^{FT}}\right)^\eta X\left(s^t\right)_0^{FT}$$

and excess demand for the US good is given by (ROW is analogous):

$$ED\left(s^t\right)_0^{FT}=x_1\left(s^t\right)_0^{FT}+d^*x_1^*\left(s^t\right)_0^{FT}-y\left(s^t\right)_0^{FT}$$

Use excess demand to inform a new guess of output prices, i.e. $p_0\left(s^t\right)_1^{FT}$: If excess demand is positive, increase the output price, if it is negative, reduce the output price. Iterate to convergence. This gives market-clearing output prices conditional on the value of θ_0^*.

Next, solve for the value of θ_0^* that exactly matches observed data at date 0. For a guess of θ_0^*, check the date-0 US ratio of net exports to GDP:

$$\frac{NX_0\left(s_0\right)^{FT}}{GDP_0\left(s_0\right)^{FT}}=\frac{p_0\left(s_0\right)^{FT}y\left(s_0\right)^{FT}-P_0\left(s_0\right)^{FT}X_0^{i,FT}}{\left(1-\mu\right)p_0\left(s_0\right)^{FT}y\left(s_0\right)^{FT}}=\kappa_0$$

If κ_0 is greater than the target, increase θ_0^*. If κ_0 is less than the target, reduce θ_0^*. Iterate to convergence. The value of e θ_0^* that matches net exports is 0.81.

D.6 Computation of trade war equilibrium

Guess a vector of output prices (i.e. $p_0\left(s^t\right)_0^{TW}$ and $p_0^*\left(s^t\right)_0^{TW}$ for all t, s^t), normalizing one value to 1. Calculate consumption prices using:

$$P_0\left(s^t\right)_0^{TW} = \left(\left(p_0\left(s^t\right)_0^{TW}\right)^{1-\eta} + \left((1+\tau)\, dp_0^*\left(s^t\right)_0^{TW}\right)^{1-\eta}\right)^{\frac{1}{1-\eta}}$$

(and similarly for $P_0^*\left(s^t\right)_0^{TW}$). Then output is:

$$y\left(s^t\right)_0^{TW} = L\left(z\left(s^t\right)\right)^{\frac{1}{1-\mu}}\left(\frac{\mu p_0\left(s^t\right)_0^{TW}}{P_0\left(s^t\right)_0^{TW}}\right)^{\frac{\mu}{1-\mu}}$$

and

$$M\left(s^t\right)_0^{TW} = \mu^{\frac{1}{1-\mu}}\left(\frac{p_0\left(s^t\right)_0^{TW} z\left(s^t\right)}{P_0\left(s^t\right)_0^{TW}}\right)^{\frac{1}{1-\mu}} L^i$$

and net foreign assets are:

$$b_0\left(s_0\right)_0^{TW} = \sum_{t=0}^{\infty}\sum_{s^t} p_0^*\left(s^t\right)_0^{TW}\alpha_0\left(s^t\right) y^*\left(s^t\right)_0^{TW} - \sum_{t=0}^{\infty}\sum_{s^t} p_0\left(s^t\right)_0^{TW}\alpha_0^*\left(s^t\right) y\left(s^t\right)_0^{TW}$$

given the portfolio $\left\{\alpha_0\left(s^t\right), \alpha_0^*\left(s^t\right)\right\}$. Now define

$$\Pi_0\left(s_0\right)_0^{TW} =$$

$$\left(\frac{1-\beta}{\theta_0(1-\beta)+\beta\left(1-\beta^T\right)}\right)^{\frac{\rho}{1-\rho}}\left[\begin{array}{c}\left(\theta_0\right)^\rho\left(P_0\left(s_0\right)_0^{TW}\right)^{1-\rho} + \\ \sum_{t=1}^T \sum_{s^t}\left(\beta^t \pi\left(s^t\right)\right)^\rho\left(P_0\left(s^t\right)_0^{TW}\right)^{1-\rho}\end{array}\right]^{\frac{1}{1-\rho}}$$

Guess that $C\left(s^t\right)_0^{TW} = 0$ for all t, s^t, i (i.e. demand only comes from materials), and similarly for ROW. Use this guess to calculate tariff revenue in the US (and similarly for $T_0^*\left(s^t\right)_{00}^{TW}$):

$$T_0\left(s^t\right)_{00}^{TW} = \frac{\left(C\left(s^t\right)_{00}^{TW} + M\left(s^t\right)_0^{TW}\right)}{\left(P_0\left(s^t\right)_0^{TW}\right)^{-\eta}}\left(\tau(1+\tau)^{-\eta}\left(dp_0^*\left(s^t\right)_0^{TW}\right)^{1-\eta}\right)$$

Then given the tariff revenue, calculate updated consumption:

$$C\left(s_0\right)_{01}^{TW} =$$

$$\left(\frac{\Pi_0\left(s_0\right)_0^{TW}\theta_0}{P_0\left(s_0\right)_0^{TW}}\frac{1-\beta}{\theta_0\left(1-\beta\right)+\beta\left(1-\beta^T\right)}\right)^{\rho}\left(\frac{b_0\left(s_0\right)_0^{TW}+\sum_{t=0}^{\infty}\sum_{s^t}\left(\left(1-\mu\right)p_0\left(s^t\right)_0^{TW}y\left(s^t\right)_0^{TW}+T_0\left(s^t\right)_{00}^{TW}\right)}{\Pi_0\left(s_0\right)_0^{TW}}\right)$$

and for $t\geq 1$

$$C\left(s^t\right)_{01}^{TW} =$$

$$\left(\frac{\Pi_0\left(s_0\right)_0^{TW}\beta^t\pi_{0,t}\left(s^t\right)}{P_0\left(s^t\right)_0^{TW}}\frac{1-\beta}{\theta_0\left(1-\beta\right)+\beta\left(1-\beta^T\right)}\right)^{\rho}\left(\frac{b_0\left(s_0\right)_0^{TW}+\sum_{t=0}^{\infty}\sum_{s^t}\left(\left(1-\mu\right)p_0\left(s^t\right)_0^{TW}y\left(s^t\right)_0^{TW}+T_0\left(s^t\right)_{00}^{TW}\right)}{\Pi_0\left(s_0\right)_0^{TW}}\right)$$

and similarly for ROW. Iterate until $C\left(s^t\right)_{0n}^{TW}=C\left(s^t\right)_{0n-1}^{TW}=C\left(s^t\right)_0^{TW}$. Then:

$$X\left(s^t\right)_0^{TW}=C\left(s^t\right)_0^{TW}+M\left(s^t\right)_0^{TW}$$

and

$$x_1\left(s^t\right)_0^{TW}=\left(\frac{P_0\left(s^t\right)_0^{TW}}{p_0\left(s^t\right)_0^{TW}}\right)^{\eta}X\left(s^t\right)_0^{TW}$$

and

$$x_2\left(s^t\right)_0^{TW}=\left(\frac{P_0\left(s^t\right)_0^{TW}}{\left(1+\tau\right)dp_0^*\left(s^t\right)_0^{TW}}\right)^{\eta}X\left(s^t\right)_0^{TW}$$

with similar expressions for ROW. Excess demand for the US good is given by (ROW is analogous):

$$ED\left(s^t\right)_0^{TW}=x_1\left(s^t\right)_0^{TW}+d^*x_1^*\left(s^t\right)_0^{TW}-y\left(s^t\right)_0^{TW}$$

Use excess demand to inform a new guess of output prices, i.e. $p_0\left(s^t\right)_1^{TW}$: If excess demand is positive, increase the output price, if it is negative, reduce the output price. Iterate to convergence.

D.7 Additional results

Tables 6, 7 and 8 contain the results for counterfactual tariffs set at 200% and 300%, as well as the full set of results for the baseline of 25%. For comparison, the tables also reports results for a version of the model where we set initial net and gross foreign assets to zero, and recalibrate θ^* so we continue to match the trade balance in 2023.

We also perform the trade war counterfactuals using a portfolio that matches the composition of actual portfolios in 2023 in terms of equity and debt. In particular, we construct a portfolio where under free trade, US equity assets, US equity liabilities, US net dollar debt and US net non-dollar debt, expressed as shares of 2023 US GDP, match their values in the data, as reported in Table 2. To model the debt part of the portfolio, we need to take a stand on maturity. The US Treasury TICS reports a weighted average maturity of US debt assets in 2023 of 7.6 years and a weighted average maturity of US debt liabilities in 2023 of 8.6 years. The maturity distribution is not reported separately for debt assets and liabilities denominated in different currencies. Since the weighted average maturity is similar for assets and liabilities, we use the average maturity for assets and liabilities (i.e. 8.1). We model debt as a bond that decays exponentially, with the rate of exponential decay chosen to match this weighted average maturity.

The portfolio is then given by $\left\{\alpha_0, \alpha_0^*\right\}$ such that:

$$\alpha_0 = \psi_0 \frac{(1-\mu)\, p_0\left(s_0\right)^{FT} y\left(s_0\right)^{FT}}{\sum_{t=0}^{T} \sum_{s^t} p_0^*\left(s^t\right)^{FT} y^*\left(s^t\right)^{FT}}$$

$$\alpha_0^* = \psi_0^* \frac{(1-\mu)\, p_0\left(s_0\right)^{FT} y\left(s_0\right)^{FT}}{\sum_{t=0}^{T} \sum_{s^t} p_0\left(s^t\right)^{FT} y\left(s^t\right)^{FT}}$$

and $\left\{\beta_0, \beta_0^*\right\}$ such that:

$$\beta_0 = \delta_0 \frac{(1-\mu)\, p_0\left(s_0\right)^{FT} y\left(s_0\right)^{FT}}{\sum_{t=0}^{T} \sum_{s^t} P_0^*\left(s^t\right)^{FT} \lambda^t}$$

$$\beta_0^* = \delta_0^* \frac{(1-\mu)\, p_0\left(s_0\right)^{FT} y\left(s_0\right)^{FT}}{\sum_{t=0}^{T} \sum_{s^t} P_0\left(s^t\right)^{FT} \lambda^t}$$

where ψ_0 is the ratio of 2023 US equity assets to 2023 US GDP, ψ_0^* is the ratio of 2023 ROW equity assets to 2023 US GDP, δ_0 is the ratio of 2023 net US non-dollar debt assets to US GDP, δ_0^* is the ratio of 2023 net ROW non-dollar debt assets to US GDP, and $\lambda = 1 - 1/8.1$.

Table 9 reports the results for the bond and equity portfolio under 25%, 200% and 300% tariffs. Results are very similar to those based on the baseline equity-only portfolio.

Table 10 reports the best response and unilaterally optimal tariff counterfactuals for the four portfolios considered in the paper, as well as for the case of zero net and gross foreign assets. Table 11 reports the best response and unilaterally optimal tariff counterfactuals using the bond and equity portfolio.

Table 6: Trade War Scenarios: 25% Tariffs

τ	τ^*	a	a^*	NX	$\Delta(p/p^*)$	$\Delta(P/P^*)$	ΔU	ΔU^*
				Baseline Portfolio				
0	0	81%	157%	-2.9%	0	0	0	0
25%	25%	83%	154%	-2.2%	-1.2%	-0.3%	−0.99 %	−0.18 %
25%	0	72%	155%	-3.3%	12%	13%	0.20%	−0.21 %
0	25%	92%	155%	-2.4%	-12%	-11%	−0.95 %	0.13%
				No Gross Positions				
0	0	0	76%	-2.9%	0	0	0	0
25%	25%	0	75%	-2.1%	-1.5%	-0.6%	−1.18 %	−0.12 %
25%	0	0	75%	-3.6%	13%	13%	0.57%	−0.32 %
0	25	0	75%	-1.9%	-12%	-12%	−1.48 %	0.29%
				Debt-Trap Portfolio				
0	0	-76%	0	-2.9%	0	0	0	0
25%	25%	-79%	0	-1.9%	-1.9%	-1.0%	−1.38 %	−0.06 %
25%	0	-68%	0	-3.9%	13%	14%	0.94%	−0.43 %
0	25%	-88%	0	-1.5%	-13%	-12%	−2.04 %	0.46%
				Contract Curve Portfolio				
0	0	314%	391%	-2.9%	0	0	0	0
25%	25%	319%	380%	-2.6%	-0.3%	0.5%	−0.52 %	−0.32 %
25%	0	285%	386%	-2.6%	11%	12%	−0.65 %	0.04%
0	25%	351%	385%	-3.4%	-9.9%	-9.5%	0.37%	−0.26 %
				Zero Initial Net and Gross Foreign Assets				
0	0	0	0	-2.9%	0	0	0	0
25%	25%	0	0	-3.2%	1.1%	1.8%	−0.86 %	−0.22 %
25%	0	0	0	-4.3%	14%	15%	0.88%	−0.44 %
0	25%	0	0	-2.3%	-12%	-11%	−1.48 %	0.32%

Notes: a, a^* and NX are date-0 values of US claims to ROW, ROW claims to US, and US net exports respectively, expressed as percentages of date-0 US GDP. $\Delta(p/p^*)$ and $\Delta(P/P^*)$ are percentage deviations of date-0 US terms-of-trade and real exchange rate from their levels at date 0 under free trade. ΔU and ΔU^* are percentage deviations of date-0 US and ROW welfare (in consumption-equivalent units) from their levels at date 0 under free trade.

Table 7: Trade War Scenarios: 200% Tariffs

τ	τ^*	a	a^*	NX	$\Delta(p/p^*)$	$\Delta(P/P^*)$	ΔU	ΔU^*
				Baseline Portfolio				
0	0	81%	157%	-2.9%	0	0	0	0
200%	200%	98%	150%	0.2%	-14.8%	-13.4%	−2.90 %	−0.93 %
200%	0	61%	152%	-1.2%	36.1%	37.7%	−1.94 %	−0.31 %
0	200%	120%	152%	-1.6%	-31.1%	-30.3%	−1.47 %	−0.45 %
				No Gross Positions				
0	0	0	76%	-2.9%	0	0	0	0
200%	200%	0	73%	0.9%	-20.4%	-19.0%	−3.87 %	−0.57 %
200%	0	0	74%	-2.0%	39.8%	41.4%	−0.96 %	−0.61 %
0	200%	0	74%	-0.2%	-34.5%	-33.6%	−3.29 %	0.12%
				Debt-Trap Portfolio				
0	0	-76%	0	-2.9%	0	0	0	0
200%	200%	-108%	0	1.9%	-27.2%	-25.9%	−5.38 %	0.01%
200%	0	-54%	0	-2.9%	44.4%	45.9%	0.18%	−0.96 %
0	200%	-126%	0	1.5%	-38.1%	-37.2%	−5.49 %	0.80%
				Contract Curve Portfolio				
0	0	314%	391%	-2.9%	0	0	0	0
200%	200%	341%	367%	-0.7%	-5.31%	-3.83%	−1.59 %	−1.43 %
200%	0	249%	378%	0.2%	29.1%	30.9%	−3.86 %	0.29%
0	200%	419%	374%	-4.3%	-23.9%	-23.2%	2.07%	−1.54 %
				Zero Initial Net and Gross Foreign Assets				
0	0	0%	0%	-2.9%	0	0	0	0
200%	200%	0	0	-1.3%	2.33%	3.83%	−3.26 %	−0.95 %
200%	0	0	0	-4.1%	51.1%	52.4%	0.16%	−0.99 %
0	200%	0	0	-1.3%	-31.7%	-30.9%	−3.31 %	0.13%

Notes: a, a^* and NX are date-0 values of US claims to ROW, ROW claims to US, and US net exports respectively, expressed as percentages of date-0 US GDP. $\Delta(p/p^*)$ and $\Delta(P/P^*)$ are percentage deviations of date-0 US terms-of-trade and real exchange rate from their levels at date 0 under free trade. ΔU and ΔU^* are percentage deviations of date-0 US and ROW welfare (in consumption-equivalent units) from their levels at date 0 under free trade.

Table 8: Trade War Scenarios: 300% Tariffs

τ	τ^*	a	a^*	NX	$\Delta(p/p^*)$	$\Delta(P/P^*)$	ΔU	ΔU^*
				Baseline Portfolio				
0	0	81%	157%	-2.9%	0	0	0	0
300%	300%	120%	149%	0.4%	-30.3%	-29.1%	−2.65 %	−1.54 %
300%	0	56%	151%	0.7%	47.9%	50.3%	−3.87 %	−0.27 %
0	300%	145%	150%	-1.4%	-43.0%	-42.2%	−1.03 %	−1.24 %
				No Gross Positions				
0	0	0	76%	-2.9%	0	0	0	0
300%	300%	0	73%	1.5%	-45.0%	-43.9%	−4.46 %	−0.65 %
300%	0	0	73%	-0.1%	56.1%	58.6%	−2.76 %	−0.65 %
0	300%	0	73%	0.8%	-51.0%	-50.2%	−4.02 %	−0.22 %
				Debt-Trap Portfolio				
0	0	-76%	0	-2.9%	0	0	0	0
300%	300%	-184%	0	4.3%	-57.4%	-56.5%	−8.78 %	1.11%
300%	0	-46%	0	-1.2%	69.2%	71.6%	−1.23 %	−1.17 %
0	300%	-193%	0	4.2%	-59.3%	-58.5%	−8.80 %	1.42%
				Contract Curve Portfolio				
0	0	314%	391%	-2.9%	0	0	0	0
300%	300%	355%	363%	-0.2%	-8.97%	-7.43%	−1.60 %	−2.08 %
300%	0	237%	375%	2.0%	35.8%	38.2%	−5.84 %	0.41%
0	300%	451%	369%	-4.7%	-29.0%	-28.4%	3.19%	−2.52 %
				Zero Initial Net and Gross Foreign Assets				
0	0	0%	0%	-2.9%	0	0	0	0
300%	300%	0	0	-0.5%	2.36%	4.05%	−4.30 %	−1.28 %
300%	0	0	0	-3.0%	91.7%	94.0%	−1.18 %	−1.22 %
0	300%	0	0	-0.7%	-45.6%	-44.8%	−4.09 %	−0.28 %

Notes: a, a^* and NX are date-0 values of US claims to ROW, ROW claims to US, and US net exports respectively, expressed as percentages of date-0 US GDP. $\Delta(p/p^*)$ and $\Delta(P/P^*)$ are percentage deviations of date-0 US terms-of-trade and real exchange rate from their levels at date 0 under free trade. ΔU and ΔU^* are percentage deviations of date-0 US and ROW welfare (in consumption-equivalent units) from their levels at date 0 under free trade.

Table 9: Bond & Equity Portfolio: 25%, 200% and 300% Tariffs

τ	τ^*	a	a^*	NX	$\Delta(p/p^*)$	$\Delta(P/P^*)$	ΔU	ΔU^*
				25% Tariffs				
0	0	81%	157%	-2.9%	0	0	0	0
25%	25%	83%	155%	-2.2%	-1.25%	-0.38%	-1.03 %	-0.17 %
25%	0	72%	156%	-3.3%	12.3%	12.8%	0.18%	-0.20 %
0	25%	92%	156%	-2.3%	-11.5%	-11.0%	-0.97 %	0.14%
				200% Tariffs				
0	0	81%	157%	-2.9%	0	0	0	0
200%	200%	98%	152%	0.3%	-15.2%	-13.7%	-2.96 %	-0.91 %
200%	0	61%	154%	-1.2%	35.8%	37.5%	-2.00 %	-0.29 %
0	200%	120%	154%	-1.5%	-31.2%	-30.4%	-1.52 %	-0.43 %
				300% Tariffs				
0	0	81%	157%	-2.9%	0	0	0	0
300%	300%	121%	151%	0.4%	-30.8%	-29.6%	-2.69 %	-1.52 %
300%	0	56%	153%	0.7%	47.4%	49.9%	-3.94 %	-0.25 %
0	300%	146%	152%	-1.3%	-43.1%	-42.3%	-1.08 %	-1.23 %

Table 10: Nash trade war and unilateral optimal tariffs

τ	τ^*	a	a^*	NX/GDP	$\Delta(p/p^*)$	$\Delta(P/P^*)$	ΔU	ΔU^*
				Baseline Portfolio				
0	0	81%	157%	-2.9%	0	0	0	0
11.5%	17%	84%	155%	-2.7%	-2.9%	-2.3%	−0.58 %	−0.03 %
15%	0	75%	156%	-3.3%	7.7%	8.0%	0.29%	−0.15 %
0	20%	90%	156%	-2.5%	-9.5%	-9.1%	−0.82 %	0.14%
				No Gross Positions				
0	0	0	76%	-2.9%	0	0	0	0
19%	37%	0	75%	-1.6%	-9.2%	-8.3%	−1.71 %	−0.03 %
23%	0	0	75%	-3.6%	12%	12%	0.57%	−0.30 %
0	37	0	75%	-1.6%	-17%	-16%	−1.95 %	0.31%
				Debt-Trap Portfolio				
0	0	-76%	0	-2.9%	0	0	0	0
35%	400%	-378%	0	9.8%	-79%	-79%	−16.0 %	3.55%
35%	0	-65%	0	-3.9%	18%	19%	0.99%	−0.54 %
0	400%	-379%	0	9.9%	-79%	-79%	−16.0 %	3.58%
				Contract Curve Portfolio				
0	0	314%	391%	-2.9%	0	0	0	0
1%	0	313%	390%	-2.9%	0.5%	0.5%	0.00%	0.00%
1%	0	313%	390%	-2.9%	0.5%	0.5%	0.00%	0.00%
0	0	314%	391%	-2.9%	0	0	0	0
				Zero Initial Net and Gross Foreign Assets				
0	0	0	0	-2.9%	0	0	0	0
34.5%	37.5%	0	0	-2.8%	0.2%	1.1%	−1.41 %	−0.34 %
35%	0	0	0	-4.5%	20%	20%	0.93%	−0.56 %
0	38%	0	0	-2.1%	-16%	-16%	−1.99 %	0.34%

Notes: The tariffs in the second row of each panel are the Nash equilibrium tariffs, capped at 400%. In the case of the net-only portfolio expressed in terms of the ROW good, this cap is binding for ROW. The tariffs in the third row of each panel are the unilaterally optimal tariffs for the US. The 400% cap is not binding in this case. The tariffs in the fourth row of each panel are the unilaterally optimal tariffs for the ROW. The 400% cap is binding in the case of the net-only portfolio expressed in terms of the ROW good.

Table 11: Bond & Equity Portfolio: Nash Trade War and Unilateral Optimal Tariffs

τ	τ^*	a	a^*	NX	$\Delta(p/p^*)$	$\Delta(P/P^*)$	ΔU	ΔU^*
				Bond and Equity Portfolio				
0%	0%	81%	157%	-2.9%	0%	0%	0 %	0 %
11%	17.5%	84%	156%	-2.6%	-3.4%	-2.8%	-0.63 %	-0.02 %
14%	0%	76%	156%	-3.3%	7.2%	7.5%	0.27%	-0.14 %
0%	20%	90%	156%	-2.4%	-9.5%	-9.1%	-0.84 %	0.14%

Notes: The tariffs in the second row are the Nash equilibrium tariffs. The tariffs in the third row are the unilaterally optimal tariffs for the US. The tariffs in the fourth row are the unilaterally optimal tariffs for the ROW.