

Tariff Wars and Net Foreign Assets*

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Abstract

This paper examines whether and how international financial claims can be honored without default once a trade war erupts. Using a two-country two-good model, we study how tariffs revalue international assets and how ex-post indebtedness depends on ex-ante gross asset positions. When gross positions are strictly positive (each country has claims on the other), sufficiently high bilateral tariffs zero out any ex-ante net debt. Conversely, this net debt cannot be zeroed out when the debtor holds no claims on the creditor. When a country has liabilities indexed to the other country's prices, as in the case of foreign currency debt, then a sufficiently large tariff levied by the lender can immiserize the borrower, generating multiple equilibria if both sides have such liabilities. We generalize the environment to an arbitrary number of goods, time, and uncertainty. In the dynamic version, under a severe trade war, domestic interest rates and the real exchange rate must decouple: interest rates align with their autarkic values, and international relative prices revalue net asset positions to zero. We calibrate a two-country model to the US net foreign asset position in 2023 and show that the welfare consequences of a tariff war that stops short of autarky depend critically on the US ex-ante gross portfolio composition.

1 Introduction

The two eras of globalization before the First World War and after the 1970s witnessed large declines in trade costs and substantial increases in cross-border asset positions. In the United States,

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both eras eventually ended in protectionism, exemplified in the US by the Smoot-Hawley tariffs in 1930 and the Trump tariffs in 2018-2025. This latter period, combining the implementation of substantial tariffs by the United States with unprecedented gross and net foreign asset positions accumulated during the prior decades of financial globalization, makes understanding the interaction between trade wars and international balance sheets a crucial question in international macroeconomics.

In this paper, we study whether and how the stock of international financial claims accumulated during a period of relatively free trade can be settled once a tariff war breaks out. Specifically, we analyze whether equilibria exist in which these claims are honored without explicit default, how international prices adjust to facilitate such settlement, and what welfare consequences follow for creditor and debtor nations.

Our main contribution is to show how different ex-ante gross portfolios that are consistent with an identical net position under free trade can give rise to very different outcomes in the event of a tariff war. Our key insight is that when tariffs restrict trade, changes in international relative prices revalue international assets. We show that the impact of this revaluation, and therefore the set of equilibria that can obtain in a tariff war, depends on the nature of the ex-ante gross positions. For some ex-ante gross positions, a severe enough tariff war may wipe out net debt, for others, valuation effects cannot eliminate net debt, and for others, a tariff war may even immiserize the debtor.

We first develop these ideas in a two-country (Home and Foreign) two-good static exchange economy, where Home is a net debtor at free trade prices. Given that tariffs drive a wedge between prices of the same good in the two locations, it matters at which prices these claims are settled. This is akin to currency denomination for assets. When Home and Foreign both hold positive claims on each other's comparative advantage good denominated in the issuer's prices, a situation we refer to as "strictly positive claims," autarky is the unique equilibrium for high enough tariffs. In this zero-trade equilibrium, international relative prices adjust to revalue ex-ante claims in such a way that Home's net debt is zeroed out without default. While this price clears the financial market, high enough tariffs ensure there is no scope for goods arbitrage. At more moderate tariff levels, a trade war may induce a revaluation of the gross asset positions that supports a balanced-trade equilibrium with positive two-way trade flows.

In the case where Home (the net debtor) holds no offsetting claims on Foreign, we show that autarky fails to be an equilibrium, and Home must repay its debt by exporting, even for arbitrarily high tariffs. This highlights that gross positions are crucial to our first result.

When Home's liabilities are denominated in Foreign prices, mirroring emerging markets' foreign-currency debt, if Foreign's tariff is high enough, there exists an equilibrium where Home must export both goods, regardless of its free-trade net position. The mechanism is self-fulfilling:

trade flowing to Foreign raises the value of Home’s liabilities (payable in Foreign) via the tariff wedge, increasing the real debt burden and forcing Home to export more to satisfy its budget constraint. In this scenario, Home is immiserized at the expense of Foreign.

Crucially, Home cannot avoid this fate by imposing retaliatory tariffs; indeed, Home’s tariffs play no role in the equilibrium outcome. Its only defense is to avoid incurring foreign currency liabilities in the first place. This highlights that the recent trend among emerging economies to hold foreign currency assets (reserves) and borrow in their domestic currency (the decline of “Original Sin”¹) has consequences for how vulnerable they are to foreign tariffs.

We extend the analysis to a two-country multiple-good static exchange economy with traded and non-traded goods, where households hold gross claims (positive or negative) on foreign goods. In an autarky allocation, domestic relative prices, determined by endowments, must value foreign claims. If both countries’ external gross claims have the same sign when valued at these domestic prices (a generalization of the simple “positive claims” case), there exists a unique international relative price that zeros out these claims, and autarky is an equilibrium for high enough tariffs, without default on any obligations.

Because this result depends only on the “same-sign” condition at autarkic prices, the logic extends naturally to dynamic settings with time and uncertainty. In a severe tariff war, domestic interest rates and the real exchange rate decouple: interest rates align with autarkic values to aggregate domestic claims, while the real exchange rate is pinned down by the condition that zeros out cross-border assets. This provides a potential rationalization for the fact that the recent tariff war affected the dollar exchange rate but had no major impact on interest rates.

For our benchmark analysis, we take existing portfolios as an inherited state and contrast equilibria with and without tariffs. This begs the question of whether such portfolios are consistent with equilibrium ex-ante when a trade war is anticipated. We answer this question in the affirmative by exploring a simple portfolio problem when agents face uncertainty over whether there will be a trade war in the next period. We show that households may be willing to hold gross positions in equilibrium and, as a reaction to the possibility of a trade war, may even increase them.

We also quantify the valuation, terms of trade, and welfare effects of a tariff war and show how these effects depend on the nature of the gross portfolio underlying a given net foreign asset position. To do so, we use a two-country (US and Rest of the World) dynamic stochastic gravity model based on Fitzgerald (2025). We calibrate to the US net foreign asset position and net exports in 2023.

We consider several alternative portfolios consistent with the same free trade net debt, includ-

¹The term “Original Sin” was coined by Eichengreen and Hausmann (1999) to refer to emerging markets’ inability to borrow in domestic currency.

ing one that closely matches the data, and a net-only portfolio where US liabilities are indexed to Rest of the World prices. The impact of a tariff war on net foreign assets, the terms of trade, and welfare varies substantially across portfolios. With the portfolio that matches the data, a tariff war can reduce US net debt, but for this to happen, the Rest of the World must retaliate. Welfare losses for the US due to a symmetric tariff war at rates of 25% total 1.2 percent of consumption, compared to total gains from trade of 1.8 percent. When US liabilities are instead indexed to Rest of the World prices, the welfare losses from retaliatory tariffs are doubled, reflecting the importance to welfare of tariff-induced portfolio revaluations.

Literature review. Our questions and the associated answers echo the Transfer Problem famously debated by Keynes (1929) and Ohlin (1929) in the aftermath of World War I. That debate turned on whether a transfer by one country to another affected the terms of trade. With identical homothetic preferences, and no tariffs or trade costs, the distribution of wealth does not influence relative prices, but this is not true when the marginal dollar is spent differently. Our analysis is closest to the treatment in Samuelson (1952, 1954) in an environment in which preferences are identical and homothetic, but where there are trade frictions. Samuelson discusses how, when tariffs are present, the required transfer may have an adverse impact on the payer’s terms of trade. Our question is the mirror image – if, under free trade, one country needs to make a net transfer to the other, can the disruption to trade from tariffs generate terms-of-trade changes that nullify the need to make the payment?²

Stockman and Dellas (1986) study an environment in which households can insure against tariff changes using financial assets that are contingent on tariff outcomes. They show that this insurance changes the welfare ranking of tariff policies.³ Devereux and Lee (1999) show how ex-ante portfolio choices by households (in assets that are not contingent on tariffs) constrain future governments’ incentives to manipulate the terms of trade through tariffs, demonstrating that trade in financial assets may weaken the incentives to tax trade in goods. This implies trade in assets is complementary to trade in goods, which provides a counterweight to the Cole and Obstfeld (1991) finding that the two are substitutes. They also find that the interaction between ex-ante financial trade and ex-post tariff policy can lead to multiple equilibria of the policy game, as households do not internalize the link between their portfolio choices and the subsequent optimal tariff rates. We also discuss how tariffs and asset positions can lead to indeterminacy, but our multiplicity arises for a constant set of tariffs and fixed asset positions. More generally, our main question (how initial net debt is settled in a tariff war) is different.

²Epifani and Gancia (2017) study global imbalances in a trade model and discuss how the price indexes react differently in a trade model with increasing returns a la Krugman.

³For a modern treatment that quantifies this force, see Caliendo, Kortum, and Parro (2025).

The role of valuation effects in understanding the behavior of net foreign asset positions dates back to the pioneering work of Lane and Milesi-Ferretti (2001), Tille (2003), and Gourinchas and Rey (2007). This initial work focused in large part on the behavior of the dollar exchange rate to account for changes in asset valuations. Recent work by Atkeson, Heathcote, and Perri (2025) highlights that even if the depreciation of the US dollar accounts for the surprising stability of the US NFA position in the early 2000s in the face of large trade deficits, the spectacular performance of the US equities relative to foreign equity markets accounts for the US NFA deterioration after 2008. We argue that valuation effects through endogenous changes in international relative prices are also important in the context of a tariff war, providing conditions on portfolios for such price adjustments to rebalance net foreign asset positions during trade disruptions.

Prompted by recent policy events, several papers explore the interactions between tariffs and trade deficits.⁴ Pujolas and Rossbach (2024) study the welfare effects of trade wars in an Armington model with trade imbalances, where countries have exogenous international net (but not gross) positions. Ignatenko, Lashkaripour, Macedoni, and Simonovska (2025) quantify the welfare effects of tariffs in a model with trade imbalances where net foreign asset positions are given but trade deficits endogenously respond to policy. Costinot and Werning (2025) analyze a dynamic model with fixed terms of trade and study the effects of a permanent increase in tariffs on the trade deficit by affecting the incentives of domestic households to save and consume. Obstfeld (2025) surveys the arguments for the determination of the U.S. trade deficit and argues that trade policies are not the main driver. In his discussion of this paper, Perri (2025) uses an international macro model to analyze the effects of tariff shocks, and their persistence, on the trade deficit, emphasizing also that the response of the economy to other shocks may be hampered by the presence of tariffs. Unlike these papers, our focus is on valuation effects and their role in determining equilibrium outcomes in tariff wars.

In a related and contemporaneous paper, Itskhoki and Mukhin (2025) study the optimal unilateral tariff in the presence of valuation effects in an Armington model.⁵ Our analysis is distinct for two reasons. First, we consider arbitrary tariff policies that may or may not be optimal (though we do discuss welfare implications). Second, we characterize how alternative gross asset positions map into the set of equilibria that can arise in a tariff war, providing conditions on portfolios

⁴There is previous work on global imbalances, trade deficits and trade frictions. See for example, Razin and Svensson (1983), Dekle, Eaton, and Kortum (2007), Fitzgerald (2012), Reyes-Heroles (2017), Kehoe, Ruhl, and Steinberg (2018), Alessandria, Bai, and Woo (2024), among others. There is also a large body of work on gross asset positions and diversification, see for example Stockman and Dellas (1989), Baxter and Jermann (1997), Heathcote and Perri, 2013, Tille (2003), Broner, Didier, Erce, and Schmukler (2013), Lee (2024), among many others. An explicit connection between trade policy and intertemporal trade is found in Costinot, Lorenzoni, and Werning (2014), and more recently in Dávila, Rodríguez-Clare, Schaab, and Tan (2025).

⁵There is a classic and very general theory on optimal tariffs emphasizing the terms-of-trade effects, but it does not consider valuation effects. See Dixit and Norman (1980) and Dixit (1985) for comprehensive summaries.

such that autarky is or is not an equilibrium, and where multiple equilibria coexist. We point out that whether liabilities are indexed to borrower country prices or lender country prices is crucial for valuation effects. We do not impose the Armington assumption, which allows us to consider situations where the pattern of trade becomes unidirectional under high tariffs. This is convenient for identifying equilibrium multiplicities in which the direction of trade is self-fulfilling, though it is not necessary.⁶ There is also an important antecedent in Alvarez, Atkeson, and Kehoe (2009), a connection we discuss later on.

Paper structure. Section 2 sets the stage by presenting three facts on international portfolios. Section 3 sets up the main model with the two (traded) goods. Section 4 specializes the model to simple assets, establishes the free-trade benchmark, and characterizes tariff-war equilibria across different portfolio configurations. It considers strictly positive gross positions, one-sided claims, and foreign-currency liabilities. This section also analyzes how an anticipated trade-war affects ex-ante portfolio choices. Section 5 relaxes the simple-asset and two-good restrictions. Subsection 5.1 extends the high-tariff results to general asset positions and provides conditions under which autarky is the unique equilibrium. Subsection 5.2 extends the analysis to multiple goods, trade costs, time, and uncertainty, emphasizing the distinct roles of exchange rates and interest rates in settling international positions. Section 6 provides a quantification of valuation effects, terms-of-trade effects, and welfare in a trade war which falls short of imposing autarky. Section 7 concludes. All proofs are collected in the appendix. An Online Appendix collects additional results and contains a description of the data and quantitative analysis.

2 International portfolios

Before showing how international portfolios determine outcomes in a tariff war, we document three salient facts about these portfolios.

Fact 1: Gross positions are much bigger than net positions. Since the collapse of the Bretton Woods system in the 1970s, international asset markets have become increasingly integrated. Cross-border liabilities accounted for just over 20% of world GDP in 1970, but more than 200% of world GDP by 2023.⁷ Financial integration has increasingly taken the form of two-way asset

⁶Related to the focus on optimal policy responses, Bianchi and Coulibaly (2025) analyze the optimal monetary policy to an increase in tariffs. See also the recent contributions of Werning, Lorenzoni, and Guerrieri (2025) and Auclert, Rognlie, and Straub (2025). For previous work on the monetary response to tariffs, we refer the reader to Erceg, Prestipino, and Raffo (2023) and Bergin and Corsetti (2023).

⁷Source: The External Wealth of Nations database; Lane and Milesi-Ferretti (2018).

Table 1: Gross and net foreign assets in OECD and non-OECD countries

	$\frac{\text{Assets} + \text{Liabilities} }{\text{GDP}}$		$\frac{ \text{NFA} }{\text{Assets} + \text{Liabilities} }$		$\frac{ \text{NFA} }{\text{GDP}}$	
	OECD	Non-OECD	OECD	Non-OECD	OECD	Non-OECD
25th percentile	2.35	0.91	0.06	0.13	0.21	0.18
50th percentile	4.00	1.27	0.09	0.34	0.43	0.39
75th percentile	6.74	2.20	0.22	0.49	0.54	0.58

Notes: Data from the External Wealth of Nations database. There are 31 non-tax-haven OECD countries with GDP greater than 0.1% of US GDP in 2023. There are 70 non-tax-haven non-OECD countries with GDP greater than 0.1% of US GDP in 2023.

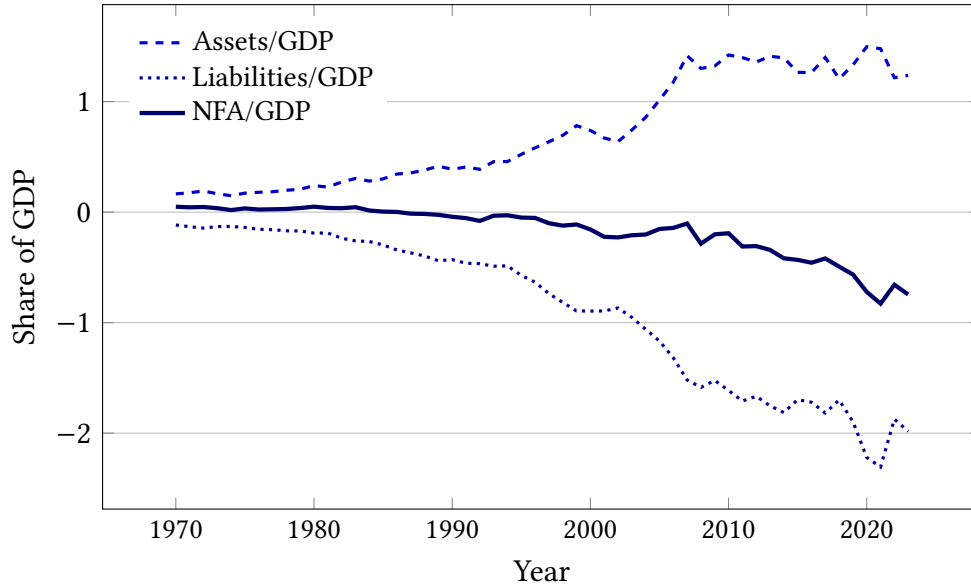
trade. Summing net foreign assets across all net debtor countries gives an indication of the minimum cross-border asset holdings necessary to account for observed net foreign asset positions (what is a liability for one country is an asset for another). In 1970, this accounted for 28% of all cross-border asset holdings. By 2023, it had fallen to just 15% of cross-border asset holdings, consistent with faster growth of gross than net positions.

For developed countries, gross foreign asset positions are particularly large. Gross positions are smaller for developing countries, but still exceed net positions. This is illustrated in Table 1, which reports the 25th, 50th and 75th percentile of gross foreign asset positions (assets + the absolute value of liabilities) relative to GDP in 2023 for non-tax-haven OECD and non-OECD countries whose 2023 GDP is greater than 0.1% of US GDP. It also reports the 25th, 50th and 75th percentile of the absolute value of net foreign assets relative to gross foreign assets for these countries.

Similar to other developed economies, the gross foreign asset position of the US is large relative to its GDP, and relative to its net foreign assets. Figure 1 plots foreign assets, foreign liabilities, and net foreign assets for the US from 1970-2023. The US has been a net debtor since 1987, but in 2023, the sum of US foreign assets and the absolute value of US foreign liabilities was 316% GDP, while US net foreign liabilities were just 75% of GDP.

Fact 2: Equity-like claims account for the majority of international liabilities. For valuation effects, it matters not just that gross asset positions differ from net positions, and are large: the nature of claims also matters. Internationally traded claims can be divided broadly into debt-like instruments (including bank loans) which may be denominated in home or foreign currency, and equity-like claims to shares of profits (this includes FDI). Throughout the period 1970-2014, debt accounted for a greater share of international liabilities than equity, but the equity share has trended upwards since the 1980s. By 2023, equity accounted for 53% of international liabilities, compared to 41% for debt (financial derivatives accounted for the remainder).

Figure 1: US gross and net foreign assets relative to GDP, 1970-2023



Notes: Data from the External Wealth of Nations database.

Since it accounts for nearly one quarter of international liabilities, and close to one sixth of international assets, the international portfolio of the United States is of particular interest. Table 2 reports the composition of the US portfolio in 2023, expressed as shares of 2023 US GDP. Although more than half of US assets and liabilities are in the form of equity, two thirds of the US negative net foreign asset position is due to its negative net debt position. Strikingly, US debt assets and debt liabilities are both largely denominated in dollars.

Table 2: US International Portfolio in 2023

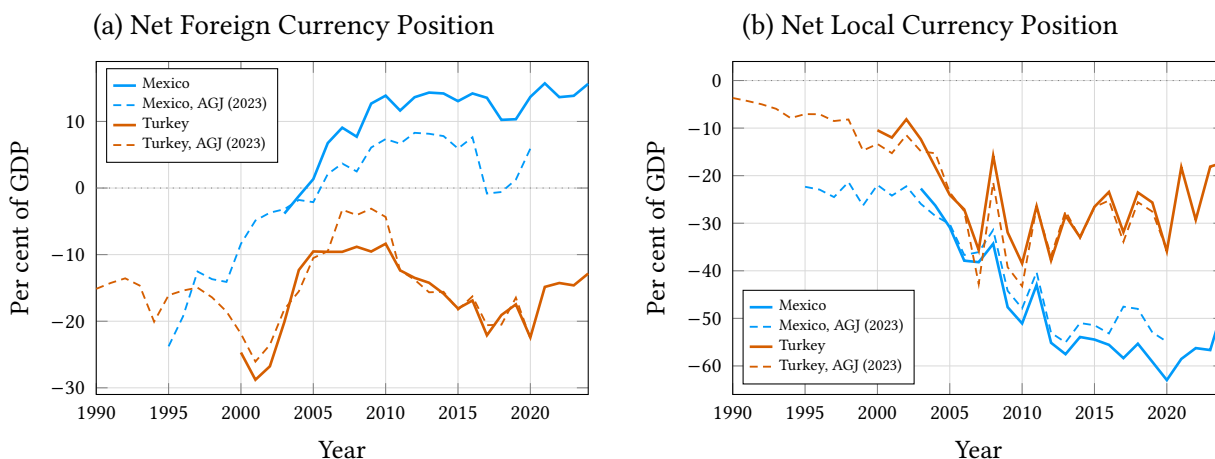
	Assets	Liabilities	Balance
Equity & FDI	0.78	1.07	-0.28
Debt (USD)	0.26	0.76	-0.50
Debt (non-USD)	0.07	0.05	0.02
Other	0.09	0.08	0.01
Total	1.20	1.96	-0.75

Notes: Data from the External Wealth of Nations database and US Treasury (currency composition of debt). Other category includes financial derivatives and foreign exchange reserves.

Fact 3: Debt may be issued in the lender's currency. The flip side of dollar-denominated debt assets on the US balance sheet is that counterparty countries have liabilities denominated in

foreign currency. In particular, many emerging markets issue external liabilities in US dollars (or other global currencies), the real burden of which increases with an appreciation of the dollar.

Figure 2: Net Foreign and Local Currency Positions: Turkey and Mexico



Notes: Panel (a) depicts the net foreign currency position and panel (b) the net local currency position, for Turkey and Mexico, as shares of GDP. For each country, the two positions combined give the Net International Investment Position (net foreign assets). Solid lines are the authors' estimates and dashed lines are those of Allen, Gautam, and Juvenal (2023). The authors' series begin when the underlying currency weights are first observed: 2000 for Turkey and 2003 for Mexico. See Online Appendix III for details.

To illustrate the importance of foreign currency liabilities for the balance sheets of countries that hold them, in Figure 2 we present positions for two large emerging markets, Turkey and Mexico. For each of these countries, we calculate the net foreign currency and net local currency positions as shares of GDP since 1990. The net foreign currency position is equal to overseas assets (foreign direct investment abroad, equity and other foreign currency portfolio investments, and foreign reserves) minus foreign currency liabilities issued by the government and private sector. The net local currency position is predominantly due to inward foreign direct and portfolio investment plus any domestic currency bonds held by foreigners. The sum of these two positions (not depicted) is equal to net foreign assets (or the Net International Investment Position). It is consistently negative for both Mexico and Turkey in the relevant period.⁸

Turkey (orange lines in Figure 2) has a consistently large negative net foreign currency position in addition to its negative net local currency position. Accounting for the negative net foreign currency position, it is well known that Turkish banks and corporations are heavy borrowers in foreign currency (see, for example, Giovanni, Kalemli-Özcan, Ulu, and Baskaya, 2022).

Mexico's pattern (blue lines of Figure 2) is common to many Latin American economies. Mex-

⁸Online Appendix III contains more detail on these calculations along with data sources. In each figure we also include the estimates from Allen, Gautam, and Juvenal (2023), which uses an alternative but closely related strategy to compute the same objects.

ico's net foreign currency position was quite negative until the early 2000s, before turning positive. The negative position in the early years reflects the fact that historically Mexico borrowed abroad primarily in dollars and had small positive positions in reserves and foreign equity. Since the mid-2000s, Mexico's increased holdings of overseas assets (including reserves, but also foreign direct investment) and relative shift towards peso debt have led its net foreign currency position to turn positive. Panel (b) shows increasing foreign holdings of Mexican peso assets, both public and private. Foreigners do not issue Mexican peso assets, and hence the net position is by definition negative.

One contribution of our paper is to highlight a previously unappreciated consequence of moving from a negative to a positive foreign currency position; namely, the consequences of a trade war are fundamentally different depending on the sign of this position, irrespective of the overall net foreign asset position. Whether the net foreign currency position – which is only one part of the net foreign asset position – is positive or negative may seem like it has only quantitative but not qualitative importance. However, in Section 4.5 we show that this intuition is incorrect. The sign is crucial: even a small negative position in foreign currency can open the door to novel equilibria that do not arise under positive foreign currency positions.

3 A Two-Good Environment

We first use a two-country two-good endowment model to study whether and how international financial claims accumulated under free trade can be settled in a tariff war. In this section, we briefly lay out the environment.

3.1 Environment

The environment is static and consists of two countries, Home and Foreign, and two goods, A and B . Using a star to denote Foreign, let endowments of A and B be denoted $Y = \{Y_A, Y_B\}$ and $Y^* = \{Y_A^*, Y_B^*\}$, with global endowments denoted by $\bar{Y}_A \equiv Y_A + Y_A^*$ and $\bar{Y}_B \equiv Y_B + Y_B^*$. Without loss, we define good A as Home's relatively abundant good:

$$\infty > \frac{Y_A}{Y_B} > \frac{\bar{Y}_A}{\bar{Y}_B} > \frac{Y_A^*}{Y_B^*} > 0. \quad (1)$$

We bound the respective endowments of both goods away from zero to ensure that relative prices are well-defined in autarky.

Preferences are assumed to be identical for Home and Foreign and characterized by a strictly increasing, strictly concave, differentiable, and homothetic utility function $u(c_A, c_B)$ over con-

sumption of the two goods.⁹ For $x = c_B/c_A$, and letting $u_i \equiv \partial u / \partial c_i$, we define the marginal rate of substitution (MRS) between B and A as:

$$g(x) \equiv \frac{u_B(1, x)}{u_A(1, x)} = \frac{u_B(c_A, c_B)}{u_A(c_A, c_B)},$$

where we use homotheticity to express the MRS as a function of the ratio of the two goods. The standard assumptions on preferences imply that $g'(x) < 0$, so the inverse $h \equiv g^{-1}$ is well defined on the range of g .

Let p_i, p_i^* denote the Home and Foreign prices of good $i \in \{A, B\}$ in units of a world numeraire currency. We restrict attention to tariffs as instruments of trade policy; export taxes are set equal to zero. Tariffs are set uniformly across all imported goods, and we let $\tau \geq 0$ and $\tau^* \geq 0$ denote the Home and Foreign tariffs.¹⁰ International goods arbitrage requires that for all traded goods, $i \in \{A, B\}$, we have:

$$\frac{p_i}{p_i^*} = \begin{cases} (1 + \tau) & \text{for Home imports: } c_i > Y_i \\ 1/(1 + \tau^*) & \text{for Foreign imports: } c_i < Y_i \end{cases}$$

with $p_i/p_i^* \in [1/(1 + \tau^*), (1 + \tau)]$ if not traded: $c_i = Y_i$.

That is, if a good is imported into Home, its price is greater in Home than Foreign by the tariff factor, and vice versa if Foreign imports the good. For nontraded goods, the price differential is bounded by the two tariffs.

Households in each country start the period with gross claims on each other. These positions are inherited as an initial state, reflecting earlier portfolio decisions. We discuss the determination of these positions in Section 4.6.

Assets are claims to goods. Since relative prices may differ across countries, it is important to be precise about which goods these are claims to and at which country's prices. Given two goods and two locations, many assets can be spanned by the following four assets: specifically, claims to goods A and B indexed to prices in Home, and claims to goods A and B indexed to prices in Foreign.

Both Home and Foreign residents may have ownership or liabilities in terms of these assets. Only the net position between Home and Foreign *within each asset* matters. We use the following sign convention: $a_A > 0$ denotes that Home has a net claim on Foreign in the asset indexed to

⁹It is not crucial that preferences are identical, and we assume it for a transparent benchmark under free-trade. Home-bias in consumption will arise due to tariffs and (in the quantitative model in Section 6) resource costs of trade.

¹⁰We extend our results to the case of trade costs in Online Appendix I.3 and include trade costs in the quantitative model in Section 6.

Table 3: Taxonomy of assets

Asset	Definition
a_A	Home asset (liability if negative) indexed to Foreign price p_A^*
a_B	Home asset (liability if negative) indexed to Foreign price p_B^*
a_A^*	Foreign asset (liability if negative) indexed to Home price p_A
a_B^*	Foreign asset (liability if negative) indexed to Home price p_B

Foreign price p_A^* . If $a_A < 0$, then Foreign has a net claim on Home in this asset. Similarly, $a_A^* > 0$ denotes that Foreign has a net claim on Home indexed to Home price p_A . We follow the same convention for good B . To the extent that the law of one price fails due to tariffs, i.e. $p_i \neq p_i^*$, a claim denominated in a particular good will have a different payout depending on which price it is indexed to. Table 3 summarizes the four cases.

From the assets in Table 3 we can construct more familiar assets. For example, an equity-like claim to a fraction γ of Foreign's total output has value $\gamma (p_A^* Y_A^* + p_B^* Y_B^*)$, which is equivalent to $a_A = \gamma Y_A^*$ and $a_B = \gamma Y_B^*$. Similarly, consider a bond that pays out in terms of a consumption basket in Foreign with fixed weights: δ_A^* and $\delta_B^* = 1 - \delta_A^*$. Ownership of a of these bonds is equivalent to a portfolio with $a_i = \delta_i^* a$, $i \in \{A, B\}$. In a nominal model, if a country's monetary authority were to hold this consumer price index (CPI) constant, this portfolio would represent a nominal bond.¹¹

With our asset notation, the budget constraints faced by Home and Foreign households are:

$$\begin{aligned} p_A(c_A + a_A^*) + p_B(c_B + a_B^*) &= p_A Y_A + p_A^* a_A + p_B Y_B + p_B^* a_B + T \\ p_A^*(c_A^* + a_A) + p_B^*(c_B^* + a_B) &= p_A^* Y_A^* + p_A a_A^* + p_B^* Y_B^* + p_B a_B^* + T^*, \end{aligned}$$

where T and T^* are government transfers in Home and Foreign, respectively. Note the presence of foreign prices in Home's budget constraint, and vice versa for the Foreign budget constraint. This reflects the asset structure discussed above.¹² The governments' budget constraints are

$$T = \tau \sum_{i \in \{A, B\}} p_i^* \max\{c_i - Y_i, 0\} \quad \text{and} \quad T^* = \tau^* \sum_{i \in \{A, B\}} p_i \max\{c_i^* - Y_i^*, 0\}.$$

¹¹If the CPI has varying weights (such as the "ideal price index"), then a bond that pays the CPI cannot be directly spanned by our simple assets, as the portfolio is implicitly indexed to tariff policy. We allow for such an asset in the quantitative model in Section 6.

¹²In standard trade models where $a_i = 0$ and $a_i^* = 0$, the levels of p_i and p_i^* do not matter for equilibrium determination; only relative prices across goods within the locations are necessary. This is Lerner symmetry: there is an equivalence in the choices of export and import taxes. In our environment with nonzero assets, p_i and p_i^* appear in the budget constraints, and Lerner symmetry fails. This failure of Lerner symmetry in the presence of cross-country asset positions appears in Costinot and Werning (2019), Barbiero, Farhi, Gopinath, and Itskhoki (2019), and, more recently, in Itskhoki and Mukhin (2025). (See also Farhi, Gopinath, and Itskhoki (2014).)

Finally, the resource constraints are $c_i + c_i^* = \bar{Y}_i$, $i \in \{A, B\}$.

The equilibrium definition is standard.

Definition 1. *Given a pair of tariffs, $(\tau, \tau^*) \geq 0$, an equilibrium consists of a non-negative consumption allocation (c_A, c_B, c_A^*, c_B^*) and a strictly positive price vector (p_A, p_B, p_A^*, p_B^*) such that: (i) households optimize subject to their budget constraints; (ii) prices satisfy goods arbitrage; (iii) the government budget constraints hold; and (iv) the aggregate resource constraints are satisfied.*

4 Simple Assets

In a tariff war, international relative price movements revalue claims. In this section, we lay out three possible cases for initial portfolios, each of which may be consistent with the same initial net debt. We characterize tariff war equilibria for the three cases, and show that revaluation of claims affects net debt very differently in each case. To ease the exposition, we restrict attention to claims indexed to the domestic prices of *abundant goods*.

Assumption 1 (Simple Claims). *Internationally traded claims to good i are indexed to prices in the country where good i is the comparative advantage good. That is, $a_A = a_B^* = 0$.*

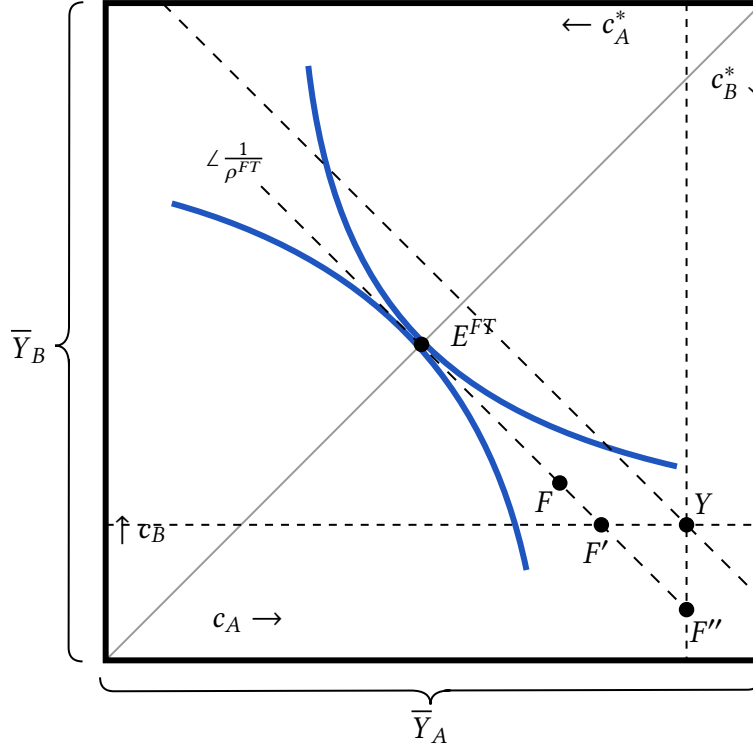
We characterize how the equilibria that arise in a tariff war, and the resulting valuation effects, depend on the signs of the positions a_A^* and a_B . In Section 5 we restore all four assets, and discuss how the results generalize.

We start by describing the free-trade benchmark, and the three possible cases for portfolios.

4.1 The Free Trade Benchmark

Under free trade, $\tau = \tau^* = 0$, prices for each good are the same in both countries, and hence marginal rates of substitution are equalized across Home and Foreign. To provide a diagrammatic intuition, we use the Edgeworth box depicted in Figure 3. The dimensions of the box are $\bar{Y}_A \times \bar{Y}_B$. Let $Y \equiv (Y_A, Y_B)$ denote Home's endowment point. Home's consumption of A and B is depicted from the southwest corner, with A on the horizontal axis and B on the vertical axis. Foreign consumption is the mirror image, with its origin in the northeast corner. Given identical, homothetic preferences, the indifference curves are tangent along the diagonal, which thus outlines the efficient contract curve. The endowment point is depicted by the point Y . The point

Figure 3: Free Trade Equilibrium



$F \equiv (Y_A - a_A^*, Y_B + a_B)$ depicts the endowment adjusted by initial asset positions in a case where $a_A^* > 0$ and $a_B > 0$. This places F to the northwest of the endowment point Y .

We define international relative prices (the terms of trade in the case depicted) as

$$\rho^{FT} \equiv \frac{p_B^{FT}}{p_A^{FT}} = g\left(\frac{\bar{Y}_B}{\bar{Y}_A}\right),$$

where the last equality uses the fact that under free trade and our preference assumptions the consumers in each country equate their MRS and hence the equilibrium lies along the diagonal.

Using the fact that under free trade the law of one price holds and tariff revenue is zero, we can divide Home's budget constraint by p_A and obtain:

$$c_A + \rho^{FT} c_B = (Y_A - a_A^*) + \rho^{FT} (Y_B + a_B).$$

In the figure, this budget constraint is represented by the line through point F with slope $-1/\rho^{FT}$. From market clearing, the Foreign budget set can be read along the same line but from the opposite origin. The free-trade equilibrium consumption allocation is where this budget line intersects the diagonal, E^{FT} . As depicted, E^{FT} is to the left and above Y , indicating that Home exports good A and imports good B .

The net foreign asset position of Home expressed in units of good A is given by $\rho^{FT} a_B - a_A^*$. This is the horizontal distance between the budget line and the line with the same slope which passes through the endowment point Y . The figure shows the case where Home is a debtor, and thus $\rho^{FT} a_B - a_A^* < 0$.

In what follows, we focus on the scenario depicted in the figure:

Assumption 2 (Free Trade Benchmark). *In the free-trade equilibrium, we have:*

- (i) *Two way trade: $c_A^{FT} < Y_A$ and $c_B^{FT} > Y_B$;*
- (ii) *Home is a net debtor: $c_A^{FT} + \rho^{FT} c_B^{FT} < Y_A + \rho^{FT} Y_B$.*

Part (i) rules out an initial asset position so indebted that it forces one country to export both goods. Having two-way trade as the baseline allows for an interesting and well defined terms of trade as reference.¹³ Part (ii) simply defines the Home country as the net debtor at free-trade prices (as in Figure 3), which is essentially a normalization.

Note that the free trade equilibrium consumption allocation can be characterized *as if* the endowment point were shifted by the initial net asset positions (for each good); that is, treating F as a translated endowment point. However equilibrium trade flows depend on the position of the endowment point Y , not the financial position F . Since tariffs apply to import flows, this characterization of equilibrium by a simple translation of the endowment point will not extend in general to the case with tariffs, as we show below.

Irrelevance of gross positions under free-trade. Let us highlight a feature of the equilibrium under free-trade: only the net foreign asset position matters. That is, only $\rho^{FT} a_B - a_A^*$ is relevant. In the figure, we display two alternative financial positions: points F' and F'' . In point F' , $a_A^* > 0$, while $a_B = 0$. In point F'' , $a_A^* = 0$ and $a_B < 0$. In both cases, Home's financial position has not changed under free trade prices (it remains a net debtor) and the equilibrium under free-trade remains at E^{FT} . More generally, all asset positions that keep Home on the same free-trade budget line lead to the same equilibrium E^{FT} under free trade.

Once tariffs are introduced, this irrelevance result no longer holds. The nature of the equilibrium response to tariffs depends sensitively on the underlying gross portfolio positions. We analyze the three possible scenarios that may arise depending on the underlying gross positions:

- (i) "Strictly positive claims" where both asset positions are strictly positive (as in point F);

¹³This would also be a natural assumption under Armington specialization.

- (ii) “Net-only claims” where one of the asset positions is zero (as in point F'); and
- (iii) “Debt-traps” where at least one of the asset positions is negative (as in point F'').

As we now demonstrate, these portfolios have starkly different implications for the equilibrium response to a tariff war. We develop this argument in several steps. First, we prove existence and uniqueness of equilibrium for “strictly positive claims” and “net-only claims” portfolios under arbitrary positive tariffs. Then we show that for high enough tariffs, revaluation of the gross portfolio can eliminate net debt under “strictly positive claims,” but not with “net-only claims.” Next we show that in the case of “debt-trap” portfolios, revaluation of assets in a trade war can increase net debt, and that when both asset positions are negative, the possibility of multiple equilibria arises. Finally, we touch on ex-ante portfolio determination, and how the possibility of a future trade war affects incentives to trade financial claims.

4.2 Tariff War Equilibria with Weakly Positive Simple Claims

We first consider the case in which each country holds weakly positive claims on the other country’s abundant good: $a_A^* \geq 0$ and $a_B \geq 0$ (recalling that $a_A = a_B^* = 0$).

Under weakly positive simple claims, tariffs (that is, $\tau, \tau^* \geq 0$) cannot reverse the free-trade direction of trade: Home weakly exports A and weakly imports B. This is intuitive given that tariffs are levied on whichever good is imported. If trade were reversed, then import tariffs would fall on the relatively abundant good, reducing the incentive to import it compared to the free trade equilibrium. In this case, the international terms of trade $\rho \equiv p_B^*/p_A$ are sufficient to characterize the country budget sets, which are given by:

$$\begin{aligned} c_A + \rho c_B &= Y_A - a_A^* + \rho(Y_B + a_B), \\ c_A^* + \rho c_B^* &= Y_A^* + a_A^* + \rho(Y_B^* - a_B). \end{aligned} \tag{2}$$

Thus, the budget sets in a tariff equilibrium are the same as under free-trade, but evaluated at a potentially different terms of trade.¹⁴

When trade is strictly positive in both goods, we have $p_B/p_A = (1+\tau)\rho$ and $p_B^*/p_A^* = \rho/(1+\tau^*)$. This implies that the households’ first-order conditions are:

$$\left(\frac{1}{1+\tau} \right) g \left(\frac{c_B}{c_A} \right) = \rho = (1+\tau^*) g \left(\frac{\bar{Y}_B - c_B}{\bar{Y}_A - c_A} \right). \tag{3}$$

¹⁴In Subsection 4.5 we show that with negative assets, the pattern of trade may reverse, and the budget sets are no longer characterized by just the terms of trade as above.

However, if a good is not actively traded, the respective MRS and equilibrium prices can take values within the intervals spanned by the tariff wedges. In particular,

$$g\left(\frac{c_B}{c_A}\right) = \frac{p_B}{p_A} = \frac{p_B}{p_B^*} \rho \quad \text{and} \quad g\left(\frac{c_B^*}{c_A^*}\right) = \frac{p_B^*}{p_A^*} = \frac{p_A}{p_A^*} \rho, \quad \text{both in } \left[\frac{\rho}{1+\tau^*}, (1+\tau)\rho\right].$$

By market clearing, $\bar{Y}_B/\bar{Y}_A$ must lie in between the two countries' consumption ratios. As a result, the free-trade relative price must also lie in between the two countries' domestic relative prices. Given that the domestic relative prices must be in $\rho/(1+\tau^*)$ and $(1+\tau)\rho$, an equilibrium value of ρ must satisfy

$$\frac{\rho^{FT}}{1+\tau} \leq \rho \leq (1+\tau^*)\rho^{FT}. \quad (4)$$

These results are formalized in Appendix B, Lemma B.1.

Let us highlight that the tariff rates do not appear in the budget sets (2). This is one of the main expositional simplifications that simple assets deliver: in terms of budget constraints, it is as if Home's endowment point has been translated from (Y_A, Y_B) to $(Y_A - a_A^*, Y_B + a_B)$, as is the case under free trade. Given this, one may think that a tariff equilibrium with financial positions is equivalent to a tariff equilibrium without financial positions, but with an appropriately translated endowment point. However, and differently from under free trade, the location of the original endowment point still matters, as we show below.

Under Assumption 1 (simple assets), if $a_B \geq 0$ and $a_A^* \geq 0$, a tariff equilibrium always exists and, under an additional gross substitutes assumption on preferences, it is unique. We show this result next:

Proposition 1 (Existence and Uniqueness under Positive Simple Claims). *Suppose Assumptions 1 and 2 hold, and*

$$a_A^* \geq 0, \quad a_B \geq 0.$$

Suppose, in addition, that $h \equiv g^{-1}$ is well defined on $(0, \infty)$. Then, for every pair of tariffs $(\tau, \tau^) \geq 0$, a tariff equilibrium exists.*

If, in addition, h is continuously differentiable and household demand satisfies the gross-substitutes condition:

$$\frac{d}{dq}[qh(q)] \leq 0 \quad \text{for all } q > 0, \quad (5)$$

then the equilibrium terms of trade and consumption allocation are unique.

Proof. In Appendix A.1. □

The gross substitute condition (5) on household demand states that the ratio of expenditure on good B to expenditure on good A does not rise with the relative price of B .¹⁵ As an example, if preferences are CES with elasticity of substitution $\sigma > 0$, gross substitutes in household demand requires $\sigma \geq 1$.

4.3 Tariff Wars with Strictly Positive Simple Claims

Suppose that each country owns a *strictly* positive claim on the other country's abundant good: $a_A^* > 0$ and $a_B > 0$. Given the restriction to simple claims, this is the case that corresponds to the portfolios of advanced economies such as the US. As we document in Section 2, the US has a gross position which greatly exceeds its net position, and its liabilities are indexed to domestic prices. Many of the insights about how tariffs and asset positions interact can be illustrated by considering a severe enough trade war that, in this strictly positive assets case, leads to autarky. We have the following result:

Proposition 2 (Autarky under Strictly Positive Claims). *Suppose that Assumptions 1 and 2 hold. Suppose also that*

$$a_A^* > 0, \quad \text{and} \quad a_B > 0.$$

Then there exist finite $\underline{\tau}, \underline{\tau}^$ such that for any $\tau \geq \underline{\tau}$ and $\tau^* \geq \underline{\tau}^*$, the autarkic allocation $c_A = Y_A, c_B = Y_B, c_A^* = Y_A^*, c_B^* = Y_B^*$ is the unique equilibrium.*

Proof. See Appendix A.2. □

The key message of this proposition is that with an initial portfolio that satisfies Assumption 1 with strictly positive claims, for high enough tariffs, autarky is the unique equilibrium, and this *does not involve default* on financial claims. The uniqueness result should not come as a surprise given Proposition 1 (but, as we show later on, uniqueness when tariffs are sufficiently high is significantly more general than stated).

An Edgeworth box representation. To understand why autarky emerges as the equilibrium allocation, and the role played by revaluation of the portfolio in this result, we make use of an Edgeworth box diagram depicted in Figure 4. As before, $Y = (Y_A, Y_B)$ is the endowment point

¹⁵In the classical general equilibrium literature, the gross substitutes is a condition on the aggregate excess demand function; see Kehoe (1998). Here, we are imposing it only on household demand. The proof shows that, with weakly positive simple claims, the gross substitutes condition in household demand delivers a monotone aggregate excess demand function, which is sufficient for uniqueness.

and $F = (Y_A - a_A^*, Y_B + a_B)$ is the endowment translated by the asset positions. Assumption 1 and strictly positive claims place F outside the shaded region.

First consider an equilibrium with positive tariffs $\{\tau > 0, \tau^* > 0\}$ and strictly positive trade in both goods. This is reflected in panel (a). Because tariffs do not reverse the pattern of trade (Lemma B.1), the equilibrium cannot be in the interior of the shaded region of Figure 4. The budget sets are characterized by $p_B^*/p_A = \rho$, the international terms of trade, and the marginal rate of substitution in each country are related to each other and to ρ by equation (3).

The ray from Home's origin traces out $c_B/c_A = h((1 + \tau)\rho)$, the first equality in equation (3), where recall that h is the inverse of the MRS function g . Similarly, the ray from Foreign's origin contains the locus $c_B^*/c_A^* = h(\rho/(1 + \tau^*))$, the second equality of (3). Consistent with tariffs distorting Home's consumption toward good A and Foreign toward B , we depict the rays below the free-trade diagonal.

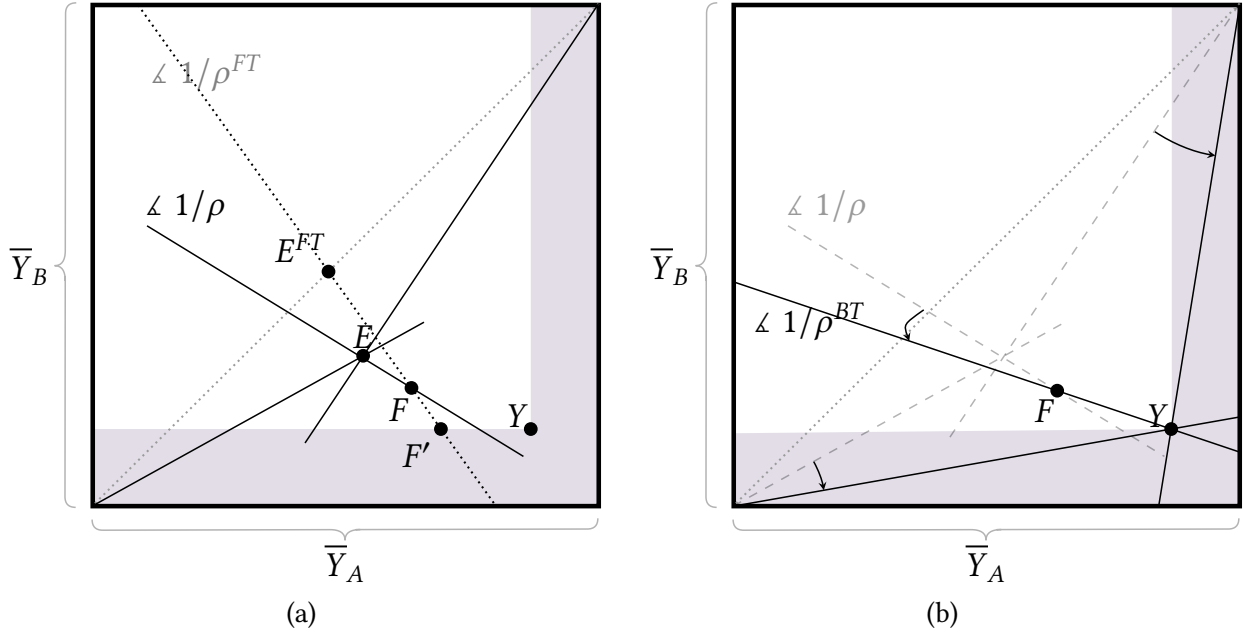
Equation (2) states that feasible consumption allocations for Home are those along the line with slope $-1/\rho$ passing through the point F . Using the resource constraint, Foreign's allocations viewed from the opposite origin lie along the same line.

The equilibrium ρ is such that these rays intersect along the budget line, depicted as point E . This intersection is the solution to three equations (the two optimality conditions in equation (3) and the Home budget constraint (2)) in three unknowns (c_A, c_B, ρ). The Foreign budget constraint is satisfied by Walras' Law. As depicted, the equilibrium terms of trade are such that Home is a net debtor, as evidenced by the fact that the endowment point Y lies beyond (to the right) of Home's budget line when evaluated at the equilibrium relative prices ρ . As shown in (2), in any equilibrium the terms of trade, $\rho = p_B^*/p_A$, play the dual role of being the international relative price of the goods, as well as being the relative price of the international claims.

Now suppose that we have an equilibrium as depicted in Figure 4 and we increase tariffs in both countries. This is now shown in panel (b). For a given ρ , from (3), the dashed rays from the origin rotate away from the diagonal, as each country's imports become more expensive. Intuitively, tariffs reduce trade and move the equilibrium closer to autarky, although some care must be taken with this intuition, as alternative equilibria arise once we relax Assumption 1 (simple claims). With that caveat, and given Assumption 1, increasing both tariffs pushes the equilibrium towards autarky. Since we have assumed positive endowments of both goods for both countries, marginal rates of substitution are bounded in autarky, and hence so too are the tariffs that implement autarky.

As noted above, the fact that imports become expensive reduces the incentives to trade. However, this begs the question of how financial claims are settled without any trade (and hence, no

Figure 4: Tariff War Equilibrium



net trade). From Home's budget constraint we obtain

$$-a_A^* + \rho a_B = c_A - Y_A + \rho(c_B - Y_B) = 0,$$

where the last equality follows from the fact that $c_A = Y_A$ and $c_B = Y_B$ in Autarky. Thus, Autarky is consistent with financial claims when

$$\rho = \rho^{BT} \equiv \frac{a_A^*}{a_B},$$

where the superscript *BT* denotes “balanced trade.” As long as both a_B and a_A^* are strictly positive, this is a valid terms of trade. That is, the equilibrium terms of trade are pinned down by the asset positions. The alert reader may notice that this would also be true if both a_B and a_A^* were strictly *negative*, but in this case, uniqueness would not be guaranteed, an issue we confront in Sections 4.5 and 5.1.

To see what is occurring to the country's net worth, recall that in Figure 4 at equilibrium E , Home is a net debtor at the relative price ρ . That is $\rho a_B < a_A^*$. As tariffs increase, ρ increases to the point that $\rho = \rho^{BT}$ such that $\rho^{BT} a_B = a_A^*$. Thus, the relative value of Home's liability *declines* to ensure that financial claims can be settled with zero net trade. To put it differently, in a sufficiently severe tariff war, valuation effects can wipe out the ex-ante net debt.

Balanced Trade with Positive Trade. Autarky is the limiting equilibrium for large enough tariffs, but for intermediate cases a trade war may generate an equilibrium in which there is two-way trade that is balanced. In Online Appendix 1.1 we show that as long as $\rho^{BT} = a_A^*/a_B$ lies between the two countries' autarkic relative prices, there exist a continuum of tariff pairs that result in a balanced-trade equilibrium. Notably, a Home tariff alone is not sufficient to achieve balanced trade: Foreign must also impose a sufficiently high tariff. That is, *the debtor country cannot close its net foreign asset position through unilateral tariffs*. In Section 6 we show in a quantitative model that in a tariff war which falls short of autarky, Foreign tariffs reduce the value of Home's net debt, while Home tariffs increase it.

Welfare. We use Figure 5 to guide our discussion of the welfare consequences of a trade war under strictly positive simple claims (i.e. a financial position given by F). The figure reproduces the free-trade equilibrium from Figure 3 and overlays the balanced trade relative price line from Figure 4 Panel (b) that passes through both F and Y . It also includes the indifference curves for Home (u) and Foreign (u^*) associated with the free-trade equilibrium. Efficiency implies that the indifference curves are tangent at the free-trade equilibrium. From Proposition 2, with financial position F , a severe enough tariff war results in the autarkic equilibrium at point Y . As we have just noted, if the financial position F lies in the cone between Home and Foreign autarky prices, there exist tariff pairs which lead to balanced-trade equilibria along an interval of the $1/\rho^{BT}$ relative price line that includes Y .

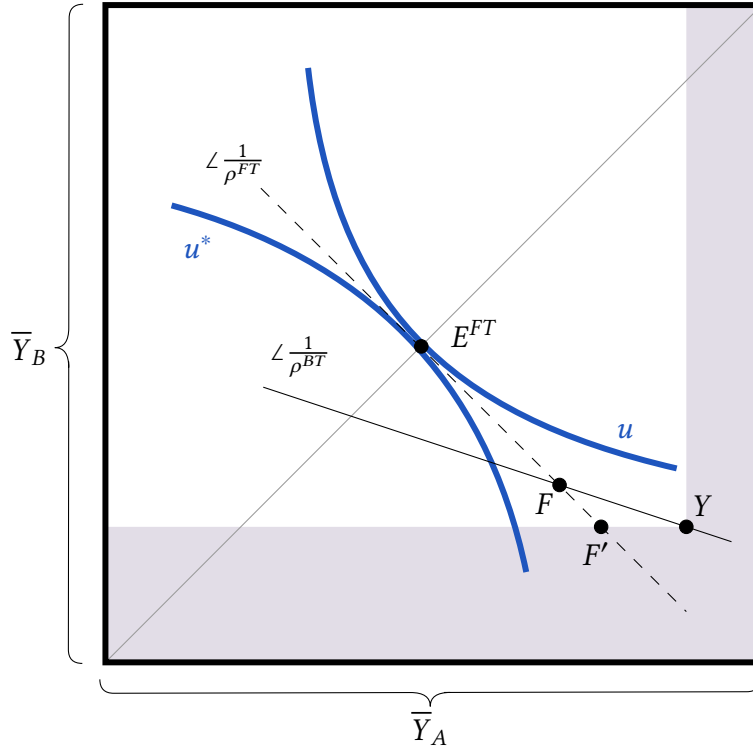
There are three effects on each country's welfare from a tariff war: (i) losses of the gains from trade, which are always negative for both countries; (ii) terms-of-trade effects on the price of imports relative to exports; and (iii) revaluation of gross asset positions.

The loss of the gains from trade is represented in the figure as movements away from the diagonal, which is the efficient contract curve given identical homothetic preferences. This loss ensures that under tariffs, at least one country (and possibly both) are worse off than under free trade.

Along the balanced-trade line, the valuation of gross asset positions is held fixed, zeroing out Foreign's claims on Home. This unambiguously benefits Home relative to free trade, while hurting Foreign. In contrast, if tariffs are such that $p_B^*/p_A < \rho^{BT}$, Home remains a net debtor. If they are such that $p_B^*/p_A > \rho^{BT}$, Home becomes a net creditor.

Along the balanced-trade line to the left of F , there may exist equilibria that make Foreign better off, while making Home strictly worse off. If they exist, these are equilibria where there is enough trade (both imports of A and exports of B) such that Foreign's gain from its improved terms of trade outweighs its losses from (i) and (iii). Along the balanced-trade line to the right of F , there may exist equilibria that make Home better off, while making Foreign strictly worse

Figure 5: Welfare Consequences



off. If they exist, these are equilibria where there is so little trade that Home's losses from the worsening of its terms of trade and from (i) are outweighed by the benefit from (iii). In Autarky, as a creditor country, Foreign unambiguously loses relative to free trade, while Home may or may not be better off. The more indebted it is, the more likely Home is better off under Autarky than free trade.

Notice that ex-post incentives to impose tariffs are influenced by the presence of gross claims. This is the insight of Devereux and Lee (1999) in the context of the optimal tariff. The gains to a country from improving its terms of trade are mitigated by the reduction in the value of its financial claims. For example, in Figure 5, if $F = E^{FT}$, there is zero incentive to manipulate the terms of trade since all of the gains from terms-of-trade effects on the price of imports relative to exports are offset by losses on financial positions. The growth in gross asset positions documented in Section 2 can therefore serve as a mechanism to reduce the incentive to engage in terms-of-trade manipulation.¹⁶

¹⁶Bagwell and Staiger (1999) argues that the World Trade Organization is designed to eliminate negative-sum terms-of-trade manipulation. Financial integration can serve as a market-based substitute for such formal institutions.

4.4 Tariff Wars with Net Only Simple Claims

Consider the case where Home's financial position is defined by a single asset: that is, $a_B = 0$ but $a_A^* > 0$. This setup is familiar from international macroeconomics, where it is often assumed that a single asset is traded internationally, with debtors holding a negative position, and creditors a positive position. This situation corresponds to point F' in Figure 4 panel (a). The outcome of a tariff war in this scenario contrasts sharply with the outcome under strictly positive positions on both sides. The next proposition shows that F' (and not autarky, as in the positive claims case) is the unique equilibrium allocation for high enough tariffs on either side:¹⁷

Proposition 3. *Suppose Assumptions 1 and 2 hold. Suppose that*

$$a_B = 0, \text{ and } 0 < a_A^*.$$

Then there exists a finite threshold κ such that for any tariff rates (τ, τ^) with $(1+\tau)(1+\tau^*) \geq \kappa$, the allocation $(c_A, c_B, c_A^*, c_B^*) = (Y_A - a_A^*, Y_B, Y_A^* + a_A^*, Y_B^*)$ is the unique equilibrium. In this equilibrium, the terms of trade, ρ , is given by:*

$$\rho = (1 + \tau^*)g\left(\frac{Y_B^*}{Y_A^* + a_A^*}\right).$$

Proof. See Appendix A.3. □

It is instructive to contrast this outcome with the autarkic outcome of Proposition 2. In particular, take the case of gross positions, $a_B > 0$ and $a_A^* > 0$, and let a_B be arbitrarily small. As in the case of Proposition 3, suppose we attempt to construct an equilibrium in which Home exports to pay a_A^* . Home's net wealth is given by

$$\rho a_B - a_A^* = (1 + \tau^*)g\left(\frac{c_B^*}{c_A^*}\right)a_B - a_A^*.$$

As τ^* becomes large, given that $a_B > 0$, Home becomes arbitrarily wealthy. The reason is that if Home is exporting to Foreign, a higher tariff in Foreign makes Home *wealthier*, as it appreciates its claim on Foreign goods. At some wealth level, Home no longer is a net debtor and no longer needs to export to Foreign. The presence of an asset paying off in Foreign prices is enough to ensure Home can settle its claims without exporting for large enough tariffs. This example shows that a net asset position composed of competing gross positions behaves fundamentally differently

¹⁷As we show in the proof, Assumption 2 implies $a_A^* < Y_A$, so the consumption allocation is indeed strictly positive.

in a trade war than if we had a single financial asset, even if the non-zero gross positions are arbitrarily small.

There is also the question of why Home does not import. Note that the conditions of Proposition 3 can be satisfied with zero Home tariffs as long as the Foreign tariff is large enough (and vice versa). However, as is clear from the expression for ρ in the proposition, a large Foreign tariff makes imports of good B very expensive (relative to Home's export good), negating the incentive to trade A for B . The only trade is the amount necessary to settle financial claims.

Welfare. Referring again to Figure 5, we can discuss welfare in a tariff war with net only simple claims, i.e. a financial position given by point F' . Following Proposition 3, starting from this financial position, a severe enough tariff war results in an equilibrium at point F' . Note that at this point, both countries are necessarily worse off relative to free trade. Because F' lies on the original free-trade budget line, it was an affordable choice for both countries under free trade, but was dominated by the allocation in which the gains from trade were fully exploited. In terms of the three channels, at F' , both countries lose the gains from trade, Home must still pay its debts via exporting, and as Foreign does not export to Home, its improved terms of trade is of no value.

4.5 Tariff Wars with Negative Simple Claims: Debt Traps

As noted in Section 2, many emerging markets have liabilities denominated in foreign currency. One way to study this in our setup is to consider the case where a borrower incurs liabilities indexed to lender country prices, a situation that is ruled out under weakly positive simple claims. Relaxing this requirement opens the door to equilibria we refer to as “debt traps”.

Assume Home has a negative net position in good B denominated in Foreign prices: $a_B < 0$. That is, to settle this claim, Home must pay Foreign $p_B^*|a_B|$ units of the numeraire currency. In this case, when Foreign tariffs are high enough, an equilibrium emerges where Home must export *both* goods A and B to pay its liabilities, breaking the no-trade-reversals result of Section 4.2. In this equilibrium, the terms of trade are such that the real value of Home's liability increases relative to free trade and it becomes immiserized by Foreign tariffs. We first state the result and then explain the economics:

Proposition 4 (Home's Debt Trap). *Suppose Assumptions 1 and 2 hold. Suppose that*

$$a_B < 0.$$

Then there exists $(\underline{\tau}^, \bar{\tau}^*)$ with $0 < \underline{\tau}^* < \bar{\tau}^* < \infty$ such that for any $\tau^* \in (\underline{\tau}^*, \bar{\tau}^*)$ there is a tariff*

equilibrium where Home exports both goods: $c_A < Y_A$, $c_B < Y_B$; relative prices are the same as under free trade: $g(c_B/c_A) = \rho^{FT}$; and Home's budget set relative to free trade is:

$$c_A - c_A^{FT} + \rho^{FT}(c_B - c_B^{FT}) = \tau^* \rho^{FT} a_B < 0.$$

Proof. Proof: See Appendix A.4. □

To see the intuition behind this result, let us guess and verify an equilibrium in which Home exports both goods. Goods arbitrage when $c_A < Y_A$ and $c_B < Y_B$ implies that $p_A^* = (1 + \tau^*)p_A$ and $p_B^* = (1 + \tau^*)p_B$. Hence, Foreign and Home households face the same relative prices, and these prices equate their marginal rates of substitution. That is, their indifference curves are tangent and hence the conjectured equilibrium allocation lies on the contract curve, i.e. along the diagonal of the Edgeworth box. We depict our candidate equilibrium as point E_1 in Figure 6. Note that this implies that

$$g(c_B/c_A) = g(c_B^*/c_A^*) = g(\bar{Y}_B/\bar{Y}_A) = \rho^{FT},$$

where the last equality follows from the definition of ρ^{FT} . Thus, households face the free trade relative price between goods B and A , but their budget constraints differ from those under free trade. Home's budget constraint in this case is:

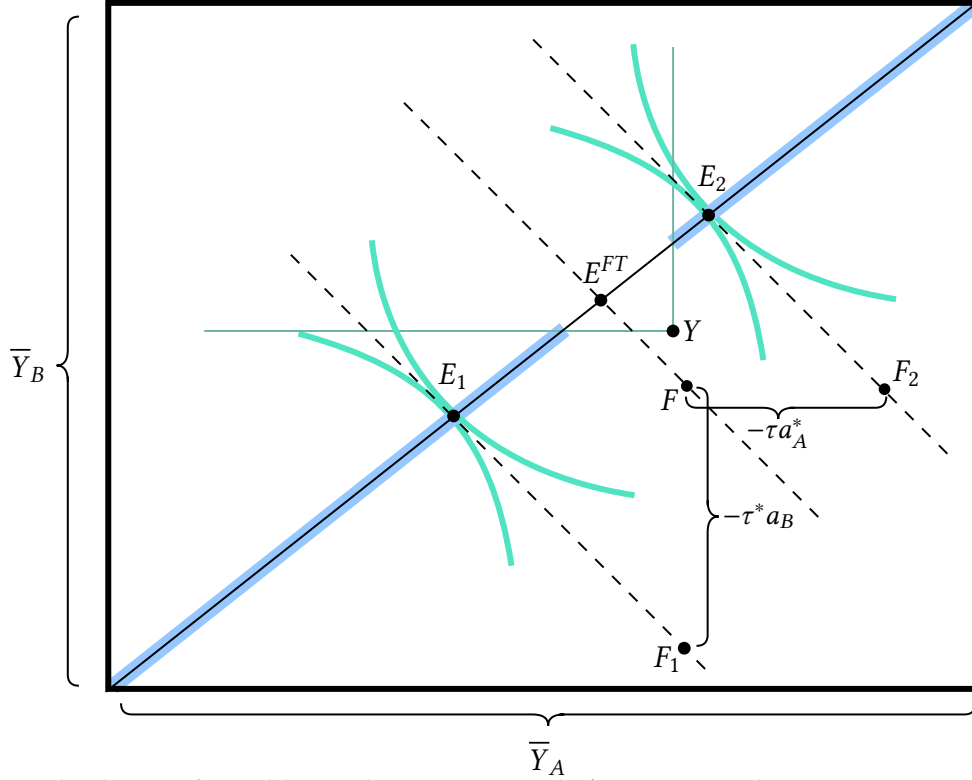
$$\begin{aligned} p_A c_A + p_B c_B &= p_A (Y_A - a_A^*) + p_B (Y_B + (1 + \tau^*) a_B) \\ \Rightarrow c_A + \rho^{FT} c_B &= Y_A - a_A^* + \rho^{FT} (Y_B + (1 + \tau^*) a_B) \\ \Rightarrow c_A + \rho^{FT} c_B &= Y_A - a_A^* + \rho^{FT} (Y_B + a_B) + \rho^{FT} \tau^* a_B. \end{aligned}$$

This is the same as the free-trade budget line but translated down by $-\tau^* a_B > 0$. Define $F_1 \equiv (Y_A - a_A^*, Y_B + (1 + \tau^*) a_B)$ as the translated financial position of Home in our conjectured equilibrium. Point E_1 in the figure must be at the intersection of the line with slope $-1/\rho^{FT}$ from F_1 and the diagonal.

The last thing to check is that E_1 is indeed in the region such that $0 < c_A < Y_A$ and $0 < c_B < Y_B$, as required for equilibrium. As we have moved along the diagonal from free trade to the left, we automatically have $c_A < c_A^{FT} < Y_A$. The lower bound on τ^* in the proposition ensures that F_1 is far enough below E^{FT} such that $c_B < Y_B$ at E_1 . The upper bound on τ^* ensures that it is still feasible for Home to pay the additional $\tau^* \rho^{FT} a_B$ relative to free trade; that is, that consumption is not forced to be negative to satisfy the budget set.

As E_1 is on the diagonal and strictly below the free trade equilibrium, it is unambiguously the case that Home suffers in welfare terms from the foreign tariff relative to free trade and Foreign

Figure 6: Tariff War: Multiplicity with Short Selling



Note: Multiplicity of equilibria when $a_B < 0$ and $a_A^* < 0$. E_1 and E_2 represent two tariff equilibria for the same tariff rates (τ, τ^*) . The shaded segments in the contract curve represent allocations that can now be attained as an equilibrium outcome for some tariff combinations of (τ, τ^*) .

gains. As $a_B < 0$, as τ^* increases, the burden of delivering good B to Foreign (at Foreign prices) increases for Home. It is “as if” Home must pay a liability in an appreciated foreign currency due to the foreign tariff. This immiserization forces Home to export both goods in order to service its debt.

This outcome is reminiscent of a country (other than the US) that owes external debt in US dollars, such as the examples discussed in Section 2. As the United States raises tariffs, the dollar appreciates, increasing the value of the debt evaluated at the debtors’ prices (or world prices). Higher tariffs in the United States thus paradoxically force the debtor to export more not less, in contrast to the case with positive assets in Proposition 2.

Symmetry and Multiplicity. Note that Proposition 4 does not rely on particular values for Home’s tariffs or Foreign’s liability. In fact, a symmetric result holds for Foreign, and in this case it is the Home tariff, τ , that determines the existence conditions. The full corollary is stated in Appendix B.

Foreign's debt trap is depicted by point E_2 in Figure 6. The point F_2 represents the position $(Y_A - (1 + \tau)a_A^*, Y_B + a_B)$, which is the free trade position translated to the right by $-\tau a_A^*$. As Home's tariff increases, Foreign's liability in good A (delivered in Home) becomes more of a burden, and it must turn to exporting both goods. This pushes the equilibrium toward Foreign's origin, reducing Foreign's welfare and increasing Home's.

The fact that τ plays no role in the condition for E_1 , and τ^* plays no role in the condition for E_2 reflects the fact that both equilibria feature only one country with imports, and the other country's tariff is not relevant. This implies that conditions for both equilibria can be simultaneously satisfied. That is, either scenario can be an equilibrium *for a fixed set of tariffs τ and τ^** .

It is worth emphasizing that these equilibria are dramatically different. In one of them, Home is poor, and consumes strictly less than its endowment of both goods. In the other, Foreign is poor, and consumes strictly less than its endowment of both goods. Which country has a debt trap is thus completely dependent on the coordination of beliefs (i.e., a sunspot). The multiplicity arises from the revaluation of net foreign asset positions due to the direction of trade. Under positive claims, these equilibria do not emerge. The reason is that in such cases, higher tariffs in Foreign make the Home wealthier (rather than poorer) when trade flows from Home to Foreign, invalidating the equilibrium conjecture that Home is poor enough. However, when a country is indebted in a "foreign currency" (i.e., a liability to be settled in the other country) it is vulnerable to a self-fulfilling welfare loss if tariffs are appropriately high.

These two equilibria are not necessarily the only ones. For example, when tariffs are large enough so that no trade is consistent with goods arbitrage, then the autarky allocation is also an equilibrium. Now note that the Home debt trap equilibrium cannot be sustained when τ^* is too large, as the equilibrium becomes infeasible (and similarly in Foreign's case). This suggests that when tariffs are sufficiently high, the only equilibrium is indeed autarky. This leads us to a more general uniqueness result than Proposition 2, something we discuss in Section 5.1.

4.6 Trade-War Anticipation and Portfolio Rebalancing

Our analysis so far takes a_A^* and a_B as given, without explaining where portfolios come from. To focus on the impact of tariffs, we have simplified by assuming that endowments are fixed. But of course if endowments are uncertain and imperfectly correlated, the standard risk-sharing rationale motivates ex-ante claims on the other country's endowments.¹⁸

That said, it is worth asking whether households still want to hold gross positions if the only source of exogenous risk is the possibility of a severe trade war that leads to autarky. In other

¹⁸In more general environments, it is also possible to rationalize negative positions indexed to foreign prices as in the literature on "Original Sin" (Eichengreen and Hausmann, 1999).

words, would news of a possible future severe trade war lead households to rebalance portfolios away from gross positions inherited for reasons outside the model? We can show that gross positions can persist, and may even increase in response to news of a possible future trade war.

To do this, we maintain the one period setup, but modify the model as follows. At the beginning of the period, there is uncertainty about the state of the world $s \in S \equiv \{FT, W\}$. If $s = FT$, the economy is in a free trade state and there are no tariffs. If $s = W$, the economy is in a trade war state and tariffs are imposed. The ex-ante probability of each state is given by $\pi(s) \in (0, 1)$ with $\pi(FT) + \pi(W) = 1$.

Households enter the period with an inherited portfolio of assets (a_A^*, a_B) and can rebalance this portfolio in a competitive asset market that operates before the state of the world is realized. The claims that are traded are the simple assets of Assumption 1. Let q denote the ex-ante price of the asset that is indexed to the Foreign country's abundant good (B) relative to the asset that is indexed to the Home country's abundant good (A). Let $(\hat{a}_A^*, \hat{a}_B)$ denote the Home household's portfolio of after-trade positions in the two assets. Note that rebalancing is a swap, and thus the ex-ante budget constraint is

$$q\hat{a}_B - \hat{a}_A^* = qa_B - a_A^*.$$

Once the state is realized, households solve the static problem of Section 3 with a portfolio given by $(\hat{a}_A^*, \hat{a}_B)$, and then consumption takes place. The formal statements of the Home and Foreign households' problems, and the market clearing conditions are given in Appendix C.

An *equilibrium in the ex-ante economy* consists of an ex-ante price q and after-trade asset positions $(\hat{a}_A^*, \hat{a}_B)$, along with ex-post goods prices and consumption allocations in each state s for both Home and Foreign households, such that households optimize, the ex-ante asset market clears, and in each state of the world, given the ex-post portfolio, the prices and consumption allocations form a tariff equilibrium as in Definition 1, with zero tariffs in the FT state and tariffs (τ, τ^*) in the W state.

We focus on the determination of $(\hat{a}_A^* > 0, \hat{a}_B > 0)$ in the case of *full-trade-war equilibria*, equilibria in which the ex-post positions are strictly positive and the tariffs are high enough that the trade-war state is autarkic, as in Proposition 2. Of course, ex-post positions are an equilibrium object, so we first guess that they are strictly positive, and then have to verify that this is true under certain conditions. The full argument is left to Appendix C, but we sketch the idea here.

Since preferences are identical and homothetic, the free trade terms of trade are given by $\rho^{FT} = g(\bar{Y}_B/\bar{Y}_A)$. In the case of a trade war, by the definition of a full-trade-war equilibrium, the consumption allocation of each country is given by its endowment, and the terms of trade are $\rho^W = \hat{a}_A^*/\hat{a}_B$. In both states, the tariff revenue is zero.

Although the assets are not indexed by the trade policy state, as long as $\rho^{FT} \neq \rho^W$, households perceive asset markets to be complete.¹⁹ Atomistic households do not see trade as infeasible in the trade war state, merely as taking place at unfavorable prices. As a result, claims on foreigners' comparative advantage good are perceived to have a positive payoff in this state.

The households' first order conditions yield the following risk sharing condition:

$$\frac{u_A(FT)}{u_A^*(FT)} = \frac{\rho^W}{\rho_{aut}^*} \frac{u_A(W)}{u_A^*(W)} \quad (\text{RS})$$

where we let $u_A(s)$ denote Home's marginal utility of good A in state $s \in \{FT, W\}$, and similarly $u_A^*(s)$ for Foreign. Note that $\rho^W/\rho_{aut}^* = p_A^*/p_A$, that is the deviation from the law of one price for good A in the trade-war state. This is a version of the Backus and Smith (1993) condition: efficient risk-sharing implies equating marginal utilities across countries and states to the real exchange rate. This implies consumption is relatively high in a state where the price of consumption is relatively low.

The ex-ante budget constraint delivers another equilibrium condition. To see this, first note that $\frac{\pi(W)u_A(W)}{\pi(FT)u_A(FT)}$ is Home's implicit price for the consumption of good A in state W relative to state FT. Then the ex-ante budget constraint can be rewritten to say that the value of Home's inherited portfolio must finance its net expenditure in the free-trade state:

$$\underbrace{\left(c_A(FT) - Y_A + \rho^{FT}(c_B(FT) - Y_B) \right)}_{\text{Home's net expenditure in FT}} = \underbrace{(\rho^{FT}a_B - a_A^*) + \frac{\pi(W)u_A(W)}{\pi(FT)u_A(FT)}(\rho^W a_B - a_A^*)}_{\text{total value of Home's portfolio}}, \quad (\text{EBC})$$

where $c_i(FT)$ denotes Home's consumption of good i in the free-trade state. In Appendix C we show that conditions (RS) and (EBC) characterize full-trade-war equilibria.

Suppose that the inherited portfolio has the following property: $a_B > 0$ and the value $\rho^W = a_A^*/a_B$ is consistent with the risk-sharing condition, (RS), given the free trade consumption allocation that would arise ex-post if the portfolio were to remain unchanged. Note that this also means that (EBC) is satisfied, for any $\pi(W)$. This free trade consumption allocation and an unchanged inherited portfolio then constitutes a *full-trade-war* equilibrium: the allocation satisfies the risk-sharing condition and the ex-ante budget constraint. Let us also highlight that in this case $\pi(W)$ has no effect in the equilibrium portfolio positions. This shows that gross positions do not need to unravel when a trade-war is anticipated.

In Appendix C we generalize this result. In Proposition C.1 we obtain general conditions for the existence of a full-trade-war equilibrium. In Corollary C.1 we characterize the cases where

¹⁹There are two assets and two states: $\rho^{FT} \neq \rho^W$ implies that the assets' payoffs are linearly independent.

the gross positions increase, remain the same, or decrease in anticipation of a trade war.

5 Generalizations and Extensions

5.1 General Asset Positions

Section 4 restricts attention to the simple claims of Assumption 1. This is mainly expositional; the description of the budget constraints makes the economy particularly amenable to graphical representation through an Edgeworth box diagram.

Proposition 1 does not easily generalize beyond Assumption 1. If either a_A or a_B^* is nonzero, then the tariff rates appear in the budget sets (Lemma B.1), and the terms of trade are no longer sufficient to summarize them. Even though we have not found a counter-example to the existence of an equilibrium when all four assets are non-negative, it is possible to construct examples of multiple equilibria in the case where a_A and a_B^* are strictly positive, even under the gross substitutes condition on household demand, (5).²⁰

However, the high tariff results in Propositions 2, 3, and 4 do have general counterparts. In the Appendix, we state and prove them as Propositions A.1, A.2, and A.3, respectively, showing that the high tariff results do hold beyond the simple claims case. As anticipated by the discussion at the end of Section 4.5, Proposition A.1 requires that each country's gross claims be non-zero and of the same sign when valued at the autarkic prices, and allows for negative claims.

5.2 Multiple Goods, Time, and Uncertainty

In Online Appendix I.2, we generalize the existence part of Proposition A.1 to multiple goods. This extension tracks textbook trade models closely (e.g., Dixit and Norman, 1980) although the existence of gross asset positions changes the analysis in interesting ways.

If the gross asset positions of each country are non-zero and of the same sign, when evaluated at the relevant country's autarkic relative prices, then under relatively mild assumptions (such as requiring that autarkic marginal rates of substitution are bounded), autarky is an equilibrium for sufficiently high tariffs. This result holds in the presence of non-tradable goods and trade costs of an arbitrary nature. The key equilibrating force is a single relative price, which can be interpreted as the exchange rate between the countries at the time a trade war is initiated.

As Dixit and Norman (1980) highlight, once the analysis contains an arbitrary number of goods, it is straightforward to introduce intertemporal trade (and uncertainty). This just re-

²⁰For example, if $u(c_A, c_B) = \log c_A + \log c_B$, $Y_A = 1$, $Y_B = 0.3$, $Y_A^* = 0.3$, $Y_B^* = 1$ and $a_A = 0.5$, $a_B = 0.02$, $a_A^* = 0.02$, $a_B^* = 0.5$, $\tau = \tau^* = 1.75$, then there are three tariff equilibria: Home imports B only ($\rho \approx 0.89$), Home exports A only ($\rho \approx 1.12$), and a symmetric two-way trade ($\rho = 1$).

quires reinterpreting different goods as different times or states, and intra-temporal prices as inter-temporal ones, as in the classic Arrow-Debreu tradition. In Online Appendix [I.4](#) we provide this reinterpretation. We summarize here the main insights. A sufficiently large (and permanent) trade war leads to autarky as long as the gross positions of each country are non-zero and share the same sign when evaluated at their respective autarky prices (and Online Appendix [II.8](#) shows the quantitative model below converging to this outcome as tariffs rise). The exchange rate at the moment the trade-war starts adjusts to revalue the net position to zero. Going forward, the autarkic interest rates determine the rates of return in each country, and the exchange rate evolves according to the interest rate parity condition. That is, equilibrium interest rates and exchange rate appreciation rates ensure that residents have no incentive to trade financial assets, while the initial exchange rate revalues the net position so that this is consistent with budget balance.²¹ In a severe enough tariff war, the exchange rate and the interest rates are not connected but rather balance distinct equilibrium forces.

6 Quantifying valuation effects in a trade war

We now quantify the terms of trade, valuation, and welfare effects of a trade war which falls short of imposing autarky, and show how these effects depend on the nature of the gross portfolio underlying a given net foreign asset position. To do so, we use a dynamic stochastic gravity model based on Fitzgerald ([2025](#)). The model has many features typical of the quantitative trade literature (such as trade costs and roundabout production) which allow it to match the pattern of gross trade. We allow productivity to be stochastic, and assume that financial markets are complete and frictionless with respect to productivity shocks. We calibrate the model to the US and a rest of the world (ROW) aggregate in 2023 under the assumption that no trade war is expected. We then use the calibrated model to calculate the impact of an unexpected tariff war for four different asset portfolios, each of which is consistent with the same (data-driven) initial net foreign asset position. One portfolio is calibrated to the actual US portfolio in 2023. Three counterfactual portfolios are chosen to illustrate the three different cases identified in Section [4](#).

²¹There is a clear antecedent in Alvarez, Atkeson, and Kehoe ([2009](#)). There, the authors analyze a two-country model where households consume only a non-traded good in each country, so by construction there cannot be international trade in goods. International financial markets are open, and thus rate of return equalization determines the required changes in exchange rates, just as here. The absence of trade in goods implies that the initial level of the exchange rate must be such that the initial net foreign asset position of each country is zero. Our result is related, with some distinctions. Our trade friction arises from the imposition of tariffs, and thus the absence of trade is not ruled out by feasibility, but rather it is an equilibrium outcome. In addition, whether autarky is the unique equilibrium that emerges in the presence of high enough tariffs requires additional restrictions on initial asset positions.

6.1 Model

There are two countries, Home (the US) and Foreign (the rest of the world). Foreign variables are denoted by stars. In each period t , the world economy experiences one event, $s_t \in S$. Denote by s^t the history of events from date 0 to date t , $\{s_0, \dots, s_t\}$. The probability at date 0 that history s^t will be realized at date t is given by $\pi(s^t)$, with $\sum_{s^t} \pi(s^t) = 1$. Since s_0 is known at date 0, $\pi(s_0) = 1$.

There is a representative agent in each country with CRRA expected utility. Date-0 utility in Home is given by:

$$U_0 = \frac{1}{1 - \frac{1}{\sigma}} \left[\theta_0 C(s_0)^{1 - \frac{1}{\sigma}} + \sum_{t=1}^{\infty} \sum_{s^t} \beta^t \pi(s^t) C(s^t)^{1 - \frac{1}{\sigma}} \right],$$

where $C(s^t)$ is Home consumption of an aggregate good at date t given s^t . The inter-temporal elasticity of substitution (and inverse risk aversion) is σ . The parameter β is the discount factor. The parameter θ_0 is a preference shifter for the initial period, which we use to target the initial period allocation in our calibration. Preferences in Foreign are similar, with $C^*(s^t)$ denoting Foreign consumption, and θ_0^* the Foreign preference shifter at date 0 (all other parameters are identical in Home and Foreign).

The representative agent in Home supplies exogenous labor L in all dates t and histories s^t . Labor supply in Foreign is L^* . At s^t , Home labor is used to produce $y(s^t)$ units of intermediate good 1:

$$y(s^t) = z(s^t) (L)^{1-\mu} M(s^t)^\mu,$$

where $z(s^t)$ is productivity, $M(s^t)$ is material input, and μ is the materials share in production. Foreign labor is used to produce intermediate good 2 using a similar technology.

The intermediate goods are aggregated into a non-traded final good, X , which is used for consumption and materials:

$$X(s^t) = \left(x_1(s^t)^{\frac{\eta-1}{\eta}} + x_2(s^t)^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} = C(s^t) + M(s^t),$$

where $x_i(s^t)$ is absorption of intermediate i in Home at s^t . The aggregator function in Foreign is identical. The Armington elasticity of substitution between the intermediates (η) determines the static trade elasticity ($\eta - 1$).

There are resource costs of trading intermediates across countries, assumed to be fixed. In order for one unit of Home's intermediate (good 1) to arrive in Foreign, $d^* \geq 1$ units must be shipped from Home. In order for one unit of Foreign's intermediate (good 2) to arrive in Home,

$d \geq 1$ units must be shipped from Foreign. The resource constraints for intermediate goods must take account of resource costs of trade:

$$y(s^t) = x_1(s^t) + d^* x_1^*(s^t), \quad \text{and} \quad y^*(s^t) = dx_2(s^t) + x_2^*(s^t).$$

Markets for labor and intermediate goods are perfectly competitive, so wages are equal to the marginal product of labor, and intermediate goods prices in the country of production are equal to marginal cost. Let $p_t(s^t)$ be the price of 1 unit of the Home intermediate (good 1) in Home at s^t in terms of date- t “dollars” (freely traded). Let $1 + \tau$ be the constant (gross) ad valorem tariff imposed by Home on imports from Foreign, and let $1 + \tau^*$ be the constant tariff imposed by Foreign on imports from Home. The price in terms of date- t dollars of 1 unit of the Home intermediate in Foreign at s^t inclusive of tariffs is $(1 + \tau^*) d^* p_t(s^t)$. The price of 1 unit of the Foreign intermediate in Home is $(1 + \tau) dp_t^*(s^t)$. Denote by $P_t(s^t)$ the price in date- t dollars of 1 unit of the final good in Home at s^t . Since the market for final goods is perfectly competitive, $P_t(s^t)$ is given by:

$$P_t(s^t) = \left(p_t(s^t)^{1-\eta} + ((1 + \tau) dp_t^*(s^t))^{1-\eta} \right)^{\frac{1}{1-\eta}},$$

with a corresponding relationship between intermediate goods prices and the price of the final good in Foreign.

Tariff revenue in each country is rebated lump sum to agents in that country. Total Home tariff revenue in terms of date- t dollars at s^t is:

$$T_t(s^t) = \tau dp_t^*(s^t) x_2(s^t),$$

with a similar expression for Foreign tariff revenue.

International Financial Positions. At date 0 agents can trade the full menu of Arrow-Debreu securities, which are claims to each of the intermediate goods, indexed by country of production and country of delivery. All agents face the same prices for these securities. Let $p_0(s^t)$ be the date-0 price of 1 unit of the Home intermediate good (good 1) in Home at s^t , and let $p_0^*(s^t)$ be the price of the Foreign intermediate (good 2) in Foreign. Goods market arbitrage implies that the date-0 price of 1 unit of the Home intermediate in Foreign is $(1 + \tau^*) d^* p_0(s^t)$, while the date-0 price of 1 unit of the Foreign intermediate in Home is $(1 + \tau) dp_0^*(s^t)$. Home tariff revenue in terms of date-0 dollars at s^t is $T_0(s^t)$, while Foreign tariff revenue is $T_0^*(s^t)$.

The two countries enter into date 0 with a predetermined portfolio of assets whose values (at equilibrium prices) are denoted by b_0 , and b_0^* , respectively. The world net financial position at

date 0 is zero:

$$b_0 + b_0^* = 0.$$

The date-0 budget constraint for Home is then:

$$\sum_{t=0}^{\infty} \sum_{s^t} (p_0(s^t) x_1(s^t) + (1 + \tau) d p_0^*(s^t) x_2(s^t)) \leq b_0 + \sum_{t=0}^{\infty} \sum_{s^t} (p_0(s^t) y(s^t) + T_0(s^t)).$$

Equilibrium. A competitive equilibrium is given by sequences of date-0 prices, intermediate goods absorption, materials and consumption such that:

- (i) Given prices, tariffs and trade costs, country i chooses absorption of each of the intermediate goods, materials, and consumption for each date and state, to maximize welfare, subject to the country- i final good resource constraint, and the country- i ex-ante budget constraint.
- (ii) Firm input demand is optimal given wages and intermediate prices.
- (iii) All intermediate goods resource constraints are satisfied, and markets clear.

In the theoretical section, we focused on tariff wars that implement autarky, and thus required that autarky prices be well defined. In this case, the Armington assumption (complete specialization) together with the CES aggregator function imply that autarky prices are not well-defined. But this is not an issue, as our interest is in tariff wars that do not lead to a complete trade shutdown.

6.2 Calibration

We treat the year 2023 as our period zero and the US and a composite of the rest of the world (ROW) as our two countries. Our approach to calibrating preferences, the Armington parameter, and the productivity processes is similar to Fitzgerald (2025), and is detailed in Online Appendix II.

To calibrate initial net foreign asset positions, we use 2023 asset data from the External Wealth of Nations dataset (Lane and Milesi-Ferretti, 2018). We compute the net positions of our two regions by summing net positions in equity and foreign direct investment (FDI) and net debt positions. Making 2023 nominal US GDP the numeraire, we define

$$b_0 \equiv \frac{(\text{US FDI} + \text{equity assets} - \text{liabilities}) + (\text{US debt assets} - \text{liabilities})}{GDP_{2023}^{US}} = -0.76.$$

This corresponds to the BEA net foreign asset position of the US, restricting attention to FDI, equities and debt. By market clearing, $b_0^* = -b_0$.

We calibrate to ensure that the initial period allocation in the model matches targeted macro aggregates observed in the 2023 data. Specifically, given b_0 as well as the structural parameters for preferences, production, and the productivity process estimated in Online Appendix II, we choose values for: period-0 relative productivity in ROW, z_0^* (normalizing $z_0 = 1$); the two trade costs, $\{d, d^*\}$; and the initial preference shifter for the ROW, θ_0^* (setting $\theta_0 = 1$); in order to match: the ratio of ROW nominal GDP to US GDP in 2023; US and ROW 2023 nominal exports as a fraction of US GDP; and the relative price levels in the two regions as measured by 2023 PPPs. Details of the computational algorithm are contained in Online Appendix II. The calibrated parameters are also reported there.

The choice of θ_0^* is used to capture the fact that the US runs a large trade deficit in 2023 despite its negative net foreign asset position. In effect, $\theta_0^* < 1$ induces a transitory “savings glut” in the rest of the world that raises relative absorption in the US in the initial period. Setting $\theta_0 = 1$ is not without loss, as alternative values of this parameter would generate alternative inter-temporal prices, and hence shift the inter-temporal budget sets given observed initial asset positions. We examine the robustness of our results to alternative choices for this and the other key parameters.

Under our calibration, gains from trade for the US, i.e. the amount by which consumption would have to be increased at all dates and states to make agents indifferent between autarky and free trade, are 1.79% of autarky consumption. For the ROW, gains from trade are 2.46% of autarky consumption. Gains from trade are smaller for the US because it is the net debtor, and we assume that in autarky it does not repay its debt.

6.3 Selecting the initial portfolios

As described above, we compute the benchmark equilibrium using the observed net foreign asset positions in 2023 expressed as a share of 2023 US GDP. In order to perform counterfactual exercises, we need to take a stand on the exact portfolio that underlies this net financial position. To illustrate the role of gross asset positions in determining the impact of a given set of tariff policies, we propose four different candidate portfolios, each of which gives rise to the same b_0 in the free trade equilibrium. However, the values of each of these portfolios may be different in a tariff equilibrium. We describe each of these portfolios in turn.

Benchmark Portfolio. The first, benchmark, portfolio matches the composition of the actual US portfolio in 2023. In particular, we construct a portfolio where under free trade, US equity assets, US equity liabilities, US net dollar debt and US net non-dollar debt, expressed as shares of 2023 US GDP, match their values in the data, as reported in Table 2. To model the debt part of the portfolio, we need to take a stand on maturity. The US Treasury TICS reports a weighted

average maturity of US debt assets in 2023 of 7.6 years and a weighted average maturity of US debt liabilities in 2023 of 8.6 years. The maturity distribution is not reported separately for debt assets and liabilities denominated in different currencies. Since the weighted average maturity is similar for assets and liabilities, we use the average of these two maturities for both assets and liabilities (i.e. 8.1). We model debt as a real bond (indexed to US or ROW consumption) that decays exponentially, with the rate of exponential decay chosen to match this weighted average maturity.

The US net foreign asset position is then given by

$$b_0 = \left(\alpha_0 \sum_{t=0}^T \sum_{s^t} p_0^* (s^t)^{FT} y^* (s^t)^{FT} + \beta_0 \sum_{t=0}^T \sum_{s^t} P_0^* (s^t)^{FT} \lambda^t \right) - \left(\alpha_0^* \sum_{t=0}^T \sum_{s^t} p_0 (s^t)^{FT} y (s^t)^{FT} + \beta_0^* \sum_{t=0}^T \sum_{s^t} P_0 (s^t)^{FT} \lambda^t \right),$$

where T is the calibration horizon, and $\{\alpha_0, \alpha_0^*\}$ are such that:

$$\alpha_0 = \psi_0 \frac{(1 - \mu) p_0 (s_0)^{FT} y (s_0)^{FT}}{\sum_{t=0}^T \sum_{s^t} p_0^* (s^t)^{FT} y^* (s^t)^{FT}}, \quad \text{and} \quad \alpha_0^* = \psi_0^* \frac{(1 - \mu) p_0 (s_0)^{FT} y (s_0)^{FT}}{\sum_{t=0}^T \sum_{s^t} p_0 (s^t)^{FT} y (s^t)^{FT}},$$

and $\{\beta_0, \beta_0^*\}$ are such that:

$$\beta_0 = \delta_0 \frac{(1 - \mu) p_0 (s_0)^{FT} y (s_0)^{FT}}{\sum_{t=0}^T \sum_{s^t} P_0^* (s^t)^{FT} \lambda^t}, \quad \text{and} \quad \beta_0^* = \delta_0^* \frac{(1 - \mu) p_0 (s_0)^{FT} y (s_0)^{FT}}{\sum_{t=0}^T \sum_{s^t} P_0 (s^t)^{FT} \lambda^t},$$

where ψ_0 is the ratio of 2023 US equity assets to 2023 US GDP, ψ_0^* is the ratio of 2023 ROW equity assets to 2023 US GDP, δ_0 is the ratio of 2023 net US non-dollar debt assets to US GDP, δ_0^* is the ratio of 2023 net ROW dollar debt assets to US GDP, and $\lambda = 1 - 1/8.1$.

No Gross Positions. The second portfolio assumes that the rest of the world holds a claim to a constant share of US output, but the US holds no claim to rest of the world output, i.e. a net-only portfolio expressed in terms of the US intermediate. This is akin to the one-asset positive claims case discussed in Section 4.4. Hence we find $\alpha_0^{*,netus} > 0$ such that:

$$-b_0 = b_0^* = \alpha_0^{*,netus} \left(\sum_{t=0}^T \sum_{s^t} p_0 (s^t)^{FT} y (s^t)^{FT} \right)$$

while $\alpha_0^{netus} = 0$. Again, the left-hand-side is given by the data described above, while the right-hand-side prices and quantities are those from the free trade equilibrium. Now $a_0 = 0$ and $a_0^* = b_0^*$.

Debt-Trap Portfolio. The third portfolio assumes that the US owes a constant share of rest of the world output, but the rest of the world holds no claim to US output, i.e. a net-only portfolio expressed in terms of the rest of the world intermediate. Hence we find $\alpha_0^{netrow} < 0$ such that:

$$b_0 = -b_0^* = \alpha_0^{netrow} \left(\sum_{t=0}^T \sum_{s^t} p_0^*(s^t)^{FT} y^*(s^t)^{FT} \right),$$

while $\alpha_0^{*,netrow} = 0$. This implies $a_0 = b_0 < 0$ while $a_0^* = 0$. This is a specific example of the negative simple claims discussed in Section 4.5.

Contract Curve Portfolio. The final portfolio (the “contract curve” portfolio) exactly delivers equilibrium absorption at free trade prices. That is, we choose $\{\alpha_0(s^t), \alpha_0^*(s^t)\}$ such that:

$$dx_2(s^t)^{FT} = \alpha_0(s^t)^{cc} y^*(s^t)^{FT}, \quad \text{and} \quad d^*x_1^*(s^t)^{FT} = \alpha_0^*(s^t)^{cc} y(s^t)^{FT}.$$

6.4 Trade War Counterfactual

We assume that before any trade takes place at date 0, the US unexpectedly announces a tariff τ on all imports, current and future, while ROW announces a tariff τ^* on all imports, present and future. We assume that the process for productivity is unaffected by the trade war, and all claims contingent on the history of realizations of productivity are honored despite the fact that tariffs have unexpectedly changed.

For our primary trade war exercise, we set tariffs equal to 25% in each country ($\tau = \tau^* = 0.25$). In the Online Appendix, we also examine the impact of a unilateral 25% tariff in the US, and a unilateral 25% tariff in ROW. For each portfolio, Table 4 reports 2023 US foreign assets and liabilities, net foreign assets, and net exports as percentages of 2023 US GDP. It also reports the percent change in the 2023 US terms of trade and consumption real exchange rate relative to free trade, and the percent change in 2023 US and ROW welfare (in consumption-equivalent units) relative to free trade.

The first panel reports results for the contract curve portfolio; the second panel is the benchmark portfolio; the third panel is the no gross positions portfolio; and the final panel is the debt-trap portfolio. Additional results for unilateral tariffs, for different values of tariffs, and for different parameter values are reported in Online Appendix II.

Consistent with our theoretical analysis, the impact of a tariff war on the terms of trade, net debt, and welfare varies across ex-ante portfolios. When the US holds substantial assets in ROW, as well as ROW holding substantial assets in the US, the movement of international relative prices in the symmetric tariff war tends to reduce US net debt. This is true in the benchmark portfolio,

Table 4: Trade War Scenarios

τ	τ^*	a	a^*	$a - a^*$	NX	$\Delta(p/p^*)$	$\Delta(P/P^*)$	ΔU	ΔU^*
Contract Curve Portfolio									
0	0	323%	399%	-76%	-2.9%	0	0	0	0
25%	25%	328%	389%	-61%	-2.6%	-0.25%	0.58%	-0.53%	-0.31 %
Benchmark Portfolio									
0	0	81%	157%	-76%	-2.9%	0	0	0	0
25%	25%	83%	154%	-71%	-2.2%	-1.20%	-0.33%	-1.04%	-0.17 %
No Gross Positions									
0	0	0	76%	-76%	-2.9%	0	0	0	0
25%	25%	0	75%	-75%	-2.1%	-1.47%	-0.60%	-1.19%	-0.12 %
Debt-Trap Portfolio									
0	0	-76%	0	-76%	-2.9%	0	0	0	0
25%	25%	-79%	0	-79%	-2.0%	-1.84%	-0.95%	-1.39%	-0.06 %

Notes: a , a^* and NX are date-0 values of US claims to ROW, ROW claims to US, and US net exports respectively, expressed as percentages of date-0 US GDP. $\Delta(p/p^*)$ and $\Delta(P/P^*)$ are percentage deviations of date-0 US terms of trade and real exchange rate from their levels at date 0 under free trade. ΔU and ΔU^* are percentage deviations of date-0 US and ROW welfare (in consumption-equivalent units) from their levels at date 0 under free trade.

and is particularly true in the contract curve portfolio. We show in the Online Appendix that with these two portfolios, reducing the value of US net debt requires retaliation on the part of ROW. With US tariffs only, the value of US net debt increases.

In our baseline calibration, a trade war with symmetric 25% tariffs reduces welfare for both parties under all the portfolios we consider (losses are minimal for ROW under the debt-trap portfolio). The welfare cost for the US is smallest under the contract curve portfolio, and greatest under the debt-trap portfolio, where losses come close to the loss from moving to full autarky. In Online Appendix II we decompose welfare changes vis-a-vis free trade into the loss of gains from trade, terms of trade, and valuation effects. Valuation effects mitigate standard losses from a trade war under the contract curve and benchmark portfolios and exacerbate them under the debt trap portfolio.

Broadly speaking, these results confirm our theoretical analysis, for levels of tariffs which do not come close to leading to autarky.²² Though, the appendix confirms that, numerically, the

²²Because under the benchmark portfolio, gross asset positions of each country are non-zero and of the same sign when evaluated at the relevant autarkic relative prices, we can calculate the shadow terms of trade in autarky, as described in the Online Appendix II. In this case: it depreciates by 44% relative to free trade. We also show numerically that the shadow terms of trade (together with a corresponding zero net asset position) is the limit of

terms of trade and net foreign assets move monotonically toward their autarky values as tariffs increase.

In Online Appendix II, we also report the Nash equilibrium and unilaterally optimal tariffs for each portfolio, as well as their implications for the terms of trade, net debt, and welfare. Valuation effects under “strictly positive claims” portfolios (benchmark and contract curve) depress optimal tariffs relative to the case of no gross positions, while in the case of the debt-trap portfolio, if unconstrained by US default, the ROW would choose to immiserize the US.

7 Conclusion

Using a series of two-country models, we study whether and how a net debt accumulated under free trade can be settled in the event of a tariff war. We show the answer depends on the composition of ex-ante international portfolios. Our key insight is that unanticipated tariffs revalue international assets, and that the impact of these valuation effects depends on ex-ante gross asset positions, not just net debt.

We consider three broad cases for international portfolios: strictly positive claims, claims on one side only, and negative claims on at least one side. The international asset portfolios of advanced economies broadly satisfy the strictly positive criterion, while emerging markets borrowing in foreign currency have some negative claims on their balance sheets.

When ex-ante gross positions are strictly positive, for high enough tariffs on both sides, valuation effects can wipe out an ex-ante net debt. This requires a real depreciation of the net debtor. However if claims are on one side only, tariff-induced valuation effects cannot wipe out an ex-ante debt. In the case where the debtor’s liabilities must be honored in the creditor country, the creditor can immiserize the debtor by levying tariffs which raise the value of the debt relative to resources for repayment. This mechanism can lead to multiple equilibria if both countries have liabilities which must be honored in the lending country.

We provide a quantitative exploration of how the impact of a tariff war depends on the nature of ex-ante gross portfolios in a model calibrated to the US net foreign asset position in 2023. At the US gross portfolio in 2023, a sufficiently severe tariff war can reduce the value of US net debt, but it requires that the rest of the world retaliates by levying tariffs on imports from the US.

equilibria as the tariffs become large enough in baseline calibration.

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A Proofs

A.1 Proof of Proposition 1

Proof. If $\tau = \tau^* = 0$, then the unique equilibrium is the free trade equilibrium with terms of trade ρ^{FT} . Suppose, therefore, that at least one tariff is strictly positive.

Let

$$\bar{r} \equiv \frac{\bar{Y}_B}{\bar{Y}_A} = h(\rho^{FT}).$$

Under $a_A = a_B^* = 0$, $a_B \geq 0$ and $a_A^* \geq 0$, condition (23) holds, so Lemma B.1 applies. By its part (ii) with $a_A = a_B^* = 0$, the country budget constraints in every admissible trade regime are

$$c_A + \rho c_B = W(\rho) \equiv Y_A - a_A^* + \rho(Y_B + a_B), \quad (6)$$

$$c_A^* + \rho c_B^* = W^*(\rho) \equiv Y_A^* + a_A^* + \rho(Y_B^* - a_B). \quad (7)$$

Consequently,

$$W(\rho) + W^*(\rho) = \bar{Y}_A + \rho \bar{Y}_B.$$

For each $\rho > 0$, define

$$C_A(\rho) \equiv \min \left\{ Y_A - a_A^* + \rho a_B, \frac{W(\rho)}{1 + \rho h((1 + \tau)\rho)} \right\}, \quad (8)$$

$$C_A^*(\rho) \equiv \max \left\{ Y_A^*, \frac{W^*(\rho)}{1 + \rho h(\rho/(1 + \tau^*))} \right\}, \quad (9)$$

and obtain $C_B(\rho)$ and $C_B^*(\rho)$ from (6)–(7). Note that here we are using the assumption that h is well defined for any strictly positive number.

Define

$$Z(\rho) \equiv C_A(\rho) + C_A^*(\rho) - \bar{Y}_A.$$

Adding the two country budget constraints gives

$$C_A(\rho) + C_A^*(\rho) + \rho [C_B(\rho) + C_B^*(\rho)] = \bar{Y}_A + \rho \bar{Y}_B.$$

Since $\bar{Y}_B = \bar{r} \bar{Y}_A$, rearranging gives

$$Z(\rho) = -\frac{\rho}{1 + \rho \bar{r}} \left\{ [C_B(\rho) - \bar{r} C_A(\rho)] + [C_B^*(\rho) - \bar{r} C_A^*(\rho)] \right\}. \quad (10)$$

By construction,

$$C_B(\rho) \geq Y_B > 0, \quad C_A^*(\rho) \geq Y_A^* > 0.$$

Thus all four consumption levels are nonnegative if and only if $C_A(\rho) \geq 0$ and $C_B^*(\rho) \geq 0$. From (8)–(9),

this is equivalent to

$$\rho \in I \equiv \left\{ \rho > 0 : Y_A - a_A^* + \rho a_B \geq 0, \quad Y_B^* - a_B + \frac{a_A^*}{\rho} \geq 0 \right\}. \quad (11)$$

The free-trade budget constraints imply

$$\begin{aligned} Y_A - a_A^* + \rho^{FT} a_B &= c_A^{FT} + \rho^{FT} (c_B^{FT} - Y_B) > 0, \\ Y_B^* - a_B + \frac{a_A^*}{\rho^{FT}} &= c_B^{*FT} + \frac{c_A^{*FT} - Y_A^*}{\rho^{FT}} > 0. \end{aligned}$$

Hence $\rho^{FT} \in I$, with both feasibility inequalities holding strictly.

By (4), any equilibrium relative price belongs to

$$J \equiv \left[\frac{\rho^{FT}}{1 + \tau}, (1 + \tau^*) \rho^{FT} \right].$$

Since $\rho^{FT} \in I \cap J$, the intersection is nonempty. Moreover, J is compact and bounded away from zero. On J , the two feasibility expressions in (11) are continuous, so their weak inequalities define a closed subset of J . Hence $I \cap J$ is compact. Furthermore, because $a_A^*, a_B \geq 0$, the first feasibility expression in (11), $Y_A - a_A^* + \rho a_B \geq 0$, is weakly increasing in ρ , while the second, $Y_B^* - a_B + a_A^*/\rho \geq 0$, is weakly decreasing. Thus $I \cap J$ is an interval. We may therefore define

$$\rho_- \equiv \min(I \cap J), \quad \rho_+ \equiv \max(I \cap J).$$

We first show that

$$Z(\rho_-) < 0.$$

The lowest point: $Z(\rho_-) < 0$. There are two cases.

Case 1: $\rho_- = \rho^{FT}/(1 + \tau) \in I$. We have then

$$h((1 + \tau)\rho_-) = h(\rho^{FT}) = \bar{r}.$$

Consider (8). If its second argument is selected, that is, $C_A(\rho_-) = \frac{W(\rho_-)}{1 + (\rho_-)h((1 + \tau)\rho_-)}$, then the budget constraint gives

$$C_B(\rho_-) = \bar{r}C_A(\rho_-).$$

If instead $C_A(\rho_-) = Y_A - a_A^* + (\rho_-)a_B$, then it must be that

$$C_A(\rho_-) \leq \frac{W(\rho_-)}{1 + \rho_- \bar{r}}.$$

Using the budget constraint, $(\rho_-)C_B(\rho_-) = W(\rho_-) - C_A(\rho_-)$, it follows that

$$C_B(\rho_-) - \bar{r}C_A(\rho_-) = \frac{W(\rho_-) - (1 + \rho_- \bar{r})C_A(\rho_-)}{\rho_-} \geq 0.$$

Thus

$$C_B(\rho_-) - \bar{r}C_A(\rho_-) \geq 0. \quad (12)$$

For Foreign's consumption, first note

$$\frac{\rho_-}{1 + \tau^*} = \frac{\rho^{FT}}{(1 + \tau)(1 + \tau^*)} < \rho^{FT},$$

where the inequality is strict because at least one tariff is positive. Since h is strictly decreasing,

$$h\left(\frac{\rho_-}{1 + \tau^*}\right) > h(\rho^{FT}) = \bar{r}.$$

Now consider (9). If $C_A^*(\rho_-) = \frac{W^*(\rho_-)}{1 + (\rho_-)h(\rho_-/(1 + \tau^*))}$ then, using the budget constraint:

$$C_B^*(\rho_-) = h\left(\frac{\rho_-}{1 + \tau^*}\right) C_A^*(\rho_-) > \bar{r}C_A^*(\rho_-).$$

If the first term of (9) is selected instead, that is, $C_A^*(\rho_-) = Y_A^*$, then

$$C_B^*(\rho_-) = Y_B^* - a_B + \frac{a_A^*}{\rho_-} \geq Y_B^* - a_B + \frac{a_A^*}{\rho^{FT}} = c_B^{*FT} + \frac{c_A^{*FT} - Y_A^*}{\rho^{FT}} > \bar{r}Y_A^* = \bar{r}C_A^*(\rho_-),$$

where the first (weak) inequality uses $\rho_- \leq \rho^{FT}$ and $a_A^* \geq 0$, and the second (strict) inequality follows from

$$c_B^{*FT} = \bar{r}c_A^{*FT} \quad \text{and} \quad c_A^{*FT} > Y_A^*.$$

Thus

$$C_B^*(\rho_-) - \bar{r}C_A^*(\rho_-) > 0. \quad (13)$$

Combining (12), (13), and (10) gives

$$Z(\rho_-) < 0.$$

Case 2: $\rho_- > \rho^{FT}/(1 + \tau)$. Since ρ_- lies strictly above the lower endpoint of J , it must equal the lower endpoint of I . Hence the first inequality in (11) binds and

$$Y_A - a_A^* + \rho_- a_B = 0.$$

This implies that

$$W(\rho_-) = Y_A - a_A^* + \rho_- (Y_B + a_B) = \rho_- Y_B > 0.$$

Thus the first argument of the minimum in (8) is zero, while the second is positive. Hence

$$C_A(\rho_-) = 0.$$

Using $a_A^* = Y_A + \rho_- a_B$, we obtain also

$$W^*(\rho_-) = \bar{Y}_A + \rho_- Y_B^*.$$

Given that $Y_A - a_A^* + \rho^{FT} a_B > 0$, we have $\rho_- < \rho^{FT}$. Therefore

$$\frac{\rho_-}{1 + \tau^*} < \rho^{FT},$$

and hence

$$h\left(\frac{\rho_-}{1 + \tau^*}\right) > h(\rho^{FT}) = \bar{r} = \frac{\bar{Y}_B}{\bar{Y}_A} > \frac{Y_B^*}{\bar{Y}_A}.$$

It follows that

$$\frac{W^*(\rho_-)}{1 + \rho_- h(\rho_-/(1 + \tau^*))} = \bar{Y}_A \frac{1 + \rho_- Y_B^*/\bar{Y}_A}{1 + \rho_- h(\rho_-/(1 + \tau^*))} < \bar{Y}_A.$$

Since $Y_A^* < \bar{Y}_A$, both arguments of the maximum in (9) are strictly below $\bar{Y}_A$. Therefore

$$C_A^*(\rho_-) < \bar{Y}_A.$$

Combining this with $C_A(\rho_-) = 0$ gives

$$Z(\rho_-) = C_A^*(\rho_-) - \bar{Y}_A < 0.$$

The highest point: $Z(\rho_+) > 0$. The argument is symmetric, with $a_B \geq 0$ replacing $a_A^* \geq 0$. If

$$\rho_+ = (1 + \tau^*)\rho^{FT},$$

then

$$C_B^*(\rho_+) - \bar{r}C_A^*(\rho_+) \leq 0, \quad C_B(\rho_+) - \bar{r}C_A(\rho_+) < 0.$$

If instead

$$\rho_+ < (1 + \tau^*)\rho^{FT},$$

then the second feasibility constraint in (11) binds, implying

$$C_B^*(\rho_+) = 0, \quad C_A(\rho_+) > Y_A.$$

The weak comparisons in the first case use $a_B \geq 0$, while the strict inequalities follow from the strict free-trade inequalities in Assumption 2. Thus, in either case,

$$Z(\rho_+) > 0.$$

Existence of a root. Thus

$$Z(\rho_-) < 0 < Z(\rho_+).$$

Since Z is continuous, there exists

$$\rho^\dagger \in (\rho_-, \rho_+) \subseteq I \cap J, \quad \text{such that } Z(\rho^\dagger) = 0.$$

The root generates an equilibrium. Set

$$c_A = C_A(\rho^\dagger), \quad c_B = C_B(\rho^\dagger), \quad c_A^* = C_A^*(\rho^\dagger), \quad c_B^* = C_B^*(\rho^\dagger).$$

Since $Z(\rho^\dagger) = 0$,

$$c_A + c_A^* = \bar{Y}_A.$$

Moreover, summing the two budget constraints gives

$$c_A + c_A^* + \rho^\dagger(c_B + c_B^*) = \bar{Y}_A + \rho^\dagger \bar{Y}_B.$$

Since $\rho^\dagger > 0$, it follows that

$$c_B + c_B^* = \bar{Y}_B.$$

Thus both resource constraints hold.

By construction,

$$c_B \geq Y_B, \quad c_A^* \geq Y_A^*.$$

The resource constraints then imply

$$c_A \leq Y_A, \quad c_B^* \leq Y_B^*.$$

Because $\rho^\dagger$ lies in the interior of $I \cap J$ and $I \cap J$ is an interval, all four constructed consumption levels are strictly positive. Hence the consumption ratios below are well defined and satisfy the trade pattern required by Lemma B.1.

It remains to show that part (iii) of Lemma B.1 holds. Define

$$q_H \equiv g\left(\frac{c_B}{c_A}\right), \quad q_F \equiv g\left(\frac{c_B^*}{c_A^*}\right),$$

and

$$\alpha \equiv \frac{q_H}{\rho^\dagger}, \quad \alpha^* \equiv \frac{q_F}{\rho^\dagger}.$$

The definitions of C_A and C_A^* imply directly that

$$\alpha \leq 1 + \tau, \quad \alpha^* \geq \frac{1}{1 + \tau^*}.$$

Because the goods markets clear, $\bar{r}$ lies between the countries' consumption ratios. Since g is decreasing, $\rho^{FT} = g(\bar{r})$ lies between q_H and q_F . Moreover, since $\rho^\dagger \in J$,

$$\frac{\rho^\dagger}{1 + \tau^*} \leq \rho^{FT} \leq (1 + \tau)\rho^\dagger. \quad (14)$$

If Foreign strictly imports A , then $c_A^* > Y_A^*$, and the second argument of the maximum in (9) must have been selected. Hence

$$c_A^* = \frac{W^*(\rho^\dagger)}{1 + \rho^\dagger h(\rho^\dagger/(1 + \tau^*))}.$$

Foreign's budget constraint therefore implies

$$\frac{c_B^*}{c_A^*} = h\left(\frac{\rho^\dagger}{1 + \tau^*}\right).$$

Consequently,

$$q_F = \frac{\rho^\dagger}{1 + \tau^*}, \quad \alpha^* = \frac{1}{1 + \tau^*}.$$

Since $\rho^\dagger \in J$,

$$q_F = \frac{\rho^\dagger}{1 + \tau^*} \leq \rho^{FT}.$$

Because ρ^{FT} lies between q_H and q_F , it follows that $q_H \geq \rho^{FT}$. Thus

$$q_H \geq \rho^{FT} \geq \frac{\rho^\dagger}{1 + \tau^*},$$

and therefore

$$\alpha = \frac{q_H}{\rho^\dagger} \geq \frac{1}{1 + \tau^*}.$$

Similarly, if Home strictly imports B , then the second argument of the minimum in (8) must have been selected. Hence

$$q_H = (1 + \tau)\rho^\dagger, \quad \alpha = 1 + \tau.$$

It follows that

$$q_F \leq \rho^{FT} \leq (1 + \tau)\rho^\dagger,$$

and therefore

$$\alpha^* \leq 1 + \tau.$$

If neither good is strictly traded, then

$$q_H = g\left(\frac{Y_B}{Y_A}\right) > \rho^{FT} > g\left(\frac{Y_B^*}{Y_A^*}\right) = q_F$$

by Assumption 2. Together with (14), this gives

$$\alpha \geq \frac{1}{1 + \tau^*}, \quad \alpha^* \leq 1 + \tau.$$

Therefore, we have shown that

$$\alpha, \alpha^* \in \left[\frac{1}{1 + \tau^*}, 1 + \tau \right],$$

with

$$\alpha = 1 + \tau \quad \text{if Home strictly imports } B, \quad \alpha^* = \frac{1}{1 + \tau^*} \quad \text{if Foreign strictly imports } A.$$

Thus all the conditions of Lemma B.1 are satisfied, and the constructed consumption allocation and $\rho^\dagger$ constitute a tariff equilibrium. This completes the proof of the existence of an equilibrium.

Uniqueness. Lemma B.1 implies that every equilibrium allocation is generated by (8)–(9) at its equilibrium value of ρ . Market clearing therefore requires $Z(\rho) = 0$. We will use this to show uniqueness. Define

$$D_H(\rho) \equiv 1 + \rho h((1 + \tau)\rho), \quad D_F(\rho) \equiv 1 + \rho h\left(\frac{\rho}{1 + \tau^*}\right).$$

Thus the second arguments in (8)–(9) are W/D_H and W^*/D_F .

Let $s(q) \equiv qh(q)$. The gross-substitutes condition implies that s is decreasing and hence

$$D'_H(\rho) \leq 0, \quad D'_F(\rho) \leq 0.$$

Moreover,

$$D_H(\rho) = 1 + \frac{s((1 + \tau)\rho)}{1 + \tau} \leq 1 + (1 + \tau^*)s\left(\frac{\rho}{1 + \tau^*}\right) = D_F(\rho).$$

Consider the four possible combinations of arguments selected in (8)–(9). Within each region,

$$Z'(\rho) = a_B \geq 0, \quad \text{if both first arguments of (8)–(9) are selected,}$$

$$Z'(\rho) = \frac{Y_B + a_B}{D_H} - \frac{WD'_H}{D_H^2} > 0, \quad \text{if only Home's second argument is selected,}$$

$$Z'(\rho) = \frac{Y_B^*}{D_F} + a_B \left(1 - \frac{1}{D_F}\right) - \frac{W^*D'_F}{D_F^2} > 0, \quad \text{if only Foreign's second argument is selected,}$$

$$Z'(\rho) = \frac{\bar{Y}_B}{D_F} + (Y_B + a_B) \left(\frac{1}{D_H} - \frac{1}{D_F}\right) - \frac{WD'_H}{D_H^2} - \frac{W^*D'_F}{D_F^2} > 0, \quad \text{if both second arguments are selected.}$$

Here the arguments ρ have been suppressed. The inequalities use

$$W(\rho), W^*(\rho) \geq 0 \text{ for } \rho \in I \cap J, \quad D_F(\rho) \geq 1, \quad a_B \geq 0, \quad D'_H(\rho), D'_F(\rho) \leq 0, \quad D_H(\rho) \leq D_F(\rho).$$

For the last inequality, note that, since $s > 0$ is weakly decreasing,

$$\frac{s((1+\tau)\rho)}{1+\tau} \leq s((1+\tau)\rho) \leq s\left(\frac{\rho}{1+\tau^*}\right) \leq (1+\tau^*)s\left(\frac{\rho}{1+\tau^*}\right).$$

The minimum and maximum of continuously differentiable functions are locally absolutely continuous. Since the preceding calculations show that $Z'(\rho) \geq 0$ wherever Z is differentiable, Z is weakly increasing on $I \cap J$, and strictly increasing except possibly when both first arguments of (8)–(9) are selected and $a_B = 0$.

In that exceptional case,

$$Z(\rho) = \rho a_B - a_A^* = -a_A^* < 0,$$

where $a_A^* > 0$ follows from Home's net-debtor condition when $a_B = 0$. Thus Z cannot attain a zero where it is constant, and therefore has at most one zero.

Since existence provides a zero of Z , we have now shown that this zero is unique. Every equilibrium must correspond to such a zero; hence the equilibrium relative price ρ and consumption allocation are unique. $\square$

A.2 Proof of Proposition 2

Note that Proposition 2 does not need to appeal to the gross substitutes condition in Proposition 1. It is a special case of a stronger result, Proposition A.1.

Proposition A.1 (Uniqueness under Same-Sign Gross Assets). *Suppose that the asset valuations satisfy:*

$$a_A + \rho^* a_B \neq 0 \text{ for all } \rho^* \in [\rho_{aut}^*, \rho^{FT}], \text{ and } a_A^* + \rho a_B^* \neq 0 \text{ for all } \rho \in [\rho^{FT}, \rho_{aut}], \quad (15)$$

where $\rho_{aut} \equiv g(Y_B/Y_A)$, $\rho_{aut}^* \equiv g(Y_B^*/Y_A^*)$ are the autarkic relative prices. Suppose also that the gross asset positions satisfy the same-sign condition:

$$(a_A^* + \rho_{aut} a_B^*) \times (a_A + \rho_{aut} a_B) > 0. \quad (16)$$

Then there exist finite $\bar{\tau}, \bar{\tau}^*$ such that for all $\tau \geq \bar{\tau}$ and $\tau^* \geq \bar{\tau}^*$, the autarky allocation is the unique equilibrium.

Proof. **Existence.** Consider the autarky allocation

$$(c_A, c_B, c_A^*, c_B^*) = (Y_A, Y_B, Y_A^*, Y_B^*).$$

Normalize $p_A = 1$ and set domestic relative prices equal to their autarkic values:

$$p_B = \rho_{aut}, \quad p_A^* = e^{BT}, \quad p_B^* = e^{BT} \rho_{aut}^*,$$

where

$$e^{BT} \equiv \frac{a_A^* + \rho_{aut} a_B^*}{a_A + \rho_{aut}^* a_B}.$$

The same-sign condition (16) guarantees that $e^{BT} > 0$. By construction,

$$p_A a_A^* + p_B a_B^* = p_A^* a_A + p_B^* a_B,$$

so both countries' budget constraints hold at the autarky allocation, with zero trade and zero tariff revenue. Moreover, the autarkic relative prices satisfy household optimality by the definitions of ρ_{aut} and ρ_{aut}^* .

It remains only to verify that international goods arbitrage is consistent with no trade. For each $i \in \{A, B\}$, this requires

$$\frac{1}{1 + \tau^*} \leq \frac{p_i}{p_i^*} \leq 1 + \tau.$$

Because all the constructed prices are finite and strictly positive, there exist finite thresholds $\bar{\tau}_E$ and $\bar{\tau}_E^*$ such that these inequalities hold for both goods whenever

$$\tau \geq \bar{\tau}_E, \quad \tau^* \geq \bar{\tau}_E^*.$$

Thus the autarky allocation is an equilibrium for all sufficiently high tariff pairs.

Uniqueness. We rule out all non-autarkic equilibria for large enough tariffs. Any non-autarkic equilibrium must involve positive trade in at least one good. We consider each possible trade pattern:

Case 1: One country exports both goods. Suppose Home exports both goods. Foreign imports both, so $p_i^* = (1 + \tau^*) p_i$ for $i = A, B$ and $T = 0$. Since the tariff wedge $(1 + \tau^*)$ is common across goods, relative prices within each country are undistorted: $p_B/p_A = p_B^*/p_A^* = \rho^{FT}$. Normalizing $p_A = 1$, Home's budget constraint is:

$$c_A + \rho^{FT} c_B = Y_A + \rho^{FT} Y_B + (1 + \tau^*)(a_A + \rho^{FT} a_B) - (a_A^* + \rho^{FT} a_B^*).$$

If $a_A + \rho^{FT} a_B \neq 0$, the right-hand side diverges to $+\infty$ or $-\infty$ as $\tau^* \rightarrow \infty$, while $c_A + \rho^{FT} c_B \in [0, \bar{Y}_A + \rho^{FT} \bar{Y}_B]$, which is a contradiction. A symmetric argument works when Foreign exports both goods, using $a_A^* + \rho^{FT} a_B^* \neq 0$ and $\tau \rightarrow \infty$.

Case 2: Equilibria with two-way trade (Home exports one good, imports the other).

Consider any equilibrium with positive bilateral trade in which Home exports good A and imports good B . Such an equilibrium must satisfy

$$\frac{p_A^*}{p_A} = 1 + \tau^*, \quad \frac{p_B}{p_B^*} = 1 + \tau.$$

Goods arbitrage therefore implies

$$\frac{p_B/p_A}{p_B^*/p_A^*} = (1 + \tau)(1 + \tau^*). \quad (17)$$

Under the assumed trade pattern, $c_A < Y_A$, $c_B > Y_B$, $c_A^* > Y_A^*$, $c_B^* < Y_B^*$. Household optimality implies

$$p_B/p_A \leq \frac{u_B(c_A, c_B)}{u_A(c_A, c_B)}.$$

This condition holds with equality when $c_A > 0$ and is the relevant inequality when $c_A = 0$. By homotheticity and the fact that g is decreasing,

$$p_B/p_A \leq \frac{u_B(c_A, c_B)}{u_A(c_A, c_B)} \leq g(Y_B/Y_A) = \rho_{aut}.$$

Similarly, Foreign household optimality implies

$$p_B^*/p_A^* \geq \frac{u_B(c_A^*, c_B^*)}{u_A(c_A^*, c_B^*)}.$$

Therefore,

$$p_B^*/p_A^* \geq \frac{u_B(c_A^*, c_B^*)}{u_A(c_A^*, c_B^*)} \geq g(Y_B^*/Y_A^*) = \rho_{aut}^*.$$

Combining these bounds with (17) requires

$$(1 + \tau)(1 + \tau^*) \leq \frac{\rho_{aut}}{\rho_{aut}^*},$$

generating a contradiction when $(1 + \tau)(1 + \tau^*) > \rho_{aut}/\rho_{aut}^*$.

Now consider the reversed pattern, in which Home imports good A and exports good B . Goods arbitrage then implies

$$\frac{p_B^*/p_A^*}{p_B/p_A} = (1 + \tau)(1 + \tau^*).$$

The direction of trade and household optimality imply

$$\frac{p_B}{p_A} \geq \rho_{aut}, \quad \frac{p_B^*}{p_A^*} \leq \rho_{aut}^*.$$

Consequently,

$$(1 + \tau)(1 + \tau^*) \leq \frac{\rho_{aut}^*}{\rho_{aut}} < 1,$$

where the strict inequality follows from comparative advantage. This contradicts $(1 + \tau)(1 + \tau^*) \geq 1$, so the reversed trade pattern is impossible for any nonnegative tariff pair.

Before considering the cases with trade in just one good, note that consumption must be strictly positive in both countries. For example, if only good A is traded and Home exports it, and $c_A = 0$, then by Home's Kuhn–Tucker condition and the strict monotonicity of the MRS, $p_B/p_A < p_B^*/p_A^*$, contradicting the no-trade condition for good B , which requires $p_B/p_A \geq p_B^*/p_A^*$. The argument when Foreign exports good A is symmetric. The same reasoning, with the goods interchanged, rules out $c_B = 0$ and $c_B^* = 0$ when only good B is traded. Cases 3 and 4 below feature interior consumption levels.

Case 3: One-way trade in good A. Only good A is traded ($c_B = Y_B$, $c_B^* = Y_B^*$). Then c_A is the only free allocation variable, with $c_A^* = \bar{Y}_A - c_A$. Define the domestic relative prices as functions of c_A :

$$\rho(c_A) \equiv g\left(\frac{Y_B}{c_A}\right), \quad \rho^*(c_A) \equiv g\left(\frac{Y_B^*}{\bar{Y}_A - c_A}\right).$$

At autarky ($c_A = Y_A$): $\rho(Y_A) = \rho_{aut}$ and $\rho^*(Y_A) = \rho_{aut}^*$.

Case 3a: $c_A < Y_A$ (Home exports A). The no-trade condition for good B requires that Foreign does not import B from Home:

$$\rho^*(c_A) \leq \rho(c_A).$$

Since $c_A < Y_A$, we have $\rho(c_A) < \rho_{aut}$ and $\rho^*(c_A) > \rho_{aut}^*$.

Moreover, ρ is increasing in c_A and ρ^* is decreasing, and they coincide at

$$\hat{c}_A = \frac{Y_B}{\bar{Y}_B} \bar{Y}_A$$

where they equal ρ^{FT} . The no trade restriction above $\rho^*(c_A) \leq \rho(c_A)$ then implies that $c_A \geq \hat{c}_A$. It follows then that

$$\rho_{aut}^* \leq \rho^*(c_A) \leq \rho^{FT} \leq \rho(c_A) \leq \rho_{aut}. \quad (18)$$

Since Foreign imports A , goods arbitrage gives $p_A^* = (1 + \tau^*) p_A$ and $T = 0$. Normalizing $p_A = 1$, Home's budget constraint is:

$$Y_A - c_A = (a_A^* + \rho(c_A) a_B^*) - (1 + \tau^*)(a_A + \rho^*(c_A) a_B). \quad (19)$$

The left-hand side satisfies $0 < Y_A - c_A \leq Y_A$. By (18), $\rho^*(c_A) \in [\rho_{aut}^*, \rho^{FT}]$.

By condition (15), $a_A + \rho^*(c_A) a_B \neq 0$ for all feasible c_A . Since this valuation is continuous on a compact interval, it is bounded away from zero. Moreover, $a_A^* + \rho(c_A) a_B^*$ is uniformly bounded by (18). It follows

that

$$(1 + \tau^*)(a_A + \rho^*(c_A)a_B)$$

diverges uniformly as $\tau^* \rightarrow \infty$, while all other terms in (19) remain bounded, yielding a contradiction for τ^* sufficiently large.

Case 3b: $c_A > Y_A$ (Home imports A). Since Home imports A, $p_A = (1 + \tau) p_A^*$. For good B not to also flow from Foreign to Home, the tariff-inclusive cost of importing B must weakly exceed Home's domestic price: $(1 + \tau) p_B^* \geq p_B$. Substituting $p_B = \rho p_A = \rho(1 + \tau) p_A^*$ and $p_B^* = \rho^* p_A^*$ gives $\rho^*(c_A) \geq \rho(c_A)$. But $c_A > Y_A$ gives $\rho(c_A) > \rho_{aut}$, while $\rho^*(c_A) < \rho_{aut}^* \leq \rho_{aut} < \rho(c_A)$, contradicting $\rho^* \geq \rho$. So this case is impossible as the comparative advantage assumption in (1) implies that $\rho_{aut}^* \leq \rho_{aut}$.

Case 4: One-way trade in good B. Suppose only good B is traded ($c_A = Y_A$, $c_A^* = Y_A^*$). Define for this case

$$\rho(c_B) \equiv g(c_B/Y_A), \quad \rho^*(c_B) \equiv g\left(\frac{\bar{Y}_B - c_B}{Y_A^*}\right).$$

If $c_B > Y_B$ (Home imports B), the no-trade condition for A requires $\rho^* \leq \rho$, and the same bounds (18) hold with $\rho(c_B) \in [\rho^{FT}, \rho_{aut}]$ and $\rho^*(c_B) \in [\rho_{aut}^*, \rho^{FT}]$. The budget constraint involves $(1 + \tau)(a_A^* + \rho(c_B) a_B^*)$. By condition (15), $a_A^* + \rho a_B^* \neq 0$ for all $\rho \in [\rho^{FT}, \rho_{aut}]$, so the same divergence argument as in Case 3a gives a contradiction for τ large enough.

If $c_B < Y_B$ (Home exports B), the no-trade condition for A requires $\rho^* \geq \rho$, but $\rho > \rho_{aut} \geq \rho_{aut}^* > \rho^*$, a contradiction, ruling out this case. $\square$

The same sign condition in this proposition requires that both countries' gross claims are on the same side of the balance sheet when evaluated at autarkic prices. Condition (15) rules out a case where the asset positions would admit an additional non-autarkic equilibrium in the limit as tariffs increase. Note that if all assets holdings are of the same sign and non-zero, conditions (15) and (16) are automatically satisfied.

Proof of Proposition 2 Now we can complete the proof of Proposition 2.

Proof. Let $\rho_{aut} \equiv g(Y_B/Y_A)$ and $\rho_{aut}^* \equiv g(Y_B^*/Y_A^*)$. Under Assumption 1, $a_A = a_B^* = 0$, so for every $\rho, \rho^* > 0$,

$$a_A + \rho^* a_B = \rho^* a_B > 0, \quad a_A^* + \rho a_B^* = a_A^* > 0,$$

where the inequalities use $a_B > 0$ and $a_A^* > 0$. Both valuations are therefore non-zero on $[\rho_{aut}^*, \rho^{FT}]$ and $[\rho^{FT}, \rho_{aut}]$. Moreover,

$$(a_A^* + \rho_{aut} a_B^*)(a_A + \rho_{aut}^* a_B) = a_A^* \rho_{aut}^* a_B > 0.$$

The conditions for Proposition A.1 then hold, and there are finite $\underline{\tau}, \bar{\tau}^*$ such that the autarky allocation

is the unique equilibrium for all $\tau \geq \underline{\tau}$ and $\tau^* \geq \underline{\tau}^*$. □

A.3 Proof of Proposition 3

We now proceed to state and prove a relaxed version of Proposition 3.

Proposition A.2. *Suppose Assumption 2 holds. Suppose that*

$$a_A = a_B = 0, \quad a_A^*, a_B^* \geq 0,$$

with at least one inequality strict. Then there exists a unique $e \in (0, Y_A)$ satisfying

$$e = a_A^* + g\left(\frac{Y_B}{Y_A - e}\right) a_B^*.$$

Moreover, there exists a finite threshold κ such that, for any tariff rates (τ, τ^) satisfying $(1 + \tau)(1 + \tau^*) \geq \kappa$, the unique equilibrium allocation is $(c_A, c_B, c_A^*, c_B^*) = (Y_A - e, Y_B, Y_A^* + e, Y_B^*)$. In this equilibrium, the terms of trade are*

$$\rho = (1 + \tau^*)g\left(\frac{Y_B^*}{Y_A^* + e}\right).$$

Proof. Let

$$\bar{r} \equiv \frac{\bar{Y}_B}{Y_A}, \quad \widehat{e} \equiv Y_A - \frac{Y_B}{\bar{r}} < Y_A.$$

By construction,

$$\frac{Y_B}{Y_A - \widehat{e}} = \bar{r}.$$

At free trade, identical homothetic preferences and market clearing imply

$$\frac{c_B^{FT}}{c_A^{FT}} = \bar{r}.$$

Since Home imports good B at free trade, $c_B^{FT} > Y_B$. Hence,

$$c_A^{FT} > Y_A - \widehat{e}.$$

Using Home's free-trade budget constraint,

$$c_A^{FT} + \rho^{FT} c_B^{FT} = Y_A + \rho^{FT} Y_B - a_A^* - \rho^{FT} a_B^*,$$

we obtain

$$a_A^* + \rho^{FT} a_B^* < \widehat{e}. \quad (20)$$

We first establish the existence and uniqueness of e . Define

$$\Phi(x) \equiv a_A^* + g\left(\frac{Y_B}{Y_A - x}\right) a_B^*, \quad x \in [0, Y_A).$$

The function Φ is continuous and weakly decreasing. Hence $\Phi(x) - x$ is continuous and strictly decreasing, which implies it has at most one zero. Moreover,

$$\Phi(0) - 0 = a_A^* + g\left(\frac{Y_B}{Y_A}\right) a_B^* > 0.$$

given that a_A^* and a_B^* are non-negative, with at least one of them strictly positive. Since $\widehat{e} < Y_A$, and

$$g\left(\frac{Y_B}{Y_A - \widehat{e}}\right) = g(\bar{r}) = \rho^{FT},$$

equation (20) gives

$$\Phi(\widehat{e}) - \widehat{e} = a_A^* + \rho^{FT} a_B^* - \widehat{e} < 0.$$

The intermediate value theorem therefore gives a unique $e \in (0, \widehat{e})$ satisfying

$$e = a_A^* + g\left(\frac{Y_B}{Y_A - e}\right) a_B^*. \quad (21)$$

In particular, $0 < e < Y_A$, and e is independent of the tariff rates.

Define

$$q_H \equiv g\left(\frac{Y_B}{Y_A - e}\right), \quad q_F \equiv g\left(\frac{Y_B^*}{Y_A^* + e}\right).$$

Because $e < \widehat{e}$,

$$\frac{Y_B}{Y_A - e} < \frac{\bar{Y}_B}{\bar{Y}_A} < \frac{Y_B^*}{Y_A^* + e}.$$

Since g is decreasing,

$$q_H > \rho^{FT} > q_F.$$

Define

$$\kappa_0 \equiv \frac{q_H}{q_F} > 1.$$

We next show that the proposed allocation is an equilibrium. Normalize $p_A = 1$ and set

$$p_A^* = 1 + \tau^*, \quad p_B = q_H, \quad p_B^* = (1 + \tau^*)q_F.$$

These prices satisfy household optimality at

$$(c_A, c_B, c_A^*, c_B^*) = (Y_A - e, Y_B, Y_A^* + e, Y_B^*).$$

Good A is exported by Home, and its arbitrage condition holds with equality. Since good B is not traded, its arbitrage conditions require

$$\frac{1}{1 + \tau^*} \leq \frac{p_B}{p_B^*} \leq 1 + \tau.$$

Using the proposed prices, these inequalities are equivalent to

$$1 \leq \kappa_0 \leq (1 + \tau)(1 + \tau^*).$$

The first inequality was established above, while the second holds when the tariff product is sufficiently large.

Home's budget constraint holds because

$$e = a_A^* + q_H a_B^*$$

by (21). The Foreign budget constraint follows from market clearing and Walras' law. Hence the proposed allocation is an equilibrium whenever

$$(1 + \tau)(1 + \tau^*) \geq \kappa_0.$$

It remains to prove uniqueness. The weak version of Lemma B.1 applies because

$$a_A + \rho^{FT} a_B = 0, \quad a_A^* + \rho^{FT} a_B^* > 0.$$

Consequently, every equilibrium satisfies

$$c_A \leq Y_A, \quad c_B \geq Y_B.$$

We consider the four possible trade regimes.

Only good A is traded. Let $x \equiv Y_A - c_A$. Since $c_B = Y_B$, Home's optimality and budget constraint imply

$$x = a_A^* + g \left(\frac{Y_B}{Y_A - x} \right) a_B^*.$$

By the uniqueness of the fixed point, $x = e$. Hence every equilibrium in this regime has the proposed allocation. Active trade in good A and Foreign optimality then imply

$$\rho = (1 + \tau^*) g \left(\frac{Y_B^*}{Y_A^* + e} \right).$$

Only good B is traded. In this regime, $c_A = Y_A$ and $c_B > Y_B$. Substituting Home's tariff revenue into its budget constraint gives

$$p_B^*(c_B - Y_B) = -p_A a_A^* - p_B a_B^*.$$

The left-hand side is strictly positive, whereas the right-hand side is strictly negative. Hence this regime is impossible.

Autarky. At the autarkic allocation, Home's budget constraint would require

$$p_A a_A^* + p_B a_B^* = 0.$$

This is impossible because prices are strictly positive, both claims are nonnegative, and at least one is strictly positive.

Two-way trade. In this regime, Home exports good A and imports good B , so

$$c_A < Y_A, \quad c_B > Y_B.$$

Consequently,

$$\frac{c_B}{c_A} > \frac{Y_B}{Y_A},$$

and, since g is decreasing,

$$g\left(\frac{c_B}{c_A}\right) < g\left(\frac{Y_B}{Y_A}\right).$$

Similarly, Foreign imports A and exports B , so

$$c_A^* > Y_A^*, \quad c_B^* < Y_B^*,$$

and hence

$$g\left(\frac{c_B^*}{c_A^*}\right) > g\left(\frac{Y_B^*}{Y_A^*}\right).$$

Active trade in both goods requires

$$(1 + \tau)(1 + \tau^*) = \frac{g(c_B/c_A)}{g(c_B^*/c_A^*)}.$$

It follows that every two-way-trade equilibrium must satisfy

$$(1 + \tau)(1 + \tau^*) < \frac{g(Y_B/Y_A)}{g(Y_B^*/Y_A^*)} \equiv \kappa.$$

Thus two-way trade is impossible whenever

$$(1 + \tau)(1 + \tau^*) \geq \kappa.$$

Finally, because $e > 0$,

$$\kappa > \frac{g(Y_B/(Y_A - e))}{g(Y_B^*/(Y_A^* + e))} = \kappa_0.$$

So we conclude that for every tariff pair satisfying

$$(1 + \tau)(1 + \tau^*) \geq \kappa,$$

all regimes other than the regime in which only good A is traded are impossible. That regime has the unique allocation and terms of trade stated in the proposition. $\square$

Proof of Proposition 3 Now we can complete the proof of Proposition 3.

Proof. Under Assumption 1, $a_A = a_B^* = 0$, and Proposition 3 sets $a_B = 0$. The hypotheses of Proposition A.2 are therefore satisfied, with $a_A^* > 0$. From its definition, $e = a_A^*$, and Proposition A.2 implies that $e \in (0, Y_A)$, and thus $a_A^* < Y_A$. The proposition then implies a unique equilibrium $(Y_A - a_A^*, Y_B, Y_A^* + a_A^*, Y_B^*)$ and $\rho = (1 + \tau^*) g(Y_B^*/(Y_A^* + a_A^*))$ for every tariff pair with $(1 + \tau)(1 + \tau^*) \geq \kappa$. $\square$

A.4 Proof of Proposition 4

We will prove a more general version of Proposition 4 that does not rely on the restriction to simple claims.

Proposition A.3 (Home's Debt Trap). *Suppose Assumption 2 holds. Suppose that*

$$a_A + \rho^{FT} a_B < 0.$$

Then there exists $(\underline{\tau}^, \bar{\tau}^*)$ with $0 < \underline{\tau}^* < \bar{\tau}^* < \infty$ such that, for any $\tau^* \in (\underline{\tau}^*, \bar{\tau}^*)$, there is a tariff equilibrium where Home exports both goods: $c_A < Y_A, c_B < Y_B$; relative prices are the same as under free trade: $g(c_B/c_A) = \rho^{FT}$; and Home's budget set relative to free trade is*

$$c_A - c_A^{FT} + \rho^{FT}(c_B - c_B^{FT}) = \tau^*(a_A + \rho^{FT} a_B) < 0.$$

Proof. Define

$$\underline{\tau}^* = \frac{(c_A^{FT} + \rho^{FT} c_B^{FT}) - (1/h(\rho^{FT}) + \rho^{FT}) Y_B}{-(a_A + \rho^{FT} a_B)}, \quad \text{and} \quad \bar{\tau}^* = \frac{c_A^{FT} + \rho^{FT} c_B^{FT}}{-(a_A + \rho^{FT} a_B)}.$$

Note that $0 < \underline{\tau}^*$. To see this

$$Y_B(1/h(\rho^{FT}) + \rho^{FT}) < c_B^{FT}(1/h(\rho^{FT}) + \rho^{FT}) = c_A^{FT} + \rho^{FT} c_B^{FT}$$

where the first inequality follows by $c_B^{FT} > Y_B$ and the equality from $c_A^{FT} = c_B^{FT}/h(\rho^{FT})$. This implies that indeed

$$0 < \frac{(c_A^{FT} + \rho^{FT} c_B^{FT}) - (1/h(\rho^{FT}) + \rho^{FT}) Y_B}{-(a_A + \rho^{FT} a_B)} = \underline{\tau}^*.$$

Note also that $\underline{\tau}^* < \bar{\tau}^*$, as the two thresholds share the same strictly positive denominator and the numerator of $\underline{\tau}^*$ is strictly positive and strictly less than that of $\bar{\tau}^*$.

In the conjectured equilibrium, Home prices are $p_B/p_A = \rho^{FT}$ and Foreign prices are $p_B^*/p_A^* = (1 + \tau^*)p_B/((1 + \tau^*)p_B) = \rho^{FT}$. Note that with these prices, goods arbitrage holds. Let c_A and c_B be such that $g(c_B/c_A) = \rho^{FT}$ and Home budget constraint holds with equality. This guarantees that Home's household optimality conditions hold. Home's budget constraint can be rewritten as:

$$c_A + \rho^{FT} c_B = Y_A + \rho^{FT} Y_B + p_A^*/p_A (a_A + \rho^{FT} a_B) - (a_A^* + \rho^{FT} a_B^*).$$

Using the fact that $p_A^*/p_A = (1 + \tau^*)$, we can rewrite this as

$$c_A + \rho^{FT} c_B = Y_A + \rho^{FT} Y_B + (1 + \tau^*)(a_A + \rho^{FT} a_B) - (a_A^* + \rho^{FT} a_B^*).$$

And using the fact that $c_A^{FT} + \rho^{FT} c_B^{FT} = Y_A + \rho^{FT} Y_B + (a_A + \rho^{FT} a_B) - (a_A^* + \rho^{FT} a_B^*)$, we can rewrite this once more as

$$c_A + \rho^{FT} c_B = c_A^{FT} + \rho^{FT} c_B^{FT} + \tau^*(a_A + \rho^{FT} a_B). \quad (22)$$

Note that $\tau^* < (c_A^{FT} + \rho^{FT} c_B^{FT})/(-(a_A + \rho^{FT} a_B))$ since $\tau^* < \bar{\tau}^*$. This implies that $c_A + \rho^{FT} c_B > 0$. Hence, expenditure is strictly positive and optimality implies that c_A and c_B are both non-negative. Next, we need to check that $c_A < Y_A$ and $c_B < Y_B$. Note that $c_A = c_B/h(\rho^{FT})$. Thus, from (22) we have that

$$c_B(1/h(\rho^{FT}) + \rho^{FT}) = c_A^{FT} + \rho^{FT} c_B^{FT} + \tau^*(a_A + \rho^{FT} a_B).$$

For $c_B < Y_B$, it suffices to show that:

$$Y_B(1/h(\rho^{FT}) + \rho^{FT}) > c_A^{FT} + \rho^{FT} c_B^{FT} + \tau^*(a_A + \rho^{FT} a_B).$$

This is the requirement that $\underline{\tau}^* < \tau^*$. For $c_A < Y_A$, recall from Assumption 2 that $c_A^{FT} < Y_A$ and $c_B^{FT} > Y_B$. Given that we have shown that $c_B < Y_B$, it follows that $c_B < c_B^{FT}$. Then, $c_A < c_A^{FT} < Y_A$, by

homotheticity. Using the resource constraints, we can obtain c_A^* and c_B^* . Note that $c_A^* = \bar{Y}_A - c_A > 0$ and $c_B^* = \bar{Y}_B - c_B > 0$. Foreign optimality condition holds as $g(c_B^*/c_A^*) = g(c_B/c_A) = g(\bar{Y}_B/\bar{Y}_A) = \rho^{FT}$. We need to check that Foreign's budget constraint holds with equality. This follows from Walras law. We have thus constructed an equilibrium with the properties of the Proposition. $\square$

Proof of Proposition 4 Now we can complete the proof of Proposition 4.

Proof. Under Assumption 1, $a_A = 0$, so $a_A + \rho^{FT} a_B = \rho^{FT} a_B$ which is strictly negative if and only if $a_B < 0$, since $\rho^{FT} > 0$. The hypothesis of Proposition A.3 therefore holds, and its implications prove the statement of Proposition 4. $\square$

B Additional Results

B.1 Equilibrium Characterization under Weakly Positive Claims

We will prove a more general version of the arguments described in Section 4.2, that does not impose the simple-claims restriction.

The characterization in Section 4.2 is a special case of the lemma below when $a_A = a_B^* = 0$.

Lemma B.1 (Positive Claims Equilibria: General Portfolios). *Suppose Assumption 2 holds, and suppose the asset positions satisfy*

$$a_A + \rho^{FT} a_B \geq 0, \quad a_A^* + \rho^{FT} a_B^* \geq 0. \quad (23)$$

Then every tariff equilibrium has strictly positive consumption. Moreover, given a pair of tariffs $(\tau, \tau^) \geq 0$, a strictly positive consumption allocation (c_A, c_B, c_A^*, c_B^*) , together with $0 < \rho < \infty$, can be supported as a tariff equilibrium if and only if there exist*

$$\alpha, \alpha^* \in \left[\frac{1}{1 + \tau^*}, 1 + \tau \right]$$

such that the following conditions hold:

(i) *Home weakly exports good A and weakly imports good B:*

$$c_A \leq Y_A, \quad c_B \geq Y_B.$$

(ii) The budget and resource constraints are satisfied:

$$c_A + \rho c_B = Y_A + \rho Y_B + \frac{a_A}{\alpha^*} + \rho a_B - a_A^* - \alpha \rho a_B^*, \quad (24)$$

$$c_A^* + \rho c_B^* = Y_A^* + \rho Y_B^* - \frac{a_A}{\alpha^*} - \rho a_B + a_A^* + \alpha \rho a_B^*, \quad (25)$$

and

$$c_A + c_A^* = \bar{Y}_A, \quad c_B + c_B^* = \bar{Y}_B. \quad (26)$$

(iii) Household optimality and goods arbitrage require

$$g(c_B/c_A) = \alpha \rho, \quad g(c_B^*/c_A^*) = \alpha^* \rho, \quad (27)$$

with

$$\alpha = 1 + \tau \quad \text{if } c_B > Y_B, \quad \alpha^* = \frac{1}{1 + \tau^*} \quad \text{if } c_A < Y_A. \quad (28)$$

The corresponding equilibrium prices are any $(p_A, p_B, p_A^*, p_B^*) \gg 0$ satisfying

$$\rho = \frac{p_B^*}{p_A}, \quad p_B = \alpha p_B^*, \quad p_A^* = \frac{p_A}{\alpha^*}.$$

Proof. We first prove interiority result, then the necessity direction, and later on the sufficiency.

Interiority of consumption. We use the standard compensated-demand implication of identical, strictly concave, homothetic preferences: after rescaling two optimal bundles onto the same indifference curve, the bundle chosen at a weakly higher relative price of B contains weakly less B and weakly more A ; if the relative prices are equal, the original bundles lie on the same ray.

Define the relative prices faced by Home and Foreign as

$$q_H \equiv \frac{p_B}{p_A}, \quad q_F \equiv \frac{p_B^*}{p_A^*}.$$

Suppose first that $c_A = 0$. If $c_B < Y_B$, Home exports both goods and therefore faces the free-trade relative price. Its budget constraint becomes

$$c_A + \rho^{FT} c_B = c_A^{FT} + \rho^{FT} c_B^{FT} + \tau^* (a_A + \rho^{FT} a_B) \geq c_A^{FT} + \rho^{FT} c_B^{FT},$$

where the inequality follows from (23). Homotheticity would then imply

$$c_B \geq c_B^{FT} > Y_B,$$

a contradiction. Hence $c_B \geq Y_B > 0$. Since Home strictly exports A , goods arbitrage implies

$$\frac{p_A}{p_A^*} = \frac{1}{1 + \tau^*}, \quad \frac{p_B}{p_B^*} \geq \frac{1}{1 + \tau^*}.$$

It follows that $q_H \geq q_F$. Rescale Foreign's bundle onto Home's indifference curve. The compensated-demand ordering requires Home to consume weakly more A than the rescaled Foreign bundle. This is impossible because $c_A = 0$, whereas resource feasibility gives $c_A^* = \bar{Y}_A > 0$. Therefore, $c_A > 0$.

A symmetric argument, interchanging Home with Foreign and goods A and B , rules out $c_B^* = 0$. Hence $c_B^* > 0$.

Suppose now that $c_B = 0$. Resource feasibility gives $c_B^* = \bar{Y}_B > 0$. Since Home strictly exports B , goods arbitrage implies

$$\frac{p_B}{p_B^*} = \frac{1}{1 + \tau^*}, \quad \frac{p_A}{p_A^*} \geq \frac{1}{1 + \tau^*}.$$

It follows that $q_H \leq q_F$. Rescale Home's bundle onto Foreign's indifference curve. The compensated-demand ordering requires Foreign to consume weakly less B than the rescaled Home bundle. This is impossible because Foreign consumes strictly positive B , whereas the rescaled Home bundle contains zero B . Therefore, $c_B > 0$.

A symmetric argument establishes that $c_A^* > 0$.

Thus every equilibrium satisfies $c_A, c_B, c_A^*, c_B^* > 0$.

Necessity of Part (i). We start by ruling out alternative patterns of trade via contradiction.

Case (a): Home imports A and weakly exports B .

Suppose $c_A > Y_A$ and $c_B \leq Y_B$. This trade pattern implies

$$\frac{c_B}{c_A} < \frac{Y_B}{Y_A} < \frac{\bar{Y}_B}{\bar{Y}_A} < \frac{Y_B^*}{Y_A^*} < \frac{c_B^*}{c_A^*}.$$

Household optimality then requires

$$\frac{p_B}{p_A} = g(c_B/c_A) > g(c_B^*/c_A^*) = \frac{p_B^*}{p_A^*}.$$

Goods arbitrage for the actively imported good A implies $p_A = (1 + \tau)p_A^*$. It follows that

$$p_B > (1 + \tau)p_B^*.$$

This violates goods arbitrage for good B , which requires $p_B \leq (1 + \tau)p_B^*$.

Case (b): Home imports both goods.

Suppose $c_A > Y_A$ and $c_B > Y_B$. In this case, $p_A = (1 + \tau)p_A^*$, and $p_B = (1 + \tau)p_B^*$. The two households therefore face the same relative prices. Household optimality, identical homothetic preferences, and

market clearing imply

$$\frac{p_B^*}{p_A^*} = \frac{p_B}{p_A} = g\left(\frac{\bar{Y}_B}{\bar{Y}_A}\right) = \rho^{FT}.$$

Foreign imports neither good, so $T^* = 0$. Its budget constraint gives

$$\begin{aligned} c_A^* + \rho^{FT} c_B^* &= Y_A^* + \rho^{FT} Y_B^* + (a_A^* + \rho^{FT} a_B^*) - (a_A + \rho^{FT} a_B) + \tau(a_A^* + \rho^{FT} a_B^*) \\ &= c_A^{*,FT} + \rho^{FT} c_B^{*,FT} + \tau(a_A^* + \rho^{FT} a_B^*) \geq c_A^{*,FT} + \rho^{FT} c_B^{*,FT}, \end{aligned}$$

where the last line follows from (23).

Foreign faces the free-trade relative price and has weakly greater wealth. Homotheticity therefore implies $c_A^* \geq c_A^{*,FT}$. But the free-trade benchmark gives $c_A^{*,FT} > Y_A^*$, contradicting the premise that Foreign exports good A.

Case (c): Home exports both goods.

Suppose $c_A < Y_A$ and $c_B < Y_B$. This case is symmetric to Case (b). In particular, $p_A^* = (1 + \tau^*)p_A$, and $p_B^* = (1 + \tau^*)p_B$, so that $p_B/p_A = \rho^{FT}$. Home's budget set is

$$\begin{aligned} c_A + \rho^{FT} c_B &= Y_A + \rho^{FT} Y_B + (a_A + \rho^{FT} a_B) - (a_A^* + \rho^{FT} a_B^*) + \tau^*(a_A + \rho^{FT} a_B) \\ &= c_A^{FT} + \rho^{FT} c_B^{FT} + \tau^*(a_A + \rho^{FT} a_B) \geq c_A^{FT} + \rho^{FT} c_B^{FT}. \end{aligned}$$

Because Home faces the free-trade relative price and has weakly greater wealth, homotheticity implies

$$c_B \geq c_B^{FT}.$$

This contradicts

$$c_B < Y_B < c_B^{FT}.$$

Case (d): Home does not trade A and exports B.

Suppose $c_A = Y_A$ and $c_B < Y_B$. As in Case (a),

$$\frac{c_B}{c_A} < \frac{Y_B}{Y_A} < \frac{\bar{Y}_B}{\bar{Y}_A} < \frac{Y_B^*}{Y_A^*} < \frac{c_B^*}{c_A^*},$$

and therefore

$$\frac{p_B}{p_A} > \frac{p_B^*}{p_A^*}.$$

Since Foreign imports good B, goods arbitrage implies $p_B^* = (1 + \tau^*)p_B$. The preceding inequality then gives

$$p_A^* > (1 + \tau^*)p_A,$$

which violates goods arbitrage for good A.

Cases (a) and (b) establish that Home weakly exports good A. Cases (c) and (d) further establish that

Home weakly imports good B . Notice that the portfolio positions enter the argument only through

$$\tau(a_A^* + \rho^{FT} a_B^*) \geq 0, \quad \tau^*(a_A + \rho^{FT} a_B) \geq 0.$$

Thus the proof uses only the weak inequalities in (23).

Necessity of Part (ii). We next establish the necessity of Part (ii). Under the trade pattern just proved, Home can only import good B . Define

$$\rho = \frac{p_B^*}{p_A}, \quad \alpha = \frac{p_B}{p_B^*}, \quad \alpha^* = \frac{p_A}{p_A^*}.$$

Home tariff revenue satisfies

$$\frac{T}{p_A} = (\alpha - 1)\rho(c_B - Y_B).$$

Indeed, when $c_B > Y_B$, we have $\alpha = 1 + \tau$; when $c_B = Y_B$, both sides are zero.

Dividing Home's household budget constraint by p_A gives

$$c_A + a_A^* + \alpha\rho(c_B + a_B^*) = Y_A + \frac{a_A}{\alpha^*} + \alpha\rho Y_B + \rho a_B + \frac{T}{p_A}.$$

Substituting for T/p_A and collecting terms yields

$$c_A + \rho c_B = Y_A + \rho Y_B + \frac{a_A}{\alpha^*} + \rho a_B - a_A^* - \alpha\rho a_B^*,$$

which is (24).

Applying the same argument to Foreign gives

$$c_A^* + \rho c_B^* = Y_A^* + \rho Y_B^* - \frac{a_A}{\alpha^*} - \rho a_B + a_A^* + \alpha\rho a_B^*.$$

Equivalently, this equation follows from the Home budget equation and the resource constraints.

Necessity of Part (iii). Retain the definitions $\alpha = p_B/p_B^*$ and $\alpha^* = p_A/p_A^*$ from part (ii). Goods arbitrage requires

$$\frac{p_B^*}{1 + \tau^*} \leq p_B \leq (1 + \tau)p_B^*,$$

with the upper inequality binding when $c_B > Y_B$. Dividing by p_A and using Home optimality gives

$$\frac{\rho}{1 + \tau^*} \leq g(c_B/c_A) \leq (1 + \tau)\rho.$$

Thus

$$g(c_B/c_A) = \alpha\rho, \quad \alpha \in \left[\frac{1}{1 + \tau^*}, 1 + \tau \right],$$

with $\alpha = 1 + \tau$ when $c_B > Y_B$.

The analogous argument for Foreign gives

$$g(c_B^*/c_A^*) = \alpha^* \rho, \quad \alpha^* \in \left[\frac{1}{1 + \tau^*}, 1 + \tau \right],$$

with $\alpha^* = \frac{1}{1 + \tau^*}$ when $c_A < Y_A$.

We have shown that a tariff equilibrium must satisfy Parts (i)–(iii).

Sufficiency. Suppose the allocation, ρ , α , and α^* satisfy the stated conditions. Choose any $p_A > 0$ and construct the remaining prices according to

$$p_B^* = \rho p_A, \quad p_B = \alpha p_B^*, \quad p_A^* = \frac{p_A}{\alpha^*}.$$

The restrictions on α and α^* , including the binding conditions in (28), ensure goods arbitrage. Equation (27), together with the maintained preference assumptions, ensures household optimization. Equations (24) and (25) recover the original household budget constraints after tariff revenue is rebated. Finally, (26) gives market clearing. Hence the constructed allocation and prices form a tariff equilibrium. $\square$

B.2 The Symmetric Debt-Trap

Here we state the symmetric version of Proposition 4:

Corollary B.1 (Foreign's Debt Trap). *Suppose that Assumptions 1 and 2 hold. Suppose that*

$$a_A^* < 0.$$

Then there exists $(\underline{\tau}, \bar{\tau})$ with $0 < \underline{\tau} < \bar{\tau} < \infty$ such that for any $\tau \in (\underline{\tau}, \bar{\tau})$ there is a tariff equilibrium where Foreign exports both goods: $c_A^ < Y_A^*$, $c_B^* < Y_B^*$; relative prices are the same as under free trade, $g(c_B^*/c_A^*) = \rho^{FT}$; and*

$$c_A^* - c_A^{*FT} + \rho^{FT}(c_B^* - c_B^{*FT}) = \tau a_A^* < 0.$$

This corollary can be generalized as in Proposition A.3. The proof is entirely symmetric.

C Anticipation and Portfolio Rebalancing

In this appendix, we provide the complete description of the model in Section 4.6.

The Home household's problem is to choose a new portfolio position x_A and x_B to solve the

following problem:

$$\begin{aligned} \max_{x_A, x_B, (c_A(s), c_B(s))_{s \in S}} \sum_{s \in S} \pi(s) u(c_A(s), c_B(s)) \quad & \text{subject to:} \\ x_A + qx_B = qa_B - a_A^* \\ 0 = p_A(s)(Y_A + x_A - c_A(s)) + p_B(s)(Y_B - c_B(s)) + p_B^*(s)x_B + T(s), \text{ for all } s \in S. \end{aligned}$$

where $p_i(s), p_i^*(s)$ are the prices of good i in state s at Home and Foreign respectively. Foreign household's problem is symmetric:

$$\begin{aligned} \max_{x_A^*, x_B^*, (c_A^*(s), c_B^*(s))_{s \in S}} \sum_{s \in S} \pi(s) u(c_A^*(s), c_B^*(s)) \quad & \text{subject to:} \\ x_A^* + qx_B^* = a_A^* - qa_B \\ 0 = p_A^*(s)(Y_A^* - c_A^*(s)) + p_B^*(s)(Y_B^* + x_B^* - c_B^*(s)) + p_A(s)x_A^* + T^*(s), \text{ for all } s \in S. \end{aligned}$$

The asset market clearing conditions are:

$$x_A + x_A^* = 0, \quad x_B + x_B^* = 0 \quad \text{and} \quad \hat{a}_A^* = x_A^*, \quad \hat{a}_B = x_B.$$

An ex-ante equilibrium is defined in the text. Recall that the objective is to characterize *full-trade-war equilibria*. These are equilibria where the tariffs are high enough that trade war state is the autarkic allocation.

In a full-trade-war equilibrium, $T(s) = T^*(s) = 0$ for $s \in S$ as in both the free-trade and autarky states there is no tariff revenue. In the free trade state, we have also that $p_B(FT) = p_B^*(FT)$. We know that, in equilibrium, the terms of trade have to be:

$$\rho(s) = \frac{p_B^*(s)}{p_A(s)} = \begin{cases} \rho^{FT} = g(\bar{Y}_B/\bar{Y}_A) & \text{if } s = FT, \\ \rho^W = \frac{\hat{a}_A^*}{\hat{a}_B} & \text{if } s = W. \end{cases}$$

where $(\hat{a}_A^*, \hat{a}_B)$ is the resulting ex-post portfolio of assets.

In the trade war state, the consumption allocation is autarky, so:

$$c_A(W) = Y_A, c_B(W) = Y_B, \quad c_A^*(W) = Y_A^*, c_B^*(W) = Y_B^*.$$

Using the households' first order conditions we can write:

$$q = \frac{\sum_{s \in S} \pi(s) u_B(s) p_B^*(s) / p_B(s)}{\sum_{s \in S} \pi(s) u_A(s)} = \frac{\sum_{s \in S} \pi(s) u_B^*(s)}{\sum_{s \in S} \pi(s) u_A^*(s) p_A(s) / p_A^*(s)}. \quad (29)$$

where $u_i(s) \equiv u_i(c_A(s), c_B(s))$ and $u_i^*(s) \equiv u_i(c_A^*(s), c_B^*(s))$.

Under free trade, $p_i(FT) = p_i^*(FT)$ for all $i \in \{A, B\}$. In the trade war

$$\frac{p_B^*(W)}{p_B(W)} = \frac{\rho^W}{\rho_{aut}}, \quad \text{and} \quad \frac{p_A(W)}{p_A^*(W)} = \frac{\rho_{aut}^*}{\rho^W}.$$

Equation (29) makes clear that the gap in the asset payoff and the corresponding price of the consumption good opens up a wedge between Foreign and Home households' valuations of the financial claims. Using (29) and the within state optimality conditions, we can obtain

$$\left(\rho^{FT} - \rho^W \right) \left(\rho^{FT} \frac{u_A(FT)}{u_B^*(FT)} - \rho^W \frac{u_A(W)}{u_B^*(W)} \right) = 0.$$

If $\rho^{FT} \neq \rho^W$, households perceive asset markets to be complete, and it follows that:²³

$$\rho^{FT} \frac{u_A(FT)}{u_B^*(FT)} = \rho^W \frac{u_A(W)}{u_B^*(W)} \Rightarrow \frac{u_A(FT)}{u_A^*(FT)} = \rho^W \frac{u_A(W)}{u_B^*(W)}.$$

where the second equality follows from the law of one price in free trade.

Given the homothetic and identical preferences, in the free-trade state, Home's consumption of both goods is a share of the aggregate endowments. Let λ denote this share. Then, the above equation reduces to:

$$\frac{u_A(\lambda \bar{Y}_A, \lambda \bar{Y}_B)}{u_A((1-\lambda) \bar{Y}_A, (1-\lambda) \bar{Y}_B)} = \frac{\rho^W}{\rho_{aut}^*} \frac{u_A(Y_A, Y_B)}{u_A(Y_A^*, Y_B^*)} \quad (\text{RS}')$$

Using the equilibrium price q from (29), together with the ex-ante budget constraint, we obtain the following condition:

$$\begin{aligned} \pi(FT) u_A(\lambda \bar{Y}_A, \lambda \bar{Y}_B) \left(\lambda \bar{Y}_A - (Y_A - a_A^*) + \rho^{FT} (\lambda \bar{Y}_B - (Y_B + a_B)) \right) \\ = \pi(W) u_A(Y_A, Y_B) (\rho^W a_B - a_A^*). \end{aligned} \quad (\text{EBC}')$$

These two equations correspond to equations (RS) and (EBC) in the text.

Equations (RS') and (EBC') jointly characterize an equilibrium λ and ρ^W . From a pair (λ, ρ^W) ,

²³As mentioned in the text, in this case there are two assets and two states: $\rho^{FT} \neq \rho^W$ implies that the assets' payoffs are linearly independent.

we can obtain the implied ex-post portfolio $(\hat{a}_A^*, \hat{a}_B)$:

$$\hat{a}_B = \frac{(\lambda \bar{Y}_A - Y_A) + \rho^{FT}(\lambda \bar{Y}_B - Y_B)}{\rho^{FT} - \rho^W} \quad \text{and} \quad \hat{a}_A^* = \rho^W \hat{a}_B. \quad (30)$$

Such pairs, together with the implied portfolio positions, can be used to construct full-trade-war equilibria.

Define λ_0 to be Home's consumption share in the free trade state if it has a zero ex-post portfolio, and define λ^{FT} to be Home's consumption share in the free trade state if the ex-post portfolio equals the ex-ante portfolio:

$$\lambda_0 \equiv \frac{Y_A + \rho^{FT} Y_B}{\bar{Y}_A + \rho^{FT} \bar{Y}_B}, \quad \lambda^{FT} \equiv \frac{Y_A - a_A^* + \rho^{FT}(Y_B + a_B)}{\bar{Y}_A + \rho^{FT} \bar{Y}_B}.$$

Define ρ_R to be the level of ρ^W that satisfies the risk sharing condition (RS') given a value of λ :

$$\rho_R(\lambda) \equiv \rho_{aut}^* \frac{u_A(Y_A^*, Y_B^*)}{u_A(Y_A, Y_B)} \frac{u_A(\lambda \bar{Y}_A, \lambda \bar{Y}_B)}{u_A((1-\lambda)\bar{Y}_A, (1-\lambda)\bar{Y}_B)},$$

Then, we have the following result:

Proposition C.1. *Suppose the Assumptions 1 and 2 hold with $a_B \geq 0$, and $\pi(W) \in (0, 1)$. Assume also that $\lim_{\lambda \downarrow 0} u_A(\lambda \bar{Y}_A, \lambda \bar{Y}_B) = \infty$. If*

$$\rho_R(\lambda_0) \leq \rho^{FT} \leq \rho_R(\lambda^{FT}), \quad (31)$$

then there is a finite tariff pair $(\bar{\tau}, \bar{\tau}^)$ such that, for every $\tau \geq \bar{\tau}$ and $\tau^* \geq \bar{\tau}^*$, an ex-ante full-trade-war equilibrium exists in which the war state is autarky and*

$$\hat{a}_A^* > 0, \quad \hat{a}_B > 0.$$

Proof. Let

$$H(\lambda) \equiv \pi(FT)u_A(\lambda \bar{Y}_A, \lambda \bar{Y}_B)(\bar{Y}_A + \rho^{FT} \bar{Y}_B)(\lambda - \lambda^{FT}) - \pi(W)u_A(Y_A, Y_B)(\rho_R(\lambda)a_B - a_A^*).$$

where we have substituted (RS') into (EBC') using the definition of ρ_R and λ^{FT} . Note that ρ_R is continuous and strictly decreasing. The limit condition in the proposition guarantees that $\lim_{\lambda \rightarrow 0} H(\lambda) = -\infty$.

Now consider $H(\lambda_0)$, in this case we have

$$H(\lambda_0) = \underbrace{\pi(FT)u_A(\lambda_0\bar{Y}_A, \lambda_0\bar{Y}_B)(\lambda_0 - \lambda^{FT})(\bar{Y}_A + \rho^{FT}\bar{Y}_B)}_{>0 \text{ given } \lambda_0 > \lambda^{FT}} - \pi(W)u_A(Y_A, Y_B)(\rho_R(\lambda_0)a_B - a_A^*).$$

Note also that

$$\rho_R(\lambda_0)a_B - a_A^* \leq \rho^{FT}a_B - a_A^* < 0,$$

by condition (31), the net-debtor assumption, and $a_B \geq 0$. And thus $H(\lambda_0) > 0$. Continuity of H then implies that there is a $\hat{\lambda} \in (0, \lambda_0)$ such that $H(\hat{\lambda}) = 0$.

Now, we can rewrite (30)

$$\hat{a}_B = \frac{(\bar{Y}_A + \rho^{FT}\bar{Y}_B)(\lambda_0 - \hat{\lambda})}{\rho_R(\hat{\lambda}) - \rho^{FT}},$$

and for $\hat{a}_B$ to be strictly positive we need $\rho_R(\hat{\lambda}) > \rho^{FT}$.

If $\hat{\lambda} < \lambda^{FT}$, then $\rho_R(\hat{\lambda}) > \rho_R(\lambda^{FT}) \geq \rho^{FT}$ where the first inequality follows from ρ_R being strictly decreasing and the second one from condition (31).

If $\hat{\lambda} > \lambda^{FT}$, it follows that

$$H(\hat{\lambda}) = \pi(FT)u_A(\hat{\lambda}\bar{Y}_A, \hat{\lambda}\bar{Y}_B)(\bar{Y}_A + \rho^{FT}\bar{Y}_B) \underbrace{(\hat{\lambda} - \lambda^{FT})}_{>0} - \pi(W)u_A(Y_A, Y_B)(\rho_R(\hat{\lambda})a_B - a_A^*) = 0,$$

and thus $\rho_R(\hat{\lambda})a_B - a_A^* > 0$. This requires that $a_B > 0$, and thus

$$\rho_R(\hat{\lambda}) > \frac{a_A^*}{a_B} > \rho^{FT},$$

where the last inequality follows by the net-debtor assumption.

The final case is one where $\hat{\lambda} = \lambda^{FT}$. In this case, $-H(\hat{\lambda}) = \pi(W)u_A(Y_A, Y_B)(\rho_R(\lambda^{FT})a_B - a_A^*) = 0$, which necessarily requires $\rho_R(\lambda^{FT}) > \rho^{FT}$ and $a_B > 0$ given the net debtor assumption.

With these, we now have found $(\hat{\lambda}, \hat{\rho}^W)$ where $\hat{\rho}^W = \rho_R(\hat{\lambda})$ such that (RS') and (EBC') hold, and the asset positions generated by (30) are both strictly positive.

Using (29) we can then obtain the price q . Then (RS') makes this price consistent with the households' first order conditions, and the (EBC') guarantees that the ex-ante swap budget constraint holds. Concavity makes these conditions sufficient for the households' problems. And because the implied portfolios $\hat{a}_A^*, \hat{a}_B$ are strictly positive, Proposition A.1 supports autarky as an outcome in the trade-war state for sufficiently high tariffs. $\square$

Condition (31) specifies that at the inherited debt allocation, the risk sharing condition calls for higher terms of trade during the trade war relative to free trade; and at the allocation with zero net assets, risk-sharing calls for lower terms of trade relative to free trade. There is a case where condition (31) automatically holds. Suppose that endowments and preferences are symmetric:

$Y_A = Y_B^*$, $Y_B = Y_A^*$, and $u(c_A, c_B) = u(c_B, c_A)$. In this case, $\lambda_0 = 1/2$ and $\rho_R(\lambda_0) = \rho^{FT} = 1$. Home being a net-debtor guarantees $\lambda^{FT} < \lambda_0$, and given that ρ_R is decreasing, it follows that $\rho_R(\lambda^{FT}) > \rho_R(\lambda_0) = \rho^{FT}$. Thus condition (31) is satisfied.

Proposition C.1 shows that anticipation of a trade war does not necessarily lead to an unraveling of the gross-positions. Quite the contrary, it may even lead them to increase:

Corollary C.1. *Suppose that the conditions of Proposition C.1 hold, and consider the full-trade-war equilibrium constructed in the proof. It follows that (i) if $a_A^* > \rho_R(\lambda^{FT})a_B$, then $\hat{a}_A^* > a_A^*$, $\hat{a}_B > a_B$; (ii) if $a_A^* = \rho_R(\lambda^{FT})a_B$, then $\hat{a}_A^* = a_A^*$, $\hat{a}_B = a_B$; and (iii) if $a_A^* < \rho_R(\lambda^{FT})a_B$, then $\hat{a}_A^* < a_A^*$, $\hat{a}_B < a_B$.*

Proof. Let $\hat{\lambda}$ be the solution to $H(\hat{\lambda}) = 0$ constructed in the proof of Proposition C.1. We have that

$$\text{sign}(\hat{\lambda} - \lambda^{FT}) = \text{sign}(\rho_R(\hat{\lambda})a_B - a_A^*).$$

Using that ρ_R is strictly decreasing, it follows that, at a root of H ,

$$\text{sign}(\hat{\lambda} - \lambda^{FT}) = \text{sign}(\rho_R(\lambda^{FT})a_B - a_A^*).$$

Using (30) and (EBC'), we obtain that

$$(\rho_R(\hat{\lambda}) - \rho^{FT})(\hat{a}_B - a_B) = -(\bar{Y}_A + \rho^{FT}\bar{Y}_B)(\hat{\lambda} - \lambda^{FT}) \left[1 + \frac{\pi(FT)u_A(\hat{\lambda}\bar{Y}_A, \hat{\lambda}\bar{Y}_B)}{\pi(W)u_A(Y_A, Y_B)} \right].$$

From the proof of Proposition C.1, $\rho_R(\hat{\lambda}) > \rho^{FT}$. Thus

$$\text{sign}(\hat{a}_B - a_B) = \text{sign}(\lambda^{FT} - \hat{\lambda}).$$

The swap budget constraint gives:

$$(\hat{a}_A^* - a_A^*) = q(\hat{a}_B - a_B).$$

Given $q > 0$, the result in the corollary follows. □